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Supertraces in String Theory*

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Abstract

We demonstrate that the spectrum of any consistent string theory in D dimensions must satisfy a number of supertrace constraints: $\mathrm{Str}\, M^{2n} = 0$ for $0 \leq n < D/2 - 1$, $n \in \mathbb{Z}$. Our results hold for a large class of string theories, including critical heterotic strings. For strings lacking spacetime supersymmetry, these supertrace constraints will be satisfied as a consequence of a hidden "misaligned supersymmetry" in the string spectrum.

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Supertraces are of interest in quantum field theories because they control the structure of the ultraviolet divergences appearing in various loop amplitudes. For example, in four-dimensional spacetime, the one-loop vacuum energy density (cosmological constant) contains quartic, quadratic and logarithmic divergences proportional to $\mathrm{Str}\, M^0$, $\mathrm{Str}\, M^2$, and $\mathrm{Str}\, M^4$, respectively. In this talk, supertraces are considered in the context of string theory, and it will be shown that at tree level, any consistent string theory satisfies $\mathrm{Str}\, M^{2n} = 0$ for $0 \leq n < D/2 - 1$, $n \in \mathbb{Z}$, where D is the number of spacetime dimensions. The details of this work may be found in Ref. [1].

In calculating supertraces for string spectra, one faces an immediate problem. In string theory, the physical spectrum contains an *infinite* number of states arranged in towers whose levels are integer-spaced (in Planck-scale units), and whose state degeneracies grow exponentially with mass. Thus, to properly define the string-theoretic supertraces over the whole string spectrum, we regulate the sum over states as follows:

$$\mathrm{Str}\, M^{2n} \;\equiv\; \lim_{\gamma\to 0} \left\{ \sum_{\substack{\text{physical}\\ \text{states}}} (-1)^F \, (M_i)^{2n} \, e^{-4\pi\gamma M_i^2} \right\} . \tag{1}$$

As is conventional, all masses are expressed in units of the Planck mass, and hence γ is a dimensionless quantity. (The factor of 4π is introduced in the exponential as a convenient normalization for what follows.)

In order to derive our supertrace constraints, we exploit the remarkable expression for the string-theoretic one-loop cosmological constant derived by Kutasov and Seiberg [2]:

$$\Lambda \;=\; \frac{\pi}{3} \lim_{\gamma\to 0} (\gamma)^{1-D/2} \sum_{\substack{\text{physical}\\ \text{states}}} (-1)^F \, e^{-4\pi\gamma M_i^2} \; . \tag{2}$$

In contrast to the orthodox one-loop formula, Eq. (2) explicitly relates Λ to the tree-level spectrum of physical string states. This formula applies for a large class of tachyon-free string theories including all unitary non-critical strings, critical Type-II strings, as well as the phenomenologically interesting case of $D > 2$ critical heterotic strings.

As stated above, Eq. (2) is applicable for string theories without physical tachyons. This is part of what defines a physically consistent string theory, and the absence of tachyons ensures that Λ is free of infrared divergences. Another remarkable feature of a consistent string theory is modular invariance, a worldsheet symmetry of the one-loop amplitudes. For the present purposes, the most important consequence of modular invariance is that the one-loop amplitudes are ultraviolet finite. Thus string consistency automatically ensures that Λ is finite. Such a result can only achieved in Eq. (2) if to leading order, as $\gamma \to 0$,

$$\sum_{\substack{\text{physical}\\\text{states}}} (-1)^F \, e^{-4\pi\gamma M_i^2} ~\sim~ \gamma^\delta \quad \text{with} \quad \delta \geq D/2-1 ~. \tag{3}$$

