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New Spacetime-Supersymmetric Methods for the Superstring

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In this talk, the new spacetime-supersymmetric description of the superstring is reviewed and some of its applications are described. These applications include the manifestly spacetime-supersymmetric calculation of scattering amplitudes, the construction of a super-Poincaré invariant open superstring field theory, and the beta-function calculation of low-energy equations of motion in superspace. Parts of this work have been done in collaboration with deBoer, van Nieuwenhuizen, Roček, Sezgin, Skenderis, Stelle, and especially, Siegel and Vafa.

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1. Introduction

Ever since the discovery of spacetime-supersymmetry in the superstring, physicists have looked for a description of the superstring where this symmetry is manifest. Just as manifest spacetime-supersymmetry simplifies calculations in supersymmetric particle theories by reducing the number of Feynmann diagrams, a manifestly spacetime-supersymmetric description of the superstring removes the need to sum over spin structures, thereby simplifying the study of multiloop scattering amplitudes. Furthermore, such a description makes it easier to analyze superstring properties which depend crucially on the presence of spacetime-supersymmetry, such as duality and finiteness.

With the exception of the work described here, the only quantizable description of the superstring with manifest spacetime-supersymmetry is the light-cone Green-Schwarz description.[1][2] This description requires light-cone gauge fixing of all superstring fields, which breaks manifest SO$(d-1,1)$ Lorentz invariance down to SO$(d-2)$. When calculating scattering amplitudes using this formalism, non-trivial interaction-point operators need to be inserted whereever strings join or split. Since the locations of these interaction points are complicated functions of the P^+ momenta of the external strings, only four-point tree and one-loop scattering amplitudes were explicitly calculated using this description (although [2]contains explicit expressions for multiloop four-point amplitudes, these expressions contain unphysical divergences when interaction-points collide).

A similar problem exists for amplitude calculations using the light-cone Ramond-Neveu-Schwarz description of the superstring, however it can be overcome by using the N=1 superconformal invariance of the underlying covariant RNS description to remove the dependence on the interaction-point locations.[3] Unfortunately, a similar procedure is not possible using the fermionic Siegel invariances of the standard covariant GS description. However in 1989, Sorokin, Tkach, Volkov, and Zheltukhin discovered that by introducing twistor-like variables into the covariant GS action, the fermionic Siegel invariances could be converted into superconformal invariances.[4] Although the number of classical superconformal invariances for the d-dimensional GS superstring is equal to $d-2$,[5] all but two of the invariances can be gauge-fixed without ghosts or global moduli. The remaining invariances form a quantum N=2 superconformal algebra with $c=3(d-6)/2$, and can be covariantly gauge-fixed with the usual N=2 ghosts when $d=10$.[6]

In a flat ten-dimensional background, the non-covariant gauge-fixing of six of the eight invariances breaks SO(9,1) Lorentz invariance down to SU(4)×U(1) (this subgroup can be slightly enlarged to a 25-dimensional subgroup which preserves pure projective spinors[7]). But for generic compactifications to four dimensions which preserve supersymmetry, these extra six invariances are not present so all of the N=1 SO(3,1) super-Poincaré invariance can be made manifest.[8] For these compactifications, the $c = 6$ N=2 superconformal generators split into a $c = -3$ four-dimensional spacetime piece, which is independent of the compactification, and a $c = 9$ piece, which is related in the usual way to a six-dimensional Calabi-Yau manifold. Also for compactifications to six dimensions, manifest SO(5,1) super-Poincaré invariance can be preserved and the generators split into a $c = 0$ six-dimensional spacetime piece and a $c = 6$ Calabi-Yau piece.[9]

It is natural to ask what is the relationship between this new N=2 GS description of the superstring and the N=1 RNS description. With a suitable field redefinition from the N=2 GS matter fields to the N=1 RNS matter and ghost fields, the N=2 superconformal generators can be expressed as the stress-tensor, the b ghost, the BRST current, and the ghost current of the RNS superstring.[10] Using the results of reference [11] for critically embedding an N=1 string into an N=2 string, this shows the equivalence of scattering amplitudes using the two different descriptions.

