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States in the BRST cohomology for G/H WZNW models. *

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WZNW models, especially gauged WZNW models, are important in the study of conformal field theories. Karabali and Schnitzer[1] initiated the study of the BRST cohomology of a WZNW model gauged by an anomaly free vector sub-group and results were given for abelian sub-groups. This result was generalized to non-abelian sub-groups for a specific set of representations [2]. The subject of this talk is the analysis of arbitrary representations [3,4].

The effective action of a G/H gauged WZNW model decomposes into three sectors; the original WZNW model based on the group G, an auxiliary WZNW model based on the sub-group H, and a Fadeev-Popov ghost part. We assume that the state space decomposes into a direct product of modules $M_\lambda^h \otimes \widetilde{M}_{\tilde{\lambda}}^h \otimes \mathcal{F}$ where M_λ^h and $\widetilde{M}_{\tilde{\lambda}}^h$ are Verma modules over affine Lie algebras $\hat{h}_k$ and $\hat{h}_{\tilde{k}}$ with levels k and $\tilde{k} = -k - 2c_h$. c_h is the quadratic Casimir in the adjoint representation of h, and λ and $\tilde{\lambda}$ are weights of the two sectors. $\mathcal{F}$ is a ghost Fock module.

Our analysis is based on the result that for an irreducible Verma module, the BRST cohomology is confined to ghost free states [2], while in the reducible case we have for negative ghost numbers the equation $\hat{Q}|S\rangle = |N\rangle$ for states $|S\rangle$ in the cohomology [3,4]. Here $|N\rangle$ is a null-vector of the Verma module. The Verma module at hand is reducible for generic weights due to prescence of null-vectors.

In order to analyse a reducible state space we perturb the weights of the Verma modules by an infinitesimal piece ϵ [5] which gives an irreducible state space. This will induce a gradation of the Verma modules $M_\lambda^h = M^{(0)} \supset M^{(1)} \supset \ldots$ where $M^{(i)}$ is the submodule which includes all states which are divisible by ϵ^i in the perturbed module. We define the irreducible sub-module $L_\lambda^h \equiv M^{(0)}/M^{(1)}$. The states in $M^{(1)}/M^{(2)}$ are denoted first generation null-vectors.

Using the perturbed Verma modules the relative cohomology is found to be trivial for non zero ghost number, and only singlets and quartets will be present [3,4]. Here we use the terminology of Kugo and Ojima [6], where singlets and singlet pairs represent non-trivial states in the cohomology, while quartets are sets of four states that will not belong to the cohomology. Now, if singlet pairs are generated as the perturbation is set to zero, then they must appear from quartets, and one must therefore identify suitable quartets. For negative ghost numbers one finds that $\hat{Q}|\bar{S}_{-q}\rangle = |\bar{N}_{-q+1}\rangle$

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and for positive ghost numbers $\hat{Q}|N_{q-1}\rangle_\epsilon = \epsilon|S_q\rangle_\epsilon$ [3,4]. Here $\hat{Q}$ is the relative BRST operator, $|\bar{S}_{-q}\rangle$ and $lim_{\epsilon\to 0}|S_q\rangle_\epsilon$ is the singlet pair, and $|\bar{N}_{-q+1}\rangle$, $lim_{\epsilon\to 0}|N_{q-1}\rangle_\epsilon$ are first generation null vectors. Subscript ϵ indicates that those states are perturbed as described above. Furthermore $|\bar{N}_{-q+1}\rangle$ and $lim_{\epsilon\to 0}|N_{q-1}\rangle_\epsilon$ are states in the cohomology in the irreducible sub-module of $M^{(1)}$. Starting at ghost number zero we may now iteratively solve the equations $\hat{Q}|\bar{S}_{-q}\rangle = |\bar{N}_{-q+1}\rangle$ and $\hat{Q}|N_{q-1}\rangle_\epsilon = \epsilon|S_q\rangle_\epsilon$ using explicit expressions for singular vectors [7].

For ghost number plus and minus one the null-states $|\bar{N}_0\rangle$ and $lim_{\epsilon\to 0}|N_0\rangle_\epsilon$ are constructed from known states in the cohomology at ghost number zero [2,4] by substituting highest weight primaries by highest weight null-vectors. One now solves $\hat{Q}|\bar{S}_{-1}\rangle = |\bar{N}_0\rangle$ and $\hat{Q}|N_0\rangle_\epsilon = \epsilon|S_1\rangle_\epsilon$ to obtain the singlet pair $|\bar{S}_{-1}\rangle$ and $lim_{\epsilon\to 0}|S_1\rangle_\epsilon$. States in the cohomology at ghost number plus and minus one are now used to get $|\bar{N}_{-1}\rangle$ and $lim_{\epsilon\to 0}|N_1\rangle_\epsilon$ in the same manner as above etc.

It is important to note that for positive ghost numbers it is sufficient to use either the original or the auxiliary sector, while for negative ghost numbers both sectors are required. Also, exchanging the sectors for positive ghost numbers yields cohomologically equivalent results [4].

This construction yields singlet pairs in the irreducible sub-module $L^h_\lambda \otimes \tilde{L}^h_{\tilde{\lambda}}$ with ghost numbers given by $p = \pm|l_\lambda - l_{\tilde{\lambda}}|$ where l_λ is the minimum number of Weyl reflexions taking us to λ starting from a dominant weight μ. $l_{\tilde{\lambda}}$ is defined in an analogous manner.

This construction remains completely valid for topological models as well as for integrable representations of the original sector. For the generic G/H model the assumption that the Verma module M^g_Λ decomposes into Verma modules M^h_λ does not always hold and some states may not stay in the cohomology. In fact, all singlet pairs for which the construction involves null-vectors of the affine Lie algebra $\hat{h}_k$ that are not null-vectors of the full affine Lie algebra $\hat{g}_k$, will not be in the cohomology. Since the construction is iterative this may in fact mean that infinitely many ghost numbers vanish.

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