With this insight in hand, it is straightforward to derive our supertrace constraints by combining Eqs. (1) and (2):

$$\begin{aligned} \text{Str}\, M^{2n} ~&=~ \lim_{\gamma\to 0} \left\lbrace \sum_i (-1)^F \, (M_i)^{2n} \, e^{-4\pi\gamma M_i^2} \right\rbrace \\ &=~ \lim_{\gamma\to 0} \left\lbrace \left(\frac{-1}{4\pi} \frac{d}{d\gamma} \right)^n \sum_i (-1)^F \, e^{-4\pi\gamma M_i^2} \right\rbrace \\ &=~ \lim_{\gamma\to 0} \left\lbrace \left(\frac{-1}{4\pi} \frac{d}{d\gamma} \right)^n \left[\frac{3}{\pi} \Lambda \, \gamma^{D/2-1} + \ldots \right] \right\rbrace \end{aligned} \tag{4}$$

We see that the right side will vanish if $n < D/2-1$. Thus we find that the spectra of all consistent unitary non-critical strings and critical Type-II and $D>2$ heterotic strings must satisfy the supertrace constraints:*

$$\text{Str}~ M^{2n} ~=~ 0 \quad \text{for} \quad 0 \leq n < D/2-1, \quad n \in \mathbb{Z} ~. \tag{5}$$

For an even number of dimensions, we also have

$$\text{Str}\, M^{D-2} ~=~ \frac{3}{\pi} \frac{(D/2-1)!}{(-4\pi)^{D/2-1}} \, \Lambda ~. \tag{6}$$

For a string theory with spacetime supersymmetry (SUSY), these relations are trivially satisfied through an exact boson/fermion degeneracy at each mass level. However, even in the absence of spacetime SUSY, these constraints will still be satisfied. These supertrace constraints are thus a generic feature of tachyon-free tree-level string vacua, with or without spacetime SUSY.

* For $D=2$ critical heterotic strings, Eq. (2) is modified to $\Lambda = \frac{\pi}{3} \lim_{\gamma\to 0} \lbrace \sum (-1)^F \, e^{-4\pi\gamma M_i^2} \rbrace - 8\pi N$, where N is the number of bosonic minus fermionic unphysical tachyons with right- and left-moving squared masses (0,$-$1) respectively. This immediately yields the supertrace formula $\text{Str}\, M^0 = \frac{3}{\pi} \Lambda + 24\, N$, which may be compared to Eq. (6).

It is interesting to consider how the spacetime bosons and fermions must be arranged throughout the mass levels of non-SUSY string models in order that Eqs. (5) and (6) are satisfied. As an explicit example of a tachyon-free, modular-invariant, non-SUSY string theory, therefore, let us consider the $SO(16) \times SO(16)$ string in ten dimensions [3]. In this case, we know that Str M^0, Str M^2, Str M^4, and Str M^6 must all vanish, and Str $M^8 \approx -0.668$ (where we have used the numerical value of Λ for this model [3]). At the massless level, one finds a surplus of 2112 fermionic states over bosonic states. At the first massive level, though, there is a surplus of 147,456 bosonic states, which more than compensates for the previous surplus of fermions. This surplus of bosons is then (over-)compensated by a surplus of 4,713,984 fermions at the next level. In fact, as shown in Fig. 1, this pattern of carefully balanced boson/fermion oscillations continues throughout the entire infinite tower of massive string states.

These boson/fermion oscillations turn out to be the signature of a hidden so-called "misaligned supersymmetry" which generically appears in the spectra of all tachyon-free modular-invariant string theories in all dimensions, and which arises even in the absence of full spacetime SUSY. Misaligned SUSY is discussed in Ref. [4]. Thus, we see that misaligned SUSY is the mechanism which allows our supertrace constraints to be satisfied without spacetime SUSY. Indeed, rather than having cancellations within each multiplet (as in ordinary softly or spontaneously broken SUSY field theories), in string theory these supertrace constraints are satisfied via cancellations between states at different energy levels across the entire infinite string spectrum. This suggests a possible new string-inspired mechanism for achieving finiteness in field theory, and for constructing a possible non-SUSY solution to the gauge hierarchy problem.