In the second section of this paper, the new N=2 description of the superstring will be reviewed for the case of four-dimensional compactifications. In the third section, vertex operators will be constructed and it will be shown how to calculate manifestly SO(3,1) super-Poincaré invariant scattering amplitudes. Certain "topological" multiloop scattering amplitudes are extremely easy to calculate using these methods.[9]In the fourth section, a new super-Poincaré invariant open superstring field theory is constructed which does not suffer from the infinite contact terms of RNS superstring field theory.[12] The new superstring field theory action resembles a WZW action, which can also be used as a string field theory action for four-dimensional self-dual Yang-Mills. In the conclusion, future applications for the new description of the superstring are discussed, which include coupling to a curved supergravity/super-Yang-Mills background and using quantum N=2 superconformal invariance to determine the low-energy equations of motion in superspace.[13]

2. The N=2 GS superstring for compactifications to four dimensions

The worldsheet variables for the four-dimensional part of the N=2 superstring consist of the spacetime variables, x^m ($m = 0$ to 3), the right-moving fermionic variables, θ^α and $\bar\theta^{\dot\alpha}$ ($\alpha, \dot\alpha = 1$ to 2), the conjugate right-moving fermionic variables, p_α and $\bar p_{\dot\alpha}$, and one right-moving bosonic variable, ρ. The chiral boson ρ is identified with $\rho \pm 2\pi$ and is related to R-transformations of four-dimensional superspace. For the heterotic superstring, one has the usual 32 left-moving fermionic variables, χ_I, while for the Type II superstring, one has the left-moving fermionic variables, $\theta^{*\alpha}$,$\bar\theta^{*\dot\alpha}$, p^*_α, $\bar p^*_{\dot\alpha}$, and one left-moving bosonic variable, ρ^*.

In conformal gauge, the worldsheet action for the heterotic superstring variables is:

$$\int d^2 z [\frac{1}{2}\partial_{\bar z} x^m \partial_z x_m + p_\alpha \partial_{\bar z}\theta^\alpha + \bar p_{\dot\alpha}\partial_{\bar z}\bar\theta^{\dot\alpha} - \frac{1}{2}\partial_{\bar z}\rho\partial_z\rho + \frac{1}{2}\chi_I\partial_{\bar z}\chi_I]. \tag{2.1}$$

The free-field OPE's for these worldsheet variables are

$$x^m(y)x^n(z) \to -\eta^{mn}\log|y-z|, \quad \rho(y)\rho(z)\to\log(y-z), \tag{2.2}$$

$$p_\alpha(y)\theta^\beta(z)\to\frac{\delta_\alpha^\beta}{y-z}, \quad \bar p_{\dot\alpha}(y)\bar\theta^{\dot\beta}(z)\to\frac{\delta_{\dot\alpha}^{\dot\beta}}{y-z}, \quad \chi_I(y)\chi_J(z)\to\frac{\delta_{IJ}}{\bar y-\bar z}.$$

Note that the chiral boson ρ can not be fermionized since $e^{i\rho(y)}\ e^{i\rho(z)}\ \to e^{2i\rho(z)}(y-z)^{-1}$ while $e^{i\rho(y)}\ e^{-i\rho(z)}\ \to (y-z)$. It has the same behavior as the negative-energy field ϕ that appears when bosonizing the RNS ghosts $\gamma=\eta e^{i\phi}$ and $\beta=\partial\xi e^{-i\phi}$[14].

