1 The Operational Amplifier

In Fig.1 the physical form of an OpAmp is shown together with a schematic sketch. This one has eight legs or pins and it is an LF411. It consists of a silicon chip with 24 transistors, 11 resistors and 1 capacitor.

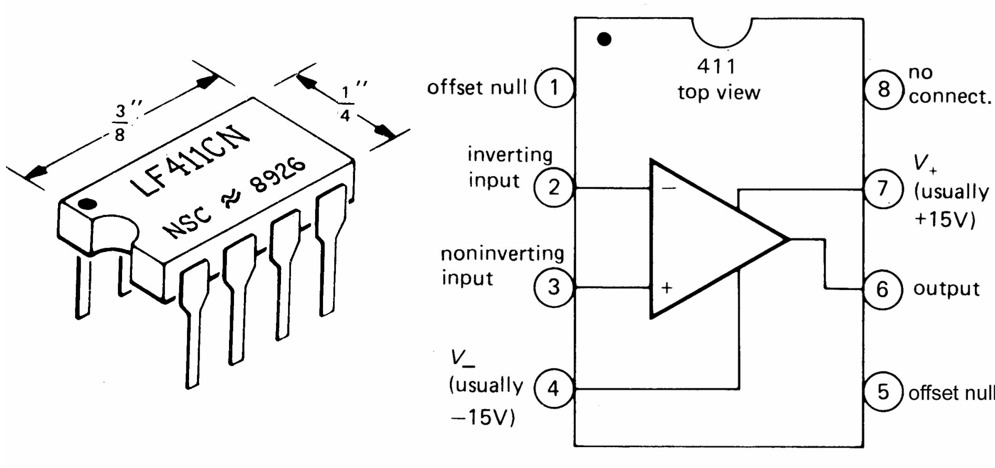


Fig.1: The operation amplifier LF411 in its physical form and its schematic sketch.

The important pins for now are no (4) and (7) which supply the positive and negative supply voltage (we will use +9V and -9V), no (2) the inverting input, no(3) the non-inverting input and no (6) the output. If the OpAmp is part of a circuit one generally draws only the two inputs (2,3) and the output (6). The main function of the OpAmp is to amplify the voltage difference between (2) and (3) and give the amplified voltage to the output (6). If the voltage at (2) is larger than the one at (3) then the output is negative, otherwise it is positive (therefore the - and + signs at the OpAmp).

For the larger part of this course we treat the OpAmp as idealized. That means:

- the amplification is infinite
- the two inputs (2) and (3) don't draw current, i.e. the input resistance is infinite (for the 411 it is $10^{12}\Omega$)
- the output resistance is zero.

We use in the first part of this course μA741 or LF356 OpAmps. For the integrator we use LF356 OpAmps

1.0.1 Inverting amplifier

The OpAmp is almost always used with (negative) feed-back. In Fig.2a a circuit with feedback is drawn, the inverted amplifier. The (+)-input B is connected to ground. The output C is connected via a resistor R_2 to the inverting input A (which is why the feedback is negative). This has the consequence that the point A is essentially at the same potential as the point B. Any difference between the potential of A and B would yield an infinite large output voltage at C. Therefore A is called a virtual ground. The analysis of the output is very simple under our simplifying assumptions. Since the potential at A is zero and since we have a bridge between "in" and "out" we have

$$\frac{R_1}{V_{in} - 0} = \frac{R_2}{0 - V_{out}}$$

$$V_{out} = -\frac{R_2}{R_1} V_{in}$$

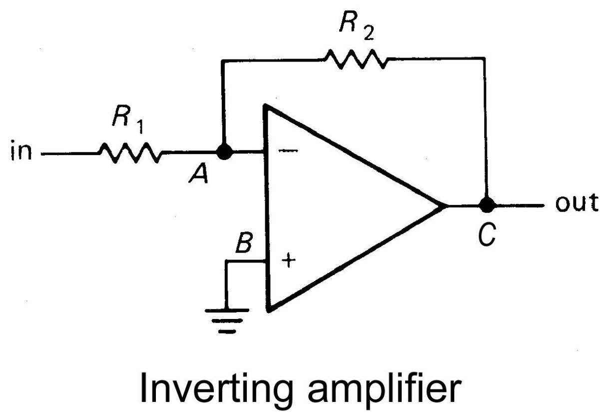


Fig.2a: Inverting amplifier, low input impedance (μA741)

This means that the output voltage is amplified by the factor R_2/R_1 and inverted. How does it work in detail? Let us assume that the (-)-input at A has a slightly positive potential v^+ with respect to ground. The the OpAmp amplifies the difference between A and B by the large factor F and inverts it because the inverting input deviates from ground. So the output voltage is $-Fv^+$. This voltage is coupled through R_2 with the input A and drives the voltage at A in the opposite direction of v^+ until it reaches zero (in case the amplification is infinite). So in case of a negative feedback (i.e. a connection between the output and the inverting input) one can assume that the potential at A is equal to the potential at B. Then one has a resistor bridge in which the inner point is at zero potential.