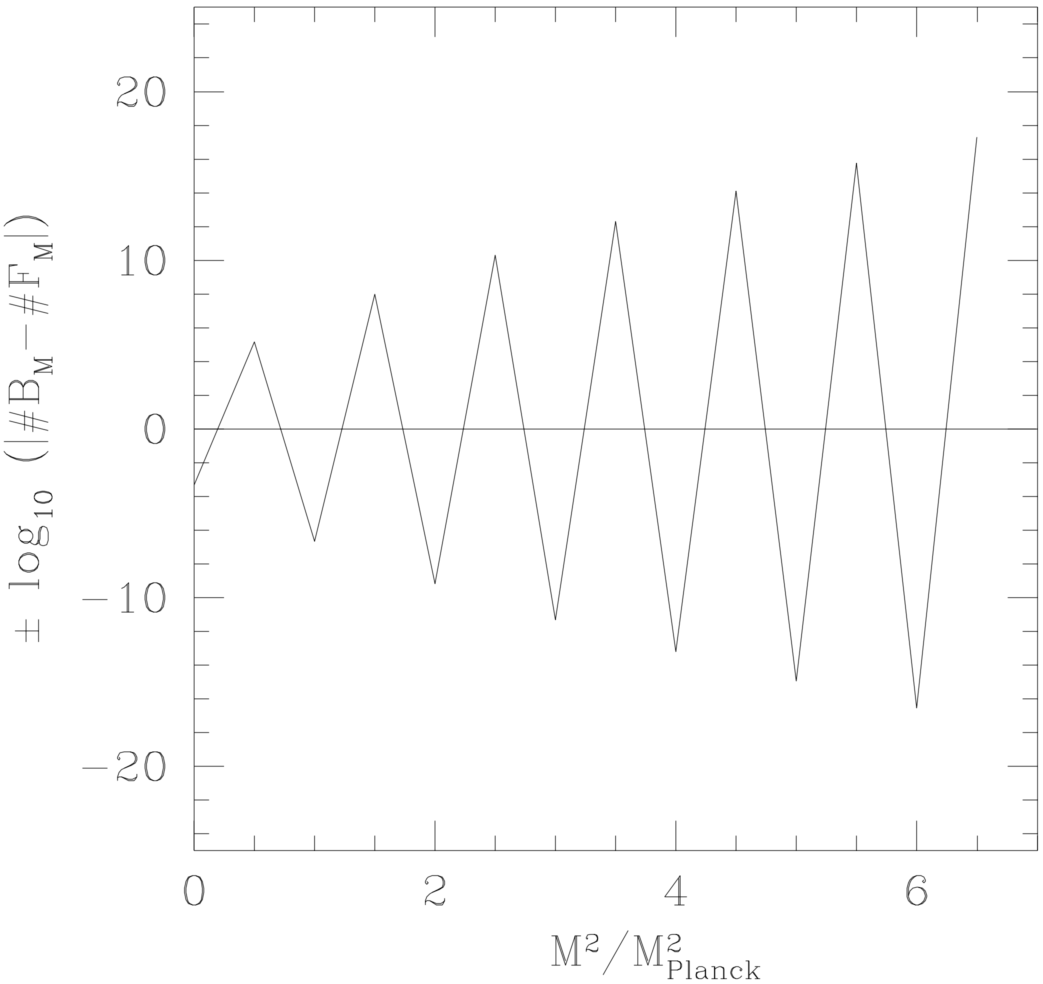

Figure 1: Boson/fermion oscillations in the $D = 10$ non-supersymmetric tachyon-free $SO(16) \times SO(16)$ heterotic string. For each value of spacetime mass-squared (either integer or half-integer in this model), we have calculated the corresponding number of spacetime bosons minus fermions, and have plotted the logarithm of the absolute value of this difference (with an overall sign reflecting whether the difference is positive or negative — *i.e.*, whether there are more bosons or fermions at that level). We have connected these points in order to stress the oscillatory behavior of the boson and fermion surpluses. These oscillations insure that Str M^0 = Str M^2 = Str M^4 = Str $M^6 = 0$ in this model, even though there is no spacetime supersymmetry.

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A non-technical introduction can also be found in:
K.R. Dienes, `hep-th/9409114` (to appear in the Proceedings of PASCOS '94);
K.R. Dienes, `hep-th/9505194` (to appear in the Proceedings of Strings '95).