These worldsheet GS variables form a representation of an $N=2$ superconformal algebra with $c=-3$. The generators of this algebra are given by:

$$L_{d=4} = -\frac{1}{2}\partial_z x^m\partial_z x_m - p_\alpha\partial_z\theta^\alpha - \bar p_{\dot\alpha}\partial_z\bar\theta^{\dot\alpha} + \frac{1}{2}\partial_z\rho\partial_z\rho \tag{2.3}$$

$$G^+_{d=4} = e^{i\rho}(d)^2, \quad G^-_{d=4} = e^{-i\rho}(\bar d)^2, \quad J_{d=4} = -i\partial_z\rho,$$

where

$$d_\alpha = p_\alpha + i\bar\theta^{\dot\alpha}\partial_z x_{\alpha\dot\alpha} + \frac{1}{2}(\bar\theta)^2\partial_z\theta_\alpha - \frac{1}{4}\theta_\alpha\partial_z(\bar\theta)^2,$$

$$\bar d_{\dot\alpha} = \bar p_{\dot\alpha} + i\theta^\alpha\partial_z x_{\alpha\dot\alpha} + \frac{1}{2}(\theta)^2\partial_z\bar\theta_{\dot\alpha} - \frac{1}{4}\bar\theta_{\dot\alpha}\partial_z(\theta)^2,$$

and $(d)^2$ means $\epsilon^{\alpha\beta}d_\alpha d_\beta$. It is straghtforward to check [15] that d_α and $d^*_{\dot\alpha}$ anticommute with the $d=4$ spacetime supersymmetries which are generated by

$$q_\alpha = \int dz[p_\alpha - i\bar\theta^{\dot\alpha}\partial_z x_{\alpha\dot\alpha} - \frac{1}{4}(\bar\theta)^2\partial_z\theta_\alpha], \tag{2.4}$$

$$\bar q_{\dot\alpha} = \int dz^-[\bar p_{\dot\alpha} - i\theta^\alpha\partial_z x_{\alpha\dot\alpha} - \frac{1}{4}(\theta)^2\partial_z\bar\theta_{\dot\alpha}],$$

and satisfy the OPE's

$$d_\alpha(y)V(x,\theta,\bar\theta)(z)\to\frac{\nabla_\alpha V(z)}{y-z},\quad d^\alpha(y)\bar d^{\dot\alpha}(z)\to\frac{2i\Pi_z^{\alpha\dot\alpha}}{y-z}, \tag{2.5}$$

where V is an arbitrary spacetime superfield, $\nabla_\alpha=\frac{\partial}{\partial\theta^\alpha}+i\bar\theta^{\dot\alpha}\frac{\partial}{\partial x^{\alpha\dot\alpha}}$, and $\Pi_z^{\alpha\dot\alpha}=\partial_z x^{\alpha\dot\alpha}-i\theta^\alpha\partial_z\bar\theta^{\dot\alpha}-i\bar\theta^{\dot\alpha}\partial_z\theta^\alpha$.

Since the Calabi-Yau manifold is descibed by an N=2 superconformal field theory with $c=9$, the combined system of the four-dimensional GS superstring and the Calabi-Yau manifold is described by an N=2 superconformal field theory with $c=6$. The generators of the corresponding N=2 algebra are given by:

$$L=L_{d=4}+\ L_{CY},\quad G^-=G^-_{d=4}+\ G^-_{CY}, \tag{2.6}$$

$$G^+=G^+_{d=4}\ +G^+_{CY},\quad J=J_{d=4}\ +J_{CY},$$

where $[L_{CY},G^-_{CY},G^+_{CY},J_{CY}]$ are the $(N=2,c=9)$ generators describing the Calabi-Yau manifold and $[L_{d=4},G^-_{d=4},G^+_{d=4},J_{d=4}]$ are the $(N=2,c=-3)$ generators defined in (2.3). Note that integral Calabi-Yau charge is required for all physical vertex operators since $J=0$ implies that Calabi-Yau charge is equal to ρ charge, which must be integral in order to avoid branch cuts with $G^\pm_{d=4}$.

As discussed in [10], there is a field transformation from N=2 GS matter fields into N=1 RNS matter and ghost fields which preserves all OPE's, maps G^- into the b ghost, and maps G^+ into the RNS BRST current. This transformation also maps q_α of (2.4)into RNS spacetime-supersymmetry generators in the $-\frac{1}{2}$ picture, and $\bar q_{\dot\alpha}$ into RNS spacetime-supersymmetry generators in the $+\frac{1}{2}$ picture. Since the generator of R-transformations,

$$R=\int dz(i\partial_z\rho+\frac{1}{2}(p_\alpha\theta^\alpha-\bar p_{\dot\alpha}\bar\theta^{\dot\alpha})) \tag{2.7}$$

is mapped into the RNS picture operator, GS R-weight is equal to RNS picture.