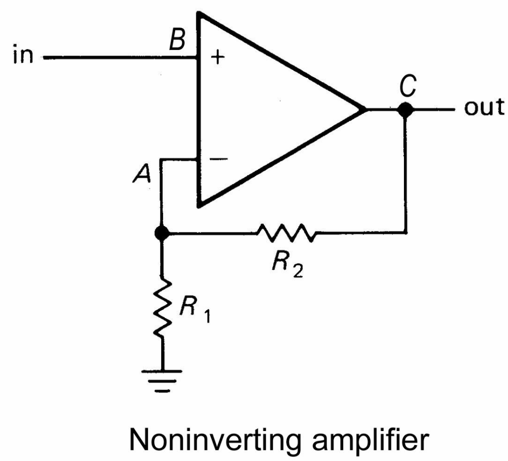


Fig.2b: Noninverting amplifier, large input impedance (μA741)

1.0.2 Noninverting amplifier

The inverting amplifier in Fig.2a has a low input impedance (a current can flow through R_1 and R_2 from input to output). This defect is removed in the inverting amplifier in Fig.2b where the input (+) is not externally connected with the output. In this case point A at (-) is at the same potential as in at B. Therefore we have a bridge with

$$\frac{V_{in} - 0}{R_1} = \frac{V_{out}}{R_1 + R_2}$$
$$V_{out} = \frac{R_1 + R_2}{R_1} V_{in}$$

The sign of the output voltage is the same as the input voltage.

1.0.3 An ac amplifier

If the signal source is ac-coupled one must provide a return to ground for the (very small) input current as shown in Fig. 3. The circuit is essentially the same as in Fig.2b. The difference is that the capacitor blocks the dc-current from reaching the OpAmp and the additional resistance of $100k\Omega$ takes care of the (very small input current into the OpAmp. The amplification is $(R_1 + R_2)/R_1 = (2.0 + 18.0)/2.0 = 10$.

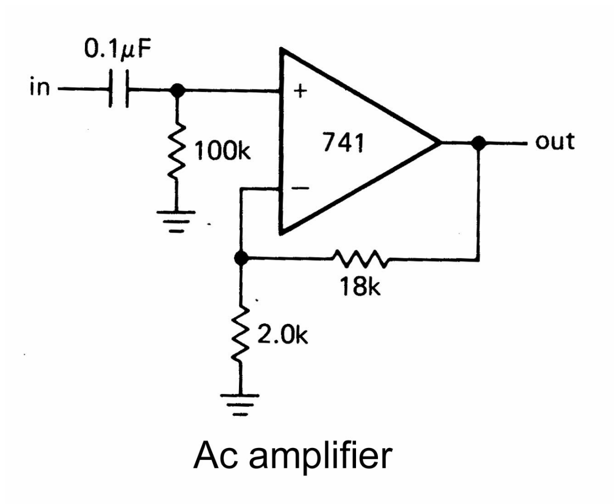


Fig.3: AC-amplifier (μA741)

The size of the capacitor determines the amplification as a function of frequency. Fig.4 is another example of an ac-amplifier.

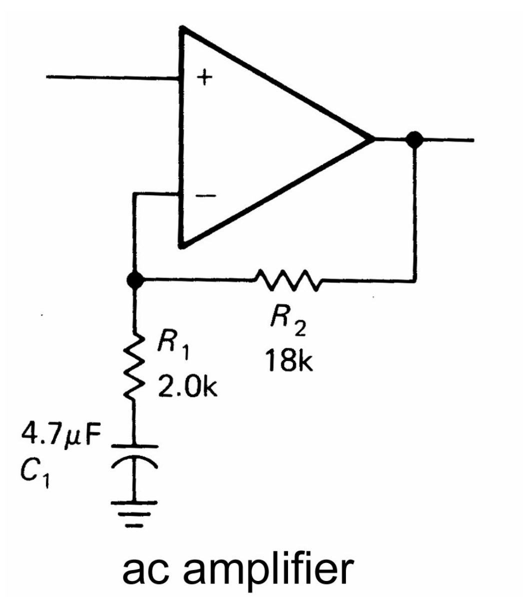


Fig.4: ac-amplifier (μA741)

1.0.4 Follower

If one connects output and (-)-input directly then the output is at the same potential as the input. It is called a voltage follower (see Fig.5). Its function is that the circuit has now a small output resistance. It corresponds to the noninverting amplifier in Fig.3 where R_1 is infinite and R_2 is zero (which yields the gain 1). It is sometimes called a buffer.

