GRAVITATIONAL DRESSING OF RENORMALIZATION GROUP β-FUNCTIONS

H. DORN
Institut für Physik, Humboldt-Universität, Invalidenstr. 110
D-10115 Berlin, Germany
E-mail: dorn@ifh.de

The coupling of 2D conformal field theories to quantized 2D gravity (gravitational dressing) is well understood, at least for central charges $c \leq 1$ or $c \geq 25$. Only recently there has been some progress in the general discussion of gravitational dressing of the larger class of renormalizable 2D field theories. One considers a 2D theory described by the action

$$S \;=\; S_c \;+\; \sum_i \lambda_i \int V_i \; d^2 z \;, \tag{1}$$

where S_c is the action of a conformal field theory with central charge c, V_i a set of marginal operators with respect to S_c which is closed under renormalization and λ_i dimensionless couplings. Then it has been shown [1,2,3,4] that the gravitational dressed RG β-functions $\bar{\beta}_i(\lambda)$ in *lowest order* are related to the original β-functions $\beta_i(\lambda)$ corresponding to the action S by the universal formula

$$\bar{\beta}_i(\lambda) \;=\; \frac{2}{\alpha Q}\, \beta_i(\lambda) \;. \tag{2}$$

α and Q are fixed by the central charge c of the unperturbed theory S_c

$$Q \;=\; ((25-c)/3)^{1/2}, \qquad \alpha(Q-\alpha) \;=\; 2 \;. \tag{3}$$

In its generalization to the case of an infinite number of couplings i.e. to generalized σ-models the dressing problem addresses the question: What critical (d+1)-dimensional string is the gravitational dressed version of what non-critical d-dimensional string?

Looking at a theory with a RG flow between two fixed points it is evident from the c-theorem that the simple result (2) cannot be valid for all higher orders. In our paper [5] we extend the $\beta \leftrightarrow \bar{\beta}$ relation to the nextleading order of perturbation theory. Based on a background-quantum split for the Liouville field, which describes the gravitational degree of freedom in conformal gauge, we construct the gravitational dressed action

$$\tilde{S} \;=\; S_c \;+\; S_L \;+\; \sum_i \lambda_i \int \tilde{V}_i \, \sqrt{\hat{g}}\; d^2 z \tag{4}$$

by the requirement of background independence. Here $S_L[\phi|\hat{g_{ab}}]$ is the Liouville action and for $\tilde{V}_i$ the ansatz

$$\tilde{V}_i(z) \;=\; e^{\delta_i \phi(z)}\; V_i(z) \tag{5}$$

is made. By this procedure the coefficients δ_i are fixed as functions of the renormalized couplings.

After this step we consider the response of the theory to a change of the cutoff in geodesic length. Out of this one can construct the gravitational dressed β-functions $\bar{\beta}$. In the simplest case of one coupling we find for $\beta(\lambda) = \beta^{(2)}\lambda^2 + \beta^{(3)}\lambda^3 + \ldots$ and an analogous expansion for $\bar{\beta}(\lambda)$

$$\bar{\beta}^{(2)} \;=\; \frac{2}{\alpha Q}\,\beta^{(2)} \qquad \text{and} \qquad \bar{\beta}^{(3)} \;=\; \frac{2}{\alpha Q}\left(\beta^{(3)} \;-\; \frac{(\beta^{(2)})^2}{Q^2}\right) \quad . \tag{6}$$

Based on the interpretation of the Liouville field as an additional target space coordinate and the identification of its constant part ϕ_0 with the logarithm of the RG scale via $\phi_0 = \frac{2}{\alpha}\log\mu$ [6,7] and refs. therein, for generalized σ-models there has been derived [6] a second order differential equation for the flow of couplings. Reduced to the one coupling case it looks like ($\cdot$ denotes differentiation with respect to ϕ_0)

$$\ddot{\lambda} \;+\; Q\,\dot{\lambda} \;=\; \beta(\lambda)\;. \tag{7}$$

Taking into account the standard suppression of one Liouville dressing exponent, i.e. one solution of the quadratic equation in (2), one finds for a first order zero of β that $\dot{\lambda}$ becomes a function of λ. We supplement eq. (7) by the requirement of a unique relation between $\dot{\lambda}$ and λ in the neighborhood of the origin also for second and higher order zeros of β. Then we are allowed to make a power series ansatz $\dot{\lambda} = \frac{\alpha}{2}\,\bar{\beta}(\lambda)$. Inserting it into (7) one reproduces (6) up to a factor 2 in front of $\frac{(\beta^{(2)})^2}{Q^2}$. This minor discrepancy raises the question of scheme dependence for the construction of $\bar{\beta}$. A source for scheme dependence of $\bar{\beta}^{(3)}$ can be found also inside our approach [5] by giving up minimal subtraction for the construction of the function $\delta(\lambda)$. Astonishingly, along this line of arguments we find scheme dependence of $\bar{\beta}^{(3)}$ although the original $\beta^{(3)}$ in the one coupling case is scheme independent.

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