3. Scattering Amplitudes

All physical states of the superstring are represented by N=2 primary fields, V, which are constructed entirely out of matter fields and are dimension zero (for the heterotic string, the left-moving part of V is dimension one). In other words, L and $G^\pm$ have only $(y-z)^{-1}$ singularities with V while J with V has no singularities. The integrated form of the vertex operators is given by $\int d^2z|G^-G^+|^2V$ where $G^\pm V$ means the residue of the single pole in the OPE of $G^\pm$ and V. Under the transformation mapping GS matter fields into RNS matter and ghost fields, V is mapped into ξW where W is an RNS BRST-invariant vertex operator whose picture is equal to its R-weight (i.e., vector bosons are in picture 0, chiral fermions are in picture $-\frac{1}{2}$, and anti-chiral fermions are in picture $+\frac{1}{2}$).

For four-dimensional massless superfields in the heterotic string, the vertex operators take the simple form $V=\partial_{\bar z}x^m E_m(x,\theta,\bar\theta)$ and $V=(\chi^J f^I_{JK}\chi^K)V_I(x,\theta,\bar\theta)$ where E_m is the prepotential for the supergravity/axion multiplets and V_I is the prepotential for the Yang-Mills multiplets (e.g., the graviton and axion fields are represented by $h_{mn}+b_{mn}=\sigma_n^{\alpha\dot\alpha}\nabla_\alpha\bar\nabla_{\dot\alpha}E_m$ and the gauge field by $A_{Im}=\sigma_m^{\alpha\dot\alpha}\nabla_\alpha\bar\nabla_{\dot\alpha}V_I$ where $\nabla_\alpha=\partial_{\theta^\alpha}+i\bar\theta^{\dot\alpha}\partial_{\alpha\dot\alpha}$, $\bar\nabla_{\dot\alpha}=\partial_{\bar\theta^{\dot\alpha}}+i\theta^\alpha\partial_{\alpha\dot\alpha}$). The condition of being primary implies the on-shell conditions

$$\nabla^2 E_m=\bar\nabla^2 E_m=\eta^{mn}\partial_m E_n=\eta^{mn}\partial_m\partial_n E_p=\nabla^2 V_I=\bar\nabla^2 V_I=\eta^{mn}\partial_m\partial_n V=0. \quad (3.1)$$

The integrated form of the above vertex operators is obtained by hitting V with G^+ and G^- and, for massless superfields, takes the form:

$$U=\int d^2z\ (\bar d^{\dot\alpha}\ \nabla^2\bar\nabla_{\dot\alpha}+d^a\ \bar\nabla^2\nabla_\alpha \quad (3.2)$$

$$+\frac{1}{2}(\partial_z\bar\theta^{\dot\alpha}\ \bar\nabla_{\dot\alpha}+\partial_z\theta^\alpha\ \nabla_\alpha+\Pi^{\alpha\dot\alpha}\ (\nabla_\alpha\bar\nabla_{\dot\alpha}-\bar\nabla_{\dot\alpha}\nabla_\alpha)))^2V.$$

A similar structure exists for four-dimensional massless fields in the Type II string, which are represented by the N=(2,2) primary field $V=E(x,\theta,\bar\theta,\theta^*,\bar\theta^*)$ (e.g., the graviphoton field strength is given by

$$F_{mn}=\sigma_{mn}^{\alpha\beta}\bar\nabla^2\nabla_\alpha(\bar\nabla^*)^2\nabla^*_\beta E+\sigma_{mn}^{\dot\alpha\dot\beta}(\nabla^2)\bar\nabla_{\dot\alpha}(\nabla^*)^2\bar\nabla^*_{\dot\beta}E.$$

Note that in both the heterotic and Type II strings, the axion and graviton vertex operators are constructed from the same prepotential. This fact will be relevant when constructing non-linear sigma models for these strings in a curved background.