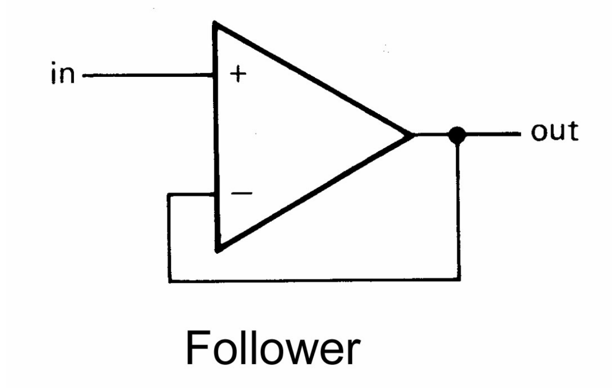


Fig.5: Follower (μA741)

1.0.5 Current source

In Fig.6 the of OpAmp as a current source is shown. The potential at (-) is identical to the one at (+), i.e. equal to V_{in}. Therefore the current through the resistor R is equal to V_{in}/R. This (constant) current is supplied by the output and flows through the load. The circuit has, however, the disadvantage that the potential of the load is floating (neither side is on ground).

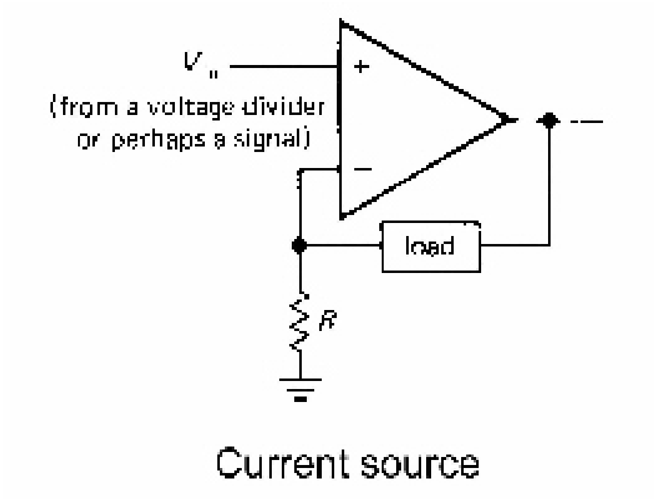
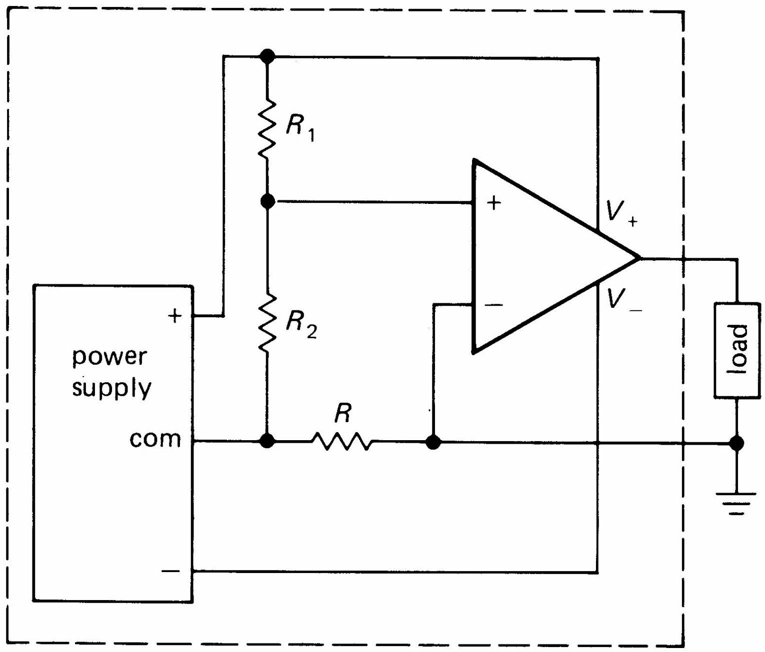


Fig.6: Current source (μA741)

Fig.7 shows a circuit where the load can be grounded. However, here the OpAmp is floating. We might later discuss other solutions for a current source.



Current source with grounded load and floating power supply.

