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S-Duality and H-Monopoles[a]

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It has been conjectured that the four dimensional heterotic string theory compactified on a torus is $SL(2,Z)$ invariant or "S-dual". The duality group, which acts on the axion, dilaton and the gauge fields, includes strong-weak coupling and electric-magnetic duality as a special case. A consequence of the conjecture is that the spectrum of BPS states, states that saturate a Bogomol'nyi bound and hence form short representations of the $N=4$ supersymmetry algebra, should be S-dual. It was pointed out by Sen[2] that this leads to specific predictions about the spectrum of magnetic monopoles some of which may be testable at weak coupling.

The four dimensional theory has an infinite tower of electrically charged BPS states in which the right movers are in their ground state. These states obey the constraint $N_L - 1 = (p_R^2 - p_L^2)/2$ and obey the mass shell condition $M = p_R^2/2$ where $(p_L, p_R) \in \Gamma_{22,6}$ with $\Gamma_{22,6}$ being an even self-dual lattice specifying the compactification (we consider generic points in the Narain moduli space $\mathcal{M}_N$ where the gauge group is $U(1)^{28}$). Acting with S-duality on these states we predict an infinite tower of magnetic monopoles and dyons in the spectrum. For $N_L = 0$ the predicted spectrum is the same as in $N=4$ super Yang-Mills theory and has been verified in the field theory limit. For $N_L > 1$ there is no region in $\mathcal{M}_N$ where the states are light and so we don't expect to find evidence for them in the field theory limit. For $N_L = 1$ the states can be light and so in particular we expect to find the magnetically charged S-duals in the field theory limit using semi-classical techniques. These are the H-monopoles, states that are magnetically charged with respect to $U(1)$'s coming from the dimensional reduction of the antisymmetric tensor field. Thus, it would seem that the spectrum of H-monopoles provides an important window into testing the S-duality conjecture in string theory over and above its validity in $N=4$ super-Yang-Mills theory.

More precisely, the S-duality conjecture predicts that there are 24 short $N=4$ multiplets of H-monopoles corresponding to the 24 ways of choosing $N_L = 1$. The classical H-monopole solutions are based on instantons on $R^3 \times S^1$ via the Bianchi identity $dH = \alpha' TrF \wedge F$. Since we are interested in the case when all the non-abelian fields are broken down to $U(1)$'s, we construct the solutions by picking an $SU(2)$ instanton with non-trivial holonomy around the S^1. The moduli space of

[a] This is based on work done in collaboration with J. Harvey; see Ref. (1) for further details and references.

H-monopoles, M, is then the moduli space of such instantons. Incorporating the fermion zero modes, one expects that the low-lying spectrum of H-monopoles is obtained by studying $N=4$ supersymmetric quantum mechanics on M.

Translation invariance implies that the moduli space is of the form $M = R^3 \times S^1 \times \tilde{M}$. The four collective coordinates parametrising $\tilde{M}$ come from the scale size of the instanton and from broken spatial rotations. From the supersymmetry we deduce that $\tilde{M}$ must be hyperKähler and from rotation invariance that it should have $SO(3)$ isometry. Finally $\tilde{M}$ should have a Z_2 orbifold singularity corresponding to instantons with zero scale size. From the analysis of Ref. (3) this restricts $\tilde{M}$ to be one of three cases[b]: 1) R^4/Z_2; 2) Taub-Nut space; 3) A one parameter family of metrics with additional curvature singularites (these singularities seem difficult to interpret and can possibly be used to rule out this case).

Quantising $N=4$ supersymmetric quantum mechanics on *any* of these candidates does not seem to lead to 24 short multiplets; what is required is the existence of 24 normalisable harmonic forms on $\tilde{M}$. In Ref. (1) we argued that if one treats collective coordinates by doing string theory instead of quantum mechanics on the moduli space and if the moduli space is R^4/Z_2 then we can see evidence for 24 states using orbifold techniques. However, there are severe infrared problems to deal with which makes the outcome of this proposal inconclusive.

In conclusion, despite naive expectations, it seems difficult to find convincing evidence for (or against) the S-duality conjecture by studying the spectrum of H-monopoles. We close with some comments. In our construction of the H-monopoles we worked near a point in $\mathcal{M}_N$ with enhanced gauge symmetry. At a generic point in $\mathcal{M}_N$ one might expect from scaling arguments that the generic H-monopoles always shrink to zero scale size. In this case a field theory analysis would not be sufficient to determine the spectrum. Note that we have not *rigorously* ruled this option out in our construction, although we think it is unlikely. It is also perhaps worth pointing out that the origin of the 24 states is intimately connected to the gauge group of the ten dimensional heterotic string being $E_8 \times E_8$ or $SO(32)$; maybe these groups should somehow enter into the analysis. If the S-duality conjecture is correct then it is clear that there is more to be understood about H-monopoles.

References

1. J. P. Gauntlett and J. A. Harvey, hep-th/9407111.
2. A. Sen, Int. J. Mod. Phys. **A9** (1994) 3707.
3. M. F. Atiyah and N. J. Hitchin, *The Geometry and Dynamics of Magnetic Monopoles* (Princeton University Press, Princeton, 1988), p. 70.

[b] This corrects a statement made in Ref. (1).