One way to calculate scattering amplitudes is to introduce N=2 ghosts, construct an N=2 BRST operator and picture-changing operators, and integrate correlation functions of BRST-invariant vertex operators on N=2 super-Riemann surfaces. However, a simpler way is to twist the N=2 string (which allows the central charge of the matter fields to cancel) and use N=4 topological techniques developed in [9]. The advantage of this topological method is that there is no need to introduce N=2 ghosts or to integrate over N=2 super-moduli. Since the N=2 GS matter system is equivalent to the N=1 RNS matter and ghost systems, it should not be surprising that there is no need to introduce a new set of ghosts.

The N=4 topological method involves twisting the $c=6$ stress-tensor L to $L+\frac{1}{2}J$, and defining two new bosonic and fermionic generators:

$$J^{++}=e^{\int^z J},\quad J^{--}=e^{-\int^z J},\quad \tilde{G}^{+}=G^{-}J^{++},\quad \tilde{G}^{-}=G^{+}J^{--} \tag{3.3}$$

where $G^{\mp}J^{\pm\pm}$ means the residue of the single pole in their OPE. Using the generators defined in (2.6),

$$J^{++}=e^{-i\rho+\int^z J_{CY}},\quad J^{--}=e^{i\rho-\int^z J_{CY}}, \tag{3.4}$$
$$\tilde{G}^{+}=e^{-2i\rho+\int^z J_{CY}}\bar{d}^2+G^{-}_{CY}e^{-i\rho+\int^z J_{CY}},\quad \tilde{G}^{-}=e^{2i\rho-\int^z J_{CY}}d^2+G^{+}_{CY}e^{i\rho-\int^z J_{CY}}.$$

It is easy to check that when $c=6$, these generators combine with the original N=2 generators of (2.6)to form a small N=4 algebra. Note that under the GS $\to$ RNS field transformation, $J^{++}=c\eta$, $J^{--}=b\xi$, $\tilde{G}^{+}=\eta$, and $\tilde{G}^{-}=bZ$ where Z is the N=1 picture-changing operator.

The condition that V is primary implies the equation of motion $\tilde{G}^{+}G^{+}V=0$ where multiplication by G^{+} or $\tilde{G}^{+}$ signifies taking its contour integral (after twisting, G^{+} and $\tilde{G}^{+}$ are spin-one). Gauge transformations of V are $\delta V=G^{+}\Lambda+\tilde{G}^{+}\bar{\Lambda}$, which leave $\tilde{G}^{+}G^{+}V$ invariant since G^{+} and $\tilde{G}^{+}$ anti-commute with each other. Since under the GS $\to$ RNS field transformation, $\tilde{G}^{+}V=W$ is the RNS vertex operator, $\tilde{G}^{+}=\eta$, and G^{+} is the RNS BRST operator, these conditions reproduce the standard cohomology of physical superstring states. Note however that there are no square-root cuts in the OPE's of (2.2), so there is no need to perform a GSO projection when using the GS variables.

The prescription for R-invariant tree-level scattering amplitudes is given by the N-point correlation function

$$A_0=<G^{+}V_1(z_1)\ \tilde{G}^{+}V_2(z_2)V_3(z_3)U_4...U_N>, \tag{3.5}$$

and for R-invariant g-loop amplitudes is given by

$$A_g=\prod_{j=1}^{3g-3}\int d^2m_j\prod_{i=1}^{g}\tilde{G}^{+}(v_i)\prod_{j=1}^{2g-2}(\mu_j\tilde{G}^{-})\prod_{j=2g-1}^{3g-2}(\mu_jG^{-})(\mu_{3g-3}J^{--})U_1...U_{2g}>, \tag{3.6}$$

where μ_j are the Beltrami differentials and $U_i=\int d^2zG^{-}G^{+}V$ (scattering amplitudes which violate R-invariance by an amount m correspond to calculations on N=2 surfaces of