Fig.7: Current source with grounded load and floating power supply (μA741)

1.0.6 Integrator

The integrator replaces one of the resistors in the inverting amplifier by a capacitor. According to Fig.8 the voltage at the capacitor is $U_2 = Q/C$ where $Q = \int I dt$ is the charge at the capacitor. To keep the input at (−) at zero we have

$$V_{in} = -RI$$
$$V_{out} = \frac{1}{C}\int I dt = \frac{1}{RC}\int V_{in} dt$$

Therefore the output is the integral of the input devided by the time constant RC.

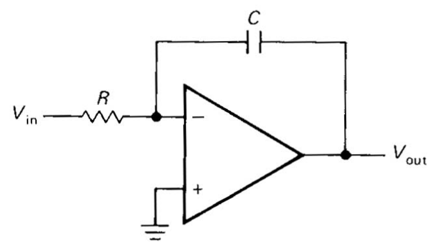


Fig.8: Integrator (LF356)

For the integrator the imperfection even of an OpAmp becomes visible. In practice the potentials at $(-)$ and $(+)$ are not exactly the same. This generates a small leakage current which charges the capacitor even if $V_{in} = 0$. In the second class we will discuss how to compensate this voltage. Presently we shunt a large resistor of $22M\Omega$ parallel to the capacitor so that the voltage at the capacitor does not grow until it is limited by the supply voltage.

A slightly different derivation of the integration uses the Fourier components of the voltages $V_i(\omega)$. The ratio of the votages V_1 and V_2 is

$$\frac{V_1(\omega)}{V_2(\omega)} = \frac{X_1}{X_2} = \frac{R}{\frac{1}{i\omega C}}$$

where X_i are the impedances of the elements. This yields

$$V_2(\omega) = \frac{1}{i\omega RC} V_1(\omega)$$
$$V_2(t) = \frac{1}{RC} \int V_1(t)\, dt$$

In our application we use $C = 2.2\mu F$ and $R = 10k\Omega$. This yields a time constant of $\tau_{RC} = 2.2 \times 10^{-6} \frac{As}{V} * 10^4 \frac{V}{A} = 0.022\, s$. This means that the integrator has a rather short time constant. In our application we want to measure the magnetic field of a permanent magnet by inserting a pickup coil into the magnetic field and integrating the induced voltage

$$\int_0^T V(t)\, dt = -\int_0^T \frac{d\Phi}{dt} dt = \Phi(0) - \Phi(T)$$

where $\Phi = nFB$, n =number of turns and F=area of pickup coil and B=magnetic field of the permanent magnet. This measurement requires a time in the range of $10 - 30$ seconds. This is a long time for such an integrator during which the imperfections of the OpAmp shows. In a real OpAmp one has (i) a voltage offset and (ii) an input bias current.

The voltage offset means that the OpAmp does not exactly amply the difference of the voltages V_3, V_2 between (2) and (3), but instead $V_3 - V_2 + V_{os}$ where V_{os} is called the offset voltage. This means that even when (2) and (3) are shortened the OpAmp amplifies the finite voltage V_{os}.

A very small but finite input bias current I_B flows through the inverting and non-inverting input. This current is required to keep the OpAmp functioning.

Both, V_{os} and I_B vary from OpAmp to OpAmp and there are high end OpAmps which optimize V_{os} or I_B. For the integrator using the LF356 the combination of V_{os} and I_B yields a quick charging of the capacitor and the output voltage reaches within 10 seconds the maximum value of about 7-8V even for a shortened input.

For the measurement of the magnetic field we use therefore an external compensation using a voltage divider as shown below.

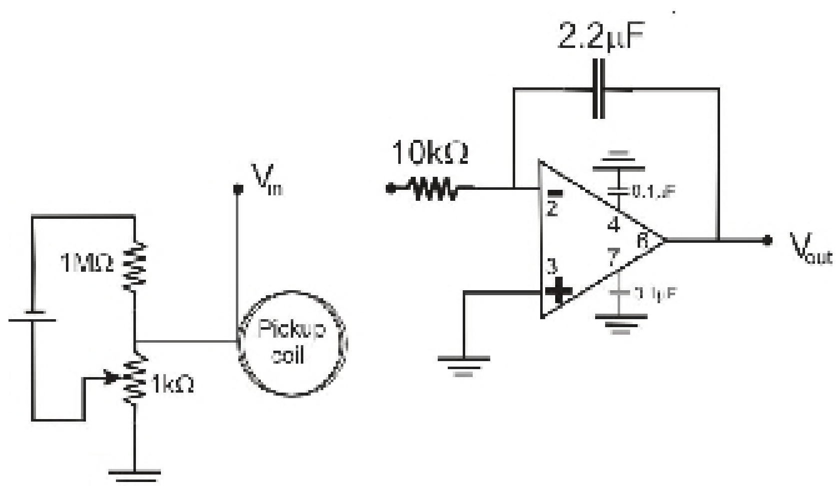


Fig.8b: Compensating input circuit plus integrator.

