# Velocity of Light

This experiment for the determination of the velocity of light uses the method of Foucault. The principle idea is beautifully simple. A light (laser) beam is reflected by a rotating mirror and travels to a fixed mirror which reflects the beam back to the rotating mirror (see Fig.1 below). If the distance between the rotating and the fixed mirror is $D$ then the time $T = \frac{2D}{c}$ passes between the first and the second reflecting from the rotating mirror. If the rotating mirror rotates with the angular frequency $\omega = 2\pi f$ then it changed its orientation by $\Delta\alpha = \omega T = 2\pi f T$. As the consequence the light beam, reflected by the rotating mirror, does not return after the second reflection along its original path but forms an angle of $2\Delta\alpha$ with the (original) incident beam. With a beam splitter one can extract the reflected beam and determine its position on a camera. If one denotes the position of the reflected beam for small rotational frequency ($f \rightarrow 0$) on the camera (i.e. the computer screen) as $(x_0, y_0)$ then one observes for finite frequency $f$ a a new position $(x, y)$ where $y \approx y_0$ and $\Delta x$ is the displacement. The computer software gives the position in units of pixel together with the conversion factor between pixel and micrometer.

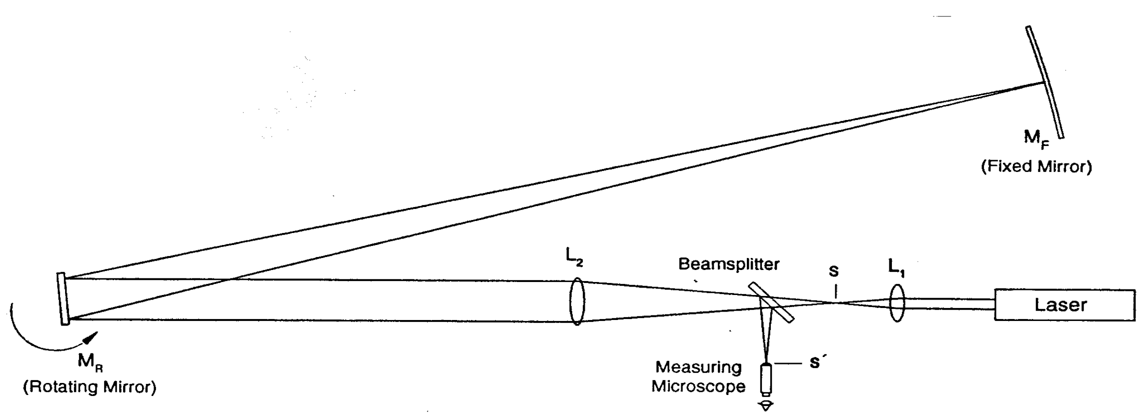


Fig.1: The principal setup of the experiment.

Before we calculate the angle $\alpha$ using the displacement, the optical setup has to be discussed in more detail. In the initial part of the optical path (before the rotating mirror) we have, going from right to left, first the laser. The laser beam passes a lens $L_1$ with a focal length of 50mm. Since the laser beam is essentially parallel this creates a point like image $\mathfrak{s}$ at in the focal plane of lens $L_1$. A second lens with a focal length of 250mm is next in the optical path. It position is chosen in such a way that it produces an image $\mathfrak{S}$ of $\mathfrak{s}$ on the fixed mirror $M_F$ (after the reflecting by the rotating mirror). Since the fixed mirror has a spherical surface with a radius of 13.5 m (which is about its optical distance from the rotating mirror) it reflects the incident beam back to the rotating mirror (after the proper alignment), independent of the position where the incident beam meets the fixed mirror. When the reflected beam returns to the rotating mirror the latter has advanced by the small angle $\Delta\alpha$. Therefore when the beam leaves the rotating mirror its direction is no longer the reverse of the original laser beam but forms an angle of $2\Delta\alpha$ with the original beam. This beam now passes through the lens $L_2$ and forms therefore an image $\mathfrak{s}_1$ in plane of the image $\mathfrak{s}$. If the rotating mirror rotates with $f = 0$ (i.e. is at rest) the images $\mathfrak{s}$ and $\mathfrak{s}_1$ coincide. It would be difficult to measure this displacement in the original beam. Therefore one uses a beam splitter to project the returning beam into a perpendicular direction. In the new image plane we place the photo-sensitive layer of a camera and record the image digital. The software permits to measure the position of the beam on the camera in units of pixels and gives the conversion factor between pixels and micrometer.

To calculate the relation between the displacement $\Delta x$ and (twice) the rotation angle $2\Delta\alpha$ it is convenient to remove the complication of the rotating mirror and the beam splitter by looking at the virtual images of the beam paths (see Fig.2).

- In Fig.2 the apparent position $\mathfrak{S}$ of the image is shown for $f = 0$ on the (virtual image of the) fixed mirror $M_2$.
- If the rotating mirror rotates with the frequency $f > 0$ then the virtual image $\mathfrak{S}_1$ is displaced by $\Delta S = (S_1 - S)$ where $\Delta S = 2\alpha D$ (D is the optical distance between rotating mirror $M_R$ and fixed mirror $M_F$.
- although the images $\mathfrak{S}$ and $\mathfrak{S}_1$ are created at different frequencies of the rotating mirror we treat them now at the two ends of a single image. The lens $L_2$ creates an image $\Delta s$ of $\Delta S$ in the plane of $\mathfrak{s}$. Now we have the relation between the object size $\Delta S = (S_1 - S)$ and image size $\Delta s = (s_1 - s)$

$$\frac{\Delta s}{A} = \frac{\Delta S}{D + A}$$

- Inserting the different relations we obtain

$$\begin{aligned} c &= \frac{2D}{T} = 4\pi\frac{D}{\alpha}f = 8\pi\frac{D^2}{\Delta S}f \\ &= \frac{8\pi D^2 A}{\Delta s\,(D + A)}f \end{aligned}$$

  where $\Delta s = \Delta x$.

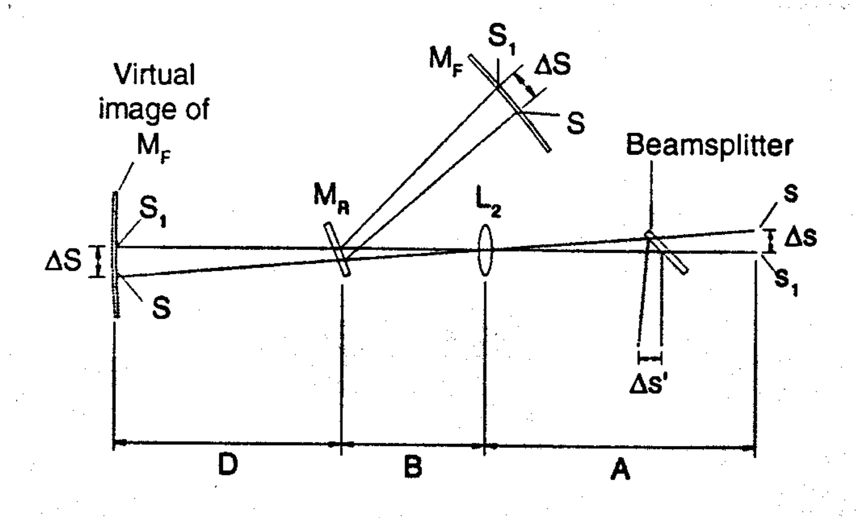


Fig.2: Definition of the different lengths and analysis of the displacement.