U(1) instanton number m, and therefore require m G^+'s to be converted to $\tilde{G}^+$'s in the above formulas). It is easy to relate these amplitudes to the standard RNS prescription by using the transformation that maps the GS matter fields into the RNS matter and ghost fields. Under this transformation, (3.5)and (3.6)get mapped into

$$A_0 =< W_1(z_1)\ ZW_2(z_2)\ \xi W_3(z_3) U_4 ... U_N >,$$

$$A_g = \prod_{j=1}^{3g-3} \int d^2 m_j \prod_{i=1}^{g} < \prod_{i=1}^{g} \eta(v_i) \prod_{j=1}^{2g-2} (\mu_j b Z) \prod_{j=2g-1}^{3g-2} (\mu_j b)(\mu_{3g-3} b\xi) U_1 ... U_{2g} >,$$

where Z is the RNS picture-changing operator and $W_i = \xi V_i$ are the RNS vertex operators (for R-invariant amplitudes, the sum of the pictures of W_i is zero). Since this RNS calculation is in the "large" Hilbert space involving the zero mode of ξ, one has to insert g zero modes for η. As was shown in reference [16], the locations of these insertions are determined by restrictions on the ϕ loop-momentum, and the combined (ξ, η) and ϕ correlation functions reproduce the usual Verlinde prescription for the (β, γ) ghost correlation function.[17] Note that without these restrictions on the ϕ loop-momentum, the amplitude is divergent since ϕ is a negative energy field (i.e. its kinetic term appears with the wrong sign).

Since ρ is also a negative-energy field, similar restrictions must be imposed on its loop-momentum in (3.6). As in the RNS case, the locations of the $\tilde{G}^+$ insertions are determined by these restrictions on the ρ momentum, and work is in progress on using this fact to obtain explicit expressions for arbitrary multiloop GS amplitudes (note that (3.5)contains no such subtleties and therefore provides the first manifestly super-Poincaré invariant formula for tree-level superstring amplitudes).

For multiloop scattering amplitudes which depend in a trivial way on the ρ variable, these subtleties can be ignored and explicit expressions can be obtained. For example, for the scattering of $2g-2$ chiral graviphotons, these expressions reproduce the "topological" amplitudes which were originally obtained in a more complicated way using the RNS formalism.[18] Similar GS techniques have also been used in six dimensions to obtain explicit topological expressions for the scattering of $4g-4$ chiral graviphotons.

4. Open superstring field theory

As was discussed in section 3, the linearized equations of motion and gauge invariances for the N=2 vertex operators are

$$\tilde{G}^+ G^+ V = 0, \quad \delta V = G^+ \Lambda + \tilde{G}^+ \bar{\Lambda}, \tag{4.1}$$

rather than the usual $QW = 0$, $\delta W = Q\Lambda$. This suggests that their non-linear generalizations in superstring field theory may look different from the Chern-Simons generalizations $QW = W^2$, $\delta W = Q\Lambda - [W, \Lambda]$.[19] This is good news since all known Chern-Simons versions of superstring field theory require midpoint operator insertions which lead to either contact terms with infinite coefficients or unphysical solutions which need to be removed by hand.[20]

By comparing with the tree-level scattering amplitude of (3.5), one can show that the correct non-linear generalization of (4.1)for four-dimensional open string superfields,

$$V(x^m(\sigma), \theta^a(\sigma), \bar{\theta}^{\dot{\alpha}}(\sigma), p^\alpha(\sigma), \bar{p}^{\dot{\alpha}}(\sigma), \partial_\sigma \rho(\sigma)),$$

is

$$\tilde{G}^+(e^{-V} G^+ e^V) = 0, \quad \delta V = (G^+ \Lambda) e^V + e^V (\tilde{G}^+ \bar{\Lambda}), \tag{4.2}$$