The potentiometer in the voltage divider is tuned to that the output voltage is zero when the magnetic field through the pickup coil is constant. Furthermore the battery voltage inputs for V$_+$ at (7) and V_- at (4) each are connected with ground through a $0.1\mu F$ capacitors. The resulting circuit permits a relatively good measurement of the integrated induced voltage.

1.0.7 Differentiator

For the differentiator one could replace the feedback resistance by a solenoid. It is, however, easier, to replace the input resistor by a capacitor. This yields

$$V_{in} = \frac{1}{C}\int I dt => \frac{dV_{in}}{dt} = \frac{1}{C} I$$
$$V_{out} = RI = RC\frac{dV_{in}}{dt}$$

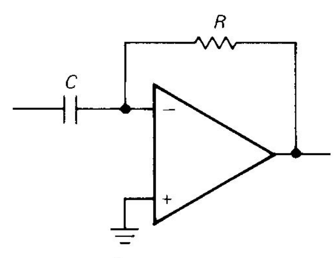


Fig.9: Differentiator (μA741)

The differentiator becomes unstable at high frequencies. Therefore one often restricts the frequency range.

1.0.8 Oscillators

Rectangular The ciruit in Fig.10 is more complicated. It generates rectangular waves. As one can see from Fig.10 there is no input voltage. Both the (+) and (−) input are connected with ground, the (+) input by a capacitor C and the $(-)$ input by a resistor of $10k\Omega$.

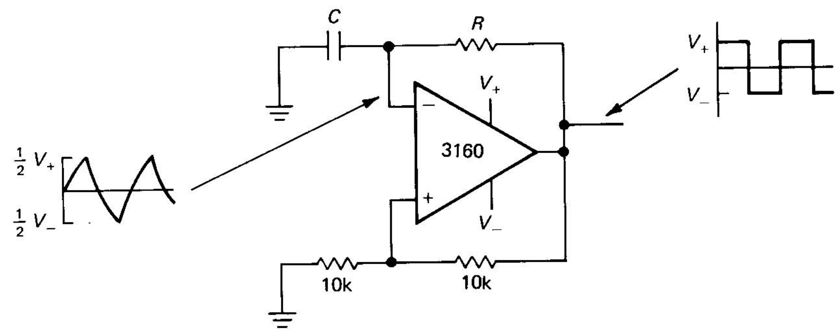


Figure 4.22
Op-amp relaxation oscillator.

Fig.10: Rectangular wave oscillator (μA741)

This is a circuit in which the potential at the $(+)$ and $(-)$ inputs cannot be kept at the same value. At the top part of the circuit we denote the voltage at the capacitor as V_1 and the voltage at the resistor as V_2, and at the bottom the voltage at the first resistor of $10k\Omega$ as V_1' and at the second one as V_2'. Then we have

$$V_1 = \frac{1}{C}\int I dt \qquad V_2 = RI$$

$$V_1' = I'R_{10} \qquad V_2' = I'R_{10}$$

The lower circuit keeps potential at the $(+), (-)$ inputs at half the output voltage. If the potential a $(+)$ and $(-)$ would be the same, i.e.$V_1 = V_1'$ then it would follow that $V_1 = V_2$ and

$$\frac{1}{C}\int I dt = RI$$

$$C\frac{dV_1}{dt} = \frac{1}{R}V_2 = \frac{1}{R}V_1$$

The output voltage increases exponentially and reaches soon the maximum value given by supply voltage.

So let us consider instead the first moment after the current in the upper part start to flow. Then we have $\frac{1}{C}\int I dt < RI$. That means that in the lower part of the circuit the $V_1' = V_2'$ while in the upper circuit $V_1 < V_2$.

This means that $V_1 < V_1'$ and as a consequence the circuit is non-linear and the output voltage reaches the maximum voltage $+V_+$ and V_1' is half the maximum value V_+. As a function of time the voltage V_1 at the capacitor grows because it integrates the current I.When the value of V_1 passes the value $V_1' = V_+/2$ then the difference between $(+)$ and $(-)$ changes sign and almost instantly the output voltage flips from largest positive V_+ to the largest negative value $-|V_-|$. Now the current reverses sign. The voltage V_1' takes the value $-|V_-|/2$ while the voltage V_1 changes from $+V_+/2$ to $-|V_-|/2$. When it passes this value of $-|V_-|/2$ then the output changes from $-|V_-|$ to $+V_+$ and the cycles starts all over. This means that the voltage at $(-)$ cycles continously between $+V_+/2$ and $-|V_-|/2$ while the voltage at the output is either at $+V_+$ or $-|V_-|$. The period is determined by the time that the capacitor needs to change its voltage between $+V_+/2$ and $-|V_-|/2$.