where all string superfields (including those coming from a Taylor series expansion of the exponential) are multiplied using Witten's half-string overlap.[19]Since the contributing pieces from G^+ and $\tilde{G}^+$ are $e^{i\rho} d^2$ and $e^{-2i\rho + J_{CY}} \bar{d}^2$, the gauge invariances for the massless component of V (which only depends on x, θ, and $\bar{\theta}$) are those of the super-Yang-Mills prepotential,

$$\delta V = (D^2 \lambda) e^V + e^V (\bar{D}^2 \bar{\lambda}),$$

where $\Lambda = e^{-i\rho}\lambda$, $\bar{\Lambda} = e^{2i\rho + J_{CY}}\bar{\lambda}$, and $\lambda, \bar{\lambda}$ are unconstrained spacetime superfields. Note that all matter string superfields are required to be U(1) neutral, so the gauge parameters carry U(1) charge -1.

The action which yields (4.2)as the equation of motion resembles the two-dimensional Wess-Zumino-Witten action where the derivatives ∂_z and $\partial_{\bar{z}}$ are replaced by the operators G^+ and $\tilde{G}^+$. This action, which includes only the contribution of the four-dimensional superfields, is

$$Tr[\, (e^{-V} G^+ e^V)(e^{-V} \tilde{G}^+ e^V) - \int_0^1 dt (e^{-tV} \partial_t e^{tV}) \{ e^{-tV} G^+ e^{tV}, e^{-tV} \tilde{G}^+ e^{tV} \}] \tag{4.3}$$

where the trace over string states is defined as in Witten's open string field theory.[19]

With a slight modification, it is also possible to include the contribution of superfields which depend on the compactification manifold. The complete SO(3,1) super-Poincaré invariant superstring field theory action for arbitrary supersymmetric compactifications can be found in reference [12]. Although this action contains terms of all orders in the string field, it differs from the RNS action in that the coefficients of all such terms are explicit and finite.

Since the action in (4.3)only requires the existence of a $c=6$ N=2 superconformal field theory, it can also be used as a string field theory action for four-dimensional super-Yang-Mills. For the N=2 string which represents self-dual Yang-Mills, the worldsheet variables are x^i (i=1 to 2), $\bar{x}_i$, ψ^i, $\bar{\psi}_i$, and the relevant superconformal generators are $G^+ = \bar{\psi}_i \partial_z x^i$ and $\tilde{G}^+ = \epsilon^{ij}\bar{\psi}_i \partial_z \bar{x}_i$ (after twisting, ψ^i is spin-one and $\bar{\psi}_i$ is spin-zero). If $\Phi(x^i(\sigma), \bar{x}_i(\sigma), \psi^i(\sigma), \bar{\psi}_i(\sigma))$ is the string field, then $\tilde{G}^+(e^{-\Phi}G^+ e^{\Phi}) = 0$ implies that its massless mode ϕ (which only depends on x and $\bar{x}$) satisfies $\partial^i(e^{-\phi}\partial_i e^{\phi}) = 0$, which is the equation of motion for the Yang scalar of self-dual Yang-Mills.[21]

5. Future Applications

In this talk, a new description of the superstring was used to calculate manifestly spacetime-supersymmetric scattering amplitudes and to construct a super-Poincaré invariant superstring field theory action which does not suffer from the contact-term problems of the RNS action. There are various further applications for this new description.

One obvious application is to couple the worldsheet GS variables to a curved supergravity/super-Yang-Mills background, and to use beta-function techniques to obtain the low-energy superstring equations of motion in superspace.[22] This is done by generalizing the heterotic worldsheet action of (2.1)to:

$$\frac{1}{\alpha'}\int d^2 z [\frac{1}{2}\Pi_z^a \Pi_{\bar{z}\,a} + d_\alpha \Pi_z^\alpha + \bar{d}_{\dot{\alpha}} \Pi_z^{\dot{\alpha}} + B_{AB}\Pi_z^A \Pi_{\bar{z}}^B - \frac{1}{2}\partial_{\bar{z}}\rho \partial_z \rho + \frac{1}{2}\chi_I D_{\bar{z}} \chi_I \tag{5.1}$$

$$+\phi(r+if) + \bar{\phi}(r-if) + \psi(e^{i\rho}d^\alpha \nabla_\alpha \phi) + +\bar{\psi}(e^{-i\rho}\bar{d}^{\dot{\alpha}}\bar{\nabla}_{\dot{\alpha}}\bar{\phi})]$$

where A are flat-space indices, M are curved space indices, $\Pi_z^A = E_M^A \partial_z Z^M$, $\Pi_{\bar{z}}^A = E_M^A \partial_{\bar{z}} Z^M$, E_M^A is the supergravity vielbein, and B_{AB} is an anti-symmetric two-form whose field strength is $H_{ABC} = \nabla_{[A}B_{BC]}$. In a flat background where $B_\alpha^m = E_\alpha^m = i\sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}}$, it is straightforward to check that this action reduces to (2.1).

However, in order to be classically superconformally invariant, the field strength H_{ABC} must be set equal to the torsion T_{ABC} (this can be seen at the level of vertex operators from the fact that g_{mn} and b_{mn} are components of the same prepotential E_m). So B_{MN} is not an independent superfield and one still needs to find a coupling for the dilaton/axion multiplet.[23]

A similar situation arises in the bosonic string where the dilaton couples to the two-dimensional curvature, so it is natural to try to couple this dilaton/axion multiplet to the N=(2,0) supercurvature. This can be done in an N=(2,0) super-reparameterization invariant manner by adding the term

$$\int d^2z d\kappa R\Phi + \int d^2z d\bar{\kappa}\bar{R}\bar{\Phi} = \tag{5.2}$$

$$\int d^2z[\Phi(r+if) + \bar{\Phi}(r-if) + \psi(e^{i\rho}d^{\alpha}\nabla_{\alpha}\Phi) + +\bar{\psi}(e^{-i\rho}\bar{d}^{\dot{\alpha}}\bar{\nabla}_{\alpha}\bar{\Phi})]$$

where κ and $\bar{\kappa}$ are the worldsheet anti-commuting parameters, $R = \psi + (r+if)\kappa$ and $\bar{R} = \bar{\psi} + (r-if)\bar{\kappa}$ are the worldsheet chiral and anti-chiral superfields describing N=(2,0) supercurvature (r is the ordinary curvature, f is the U(1) field strength, and $\psi, \bar{\psi}$ are the gravitino field strengths), $\Phi(x,\theta)$ and $\bar{\Phi}(x,\bar{\theta})$ are spacetime chiral and anti-chiral superfields for the dilaton multiplet (the dilaton is $(\Phi+\bar{\Phi})|_{\theta=\bar{\theta}=0}$ and the axion is $i(\Phi-\bar{\Phi})|_{\theta=\bar{\theta}=0}$).[24]

The term $\int d^2z d\kappa R\Phi$ makes sense since G^- has no singularity with Φ, and therefore Φ is worldsheet chiral as well as spacetime chiral. Note that the dilaton zero modes couples to the Euler number of the surface and the axion zero mode couples to the U(1) instanton number. So just as the string coupling constant can be absorbed into the dilaton field, the string theta-parameter (which counts the violation of R-invariance in scattering amplitudes) can be absorbed into the axion field.

As in the bosonic string, the supercurvature term (5.2)is not classically superconformally invariant, but because it is higher order in α', its classical variation is expected to cancel the quantum variation of (5.1). Work is in progress on checking that the sum of (5.1)and (5.2)is superconformally invariant at the quantum level when the low-energy equations of motion are imposed on the supergravity/super-Yang-Mills background.[22]Note that the compactification-dependent fields have been frozen in this sigma model, so the only effect of the compactification is to increase the central charge by +9.

Other possible applications of this new description include the coupling of Type II strings in N=2 d=4 supergravity backgrounds[25], comparison of multiloop scattering amplitudes and effective actions for different six-dimensional superstrings[26], and the construction of a super-Poincaré invariant version of closed superstring field theory.

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