To determine the period we consider the cycle which start with $V_1 = -|V_-|/2$, V$_2 = -|V_-|$ and ends with $V_1 = +V_+/2$,V$_2 = +|V_+|$. The condition is

$$\begin{aligned}
V_1(t) + V_2(t) &= \frac{1}{C}\int I(t)\,dt + I(t)\,R = V_+ \quad = const \\
V_1(0) &= -|V_-|/2
\end{aligned}$$

The current $I(t)$ follows the time dependence $I(t) = I_0 e^{-t/\tau}$. The charge at the capacitor at the time $t = 0$ is $Q(0) = CV_1(0) = -C|V_-|/2$.

$$\begin{aligned}
-\frac{1}{2}|V_-| + \frac{1}{C}\int_0^t I_0 e^{-t'/\tau}dt' + I_0 e^{-t/\tau}R &= V_+ \\
\frac{\tau}{C}I_0\left(1 - e^{-t/\tau}\right) + I_0 e^{-t/\tau}R &= V_+ + \frac{1}{2}|V_-| \\
I_0 - I_0 e^{-t/\tau} + \frac{RC}{\tau}I_0 e^{-t/\tau} &= \frac{C(V_+ + |V_-|/2)}{\tau} \\
\left(\frac{CR}{\tau} - 1\right)e^{-\dfrac{t}{\tau}}I_0 &= \frac{C(V_+ + |V_-|/2)}{\tau} - I_0
\end{aligned}$$

This yields $\tau = RC$ and $I_0 = \dfrac{C(V_+ + |V_-|/2)}{\tau} = \dfrac{(V_+ + |V_-|/2)}{R}$ so that

$I = \dfrac{(V_+ + |V_-|/2)}{R} e^{-t/RC}$. Therefore

$$\begin{aligned} Q(t) &= Q(0) + \int_0^t I(t')\, dt' \\ &= -C\,|V_-|/2 + C\,(V_+ + |V_-|/2)\left(1 - e^{-t/RC}\right) \end{aligned}$$

and the voltage $V_1(t)$ is

$$V_1(t) = \frac{Q(t)}{C} = -|V_-|/2 + (V_+ + |V_-|/2)\left(1 - e^{-t/RC}\right)$$

At the time t_1 the voltage reaches the value $V_+/2$.

$$\begin{aligned} -|V_-|/2 + (V_+ + |V_-|/2)\left(1 - e^{-t_1/RC}\right) &= V_+/2 \\ \left(1 - e^{-t_1/RC}\right) &= \frac{(V_+ + |V_-|)}{(2V_+ + |V_-|)} \\ e^{-t_1/RC} &= \frac{(2V_+ + |V_-|) - (V_+ + |V_-|)}{(2V_+ + |V_-|)} \\ t_1 &= -RC \ln \frac{V_+}{(2V_+ + |V_-|)} \end{aligned}$$

Similarly one obtains t_2 for the other half cycle

$$t_2 = -RC \ln\left(\frac{|V_-|}{2\,|V_-| + V_+}\right)$$

and the period is

$$T = t_1 + t_2$$

If the two supply voltage are equal one obtains

$$T = -2RC \ln\left(\frac{1}{3}\right) = 2.197\,RC$$

Sine wave oscillator Fig.11 shows the circuit of an oscillator which generates an ac signal proportional to $\sin(\omega t)$. In this oscillator the potential of the (+) and (−) input are identical. We consider the impedance of an oscillation with the frequency ω. The impedences of the lower circuit are $X_1' = \left(\dfrac{1}{R} + i\omega C\right)^{-1}$ and $X_2 = \left(R + \dfrac{1}{i\omega C}\right)$. The impedances of the upper circuit are $X_1 = R_{adj}$ and $X_2 = R_{.75}$. The resistance of the lamp decreases with increasing current and adjusts to satisfy the stationary condition.

In the linear range one obtains

$$\frac{R_{adj}}{\left(\frac{1}{R} + i\omega C\right)^{-1}} = \frac{R_{.75}}{\left(R + \frac{1}{i\omega C}\right)}$$

$$\left(R + \frac{1}{i\omega C}\right)\left(\frac{1}{R} + i\omega C\right) = \frac{R_{.75}}{R_{adj}}$$

$$2 + i\left(CR\omega - \frac{1}{CR\omega}\right) = \frac{R_{.75}}{R_{adj}}$$

For the two $(+), (-)$ inputs to be in phase the imaginary term on the left must be zero or

$$CR\omega = \frac{1}{CR\omega}$$

$$\omega = \frac{1}{RC}$$

In that case the resistance adjusts to

$$R_{adj} = \frac{1}{2} R_{.75}$$

The filament of the lamp is a fairly pure metallic wire and its resistance increases with rising temperature (a factor 10 increase between room temperature and normal operating temperature of about about 2800K for a lamp). It is this resistance change which the rms current adjusts in such a way that the resistance of the lamp is equal to $\frac{1}{2}R_{.75}$ and stabilizes the oscillation.. It is important for a harmonic oscillation that the frequency is sufficiently high so that the resistance of the lamp is essentially constant during a cycle.

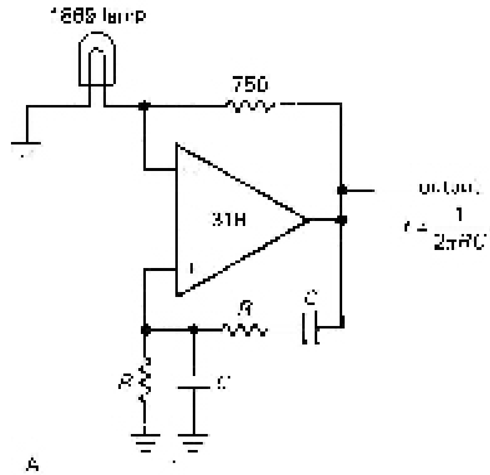


Fig.11: Sine wave oscillator

1.0.9 Finite amplification

Now we consider the large but finite amplification F of the OpAmp. As an example we take the inverting amplifier in Fig.2a. The voltage at the point **A** is now not exactly equal to the voltage at point **B**, $V_A \neq V_B = 0$. Instead we have

$$V_{out} = F\,(V_B - V_A) = -FV_A$$
$$V_A = -\frac{1}{F}V_{out}$$

Furthermore the current I, flowing from the input via $R_1$and R_2 to the output, yields the potential differences

$$V_{in} - V_A = IR_1$$
$$V_A - V_{out} = IR_2$$

This yields

$$\frac{V_A - V_{out}}{R_2} = \frac{V_{in} - V_A}{R_1}$$
$$V_A - V_{out} = \frac{R_2}{R_1}V_{in} - \frac{R_2}{R_1}V_A$$

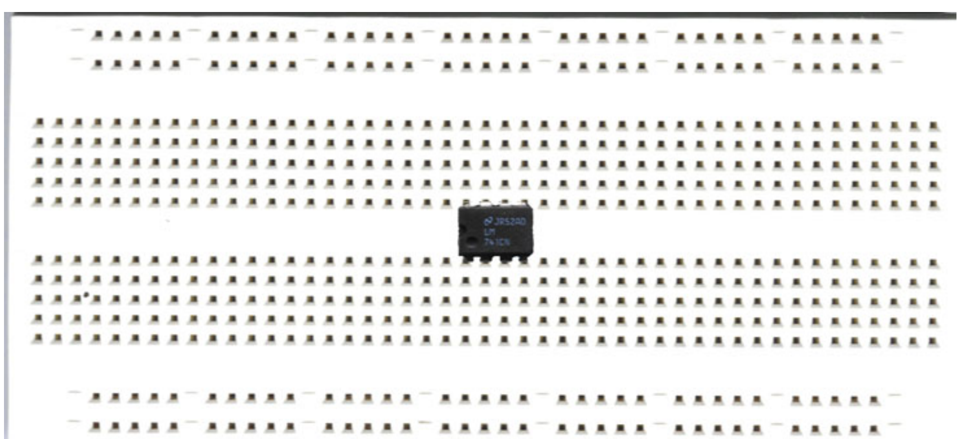
Setting $r = R_2/R_1$ we find

$$\begin{aligned} V_A - V_{out} + rV_A &= rV_{in} \\ -\frac{1}{F}V_{out} - V_{out} + r\left(-\frac{1}{F}V_{out}\right) &= rV_{in} \\ -V_{out}\frac{F+r+1}{F} &= rV_{in} \\ V_{out} &= -\frac{r}{1+\frac{r+1}{F}}V_{in} \end{aligned}$$

So when we have $F \approx 10^6$ then $r = R_2/R_1$ can still have a value of 10^3 and the denominator is 0.999, differing only by 10^{-3} from the value one.

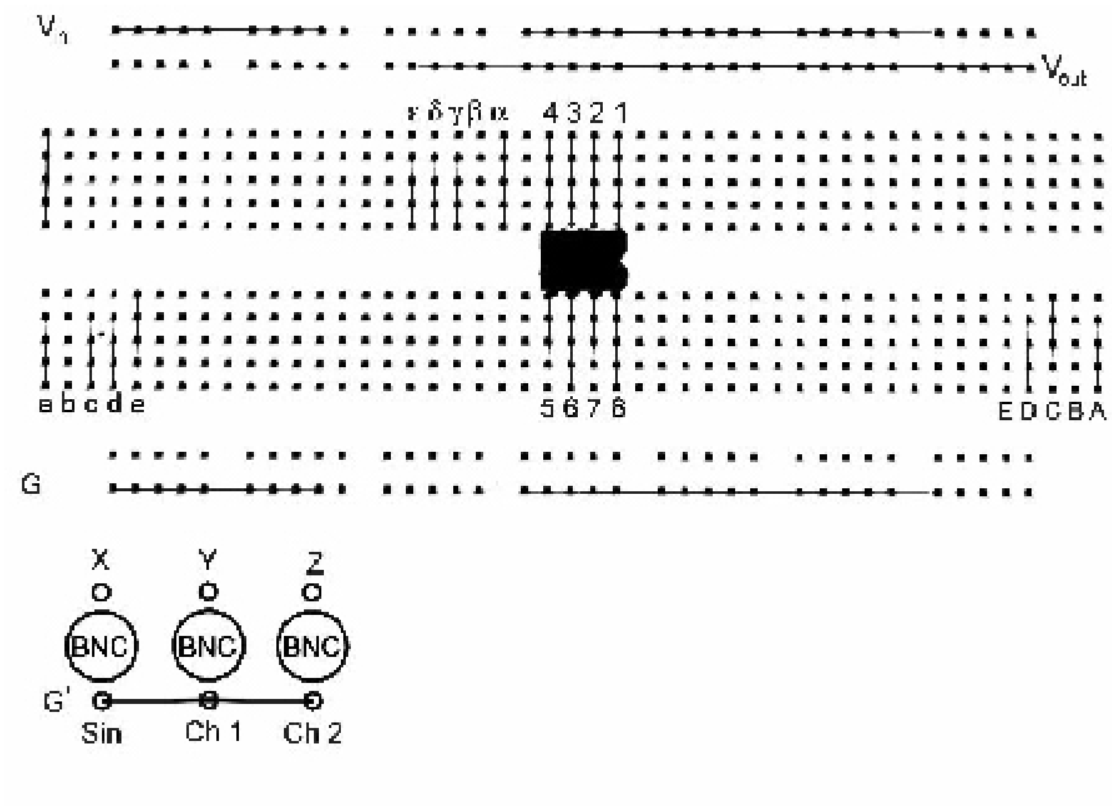
1.0.10 Assembling a circuit

In Fig.12a "bread board" is shown. Each dark point is a contact in which a lead can be inserted. In the center an operational amplifier with 8 electrode is inserted, the half circle mark pointing to the right, which means that the leads 1,2,3,4 are at the top (from right to left) and the leads 5,6,7,8 at the bottom (from left to right). These leads are vertically connected to the contact above (and below respectively). In Fig.12b a partly disassembled bread board is pictured. The dark strips are the electrical connections. They run in the central part vertically while the two top and bottom arrays of contacts are horizontally connected.



Here we describe the first preparation of a circuit, choosing the inverted amplifier. After the OpAmp is inserted in the center, defining electrodes 1 to 8, we proceed in the following way.

- Define the bottom horizontal contact array as ground.
- Define the top horizontal contacts array as input.
- Define the second horizontal contacts array as output.
- Take one battery and connect the positive pole with (7) and the negative pole with ground (yielding $V_+ = 9V$).
- Take the second battery and connect the negative pole with (4) and the positive pole with ground (yielding $V_- = -9V$).
- Use resistor R_1 to connect input with (2).
- Use resistor R_2 to connect (2) with (6).
- Connect (6) with output.
- Insert a red input wire into input and a black input wire into ground.
- Insert a red output wire into output and a black output wire into ground.



V1
Vout
ε δ γ β α
4 3 2 1
a b c d e
5 6 7 8
E D C B A
G
X
Y
Z
BNC
BNC
BNC
G'
Sin
Ch 1
Ch 2