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Spacetime Properties of (1,0) String Vacua*

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Abstract

We discuss one of the generic spacetime consequences of having (1,0) world-sheet supersymmetry in tachyon-free string theory, namely the appearance of a "misaligned supersymmetry" in the corresponding spacetime spectrum. Misaligned supersymmetry is a universal property of (1,0) string vacua which describes how the arrangement of bosonic and fermionic states at all string energy levels conspires to preserve finite string amplitudes, even in the absence of full spacetime supersymmetry. Misaligned supersymmetry also constrains the degree to which spacetime supersymmetry can be broken without breaking modular invariance, and is responsible for the vanishing of various mass supertraces evaluated over the infinite string spectrum.

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In string theory, the interplay between worldsheet symmetries and their consequences in spacetime remains largely mysterious. Certain results, however, indicate strong connections between the two. For example, it is well-known that $N=4$ supersymmetry on the worldsheet implies $N=2$ supersymmetry in spacetime, and likewise it has been demonstrated that $N=2$ supersymmetry on the worldsheet implies $N=1$ supersymmetry in spacetime. In this talk, we shall consider the more general situation from which these results might follow as special cases. In particular, we shall discuss some of the spacetime consequences of $N=1$ worldsheet supersymmetry.

There are various reasons why this is an important question. $N=1$ worldsheet supersymmetry is the defining property of the superstring and heterotic string theories, and it is in fact this feature which is single-handedly responsible for the introduction of spacetime fermions into the resulting string spectrum. It is therefore interesting to determine whether $N=1$ worldsheet supersymmetry is also sufficiently powerful to constrain the *distribution* of these fermions relative to the bosons. Clearly we do not expect exact boson/fermion degeneracies, as occur in the more restrictive cases with spacetime supersymmetry resulting from $N=2$ or $N=4$ worldsheet supersymmetry, but we might expect that some more general constraints nevertheless control their distribution. Another reason for investigating this issue is to uncover some of the hidden stringy mechanisms whereby super- or heterotic string theories achieve finiteness even without spacetime supersymmetry. For example, it is well-known that the string one-loop vacuum energy (cosmological constant) Λ is a finite quantity in these theories, *even without spacetime supersymmetry*; in an ordinary non-supersymmetric field theory this quantity would diverge. Clearly, the string theory differs from field theory in providing infinite towers of massive (Planck-scale) states, and it is well-understood how, through the requirement of modular invariance, the presence of this tower succeeds in removing the ultraviolet divergences. What is perhaps less clear, by contrast, is how the bosonic and fermionic states ultimately conspire to arrange themselves level-by-level throughout this tower in order to achieve this remarkable result.

Recently it has been shown [1] that the answers to these questions involve a hidden so-called "misaligned supersymmetry" which persists in the string spectrum, even without full spacetime supersymmetry. Indeed, this "misaligned supersymmetry" is a generic property of modular-invariant tachyon-free $(1,0)$ string vacua, and appears for *any* spacetime dimension $D > 2$ and for *any* compactification mechanism. Furthermore, it has been shown that this misaligned supersymmetry is the underlying symmetry responsible for the finiteness of the cosmological constant in the absence full spacetime supersymmetry, and in fact explains how string finiteness is ultimately reconciled with the presence of exponentially growing numbers of string states throughout the infinite towers in the string spectrum. Moreover, misaligned supersymmetry in principle also constrains the degree to which spacetime supersymmetry may be broken in string theory without destroying modular invariance and the resulting finiteness of string amplitudes. Indeed, misaligned supersymmetry is sufficiently powerful to guarantee the vanishing of various mass supertraces $\mathrm{Str}\, M^n$ in string theory, even without spacetime supersymmetry [2]. Thus, in some sense, misaligned supersymmetry lies at the root of many of the remarkable properties that string theory exhibits.

In the remainder of this talk, we shall outline some spacetime consequences of misaligned supersymmetry. Further discussion and details can be found in Ref. [1].

We begin by considering how states are typically arranged in string theory. In general, the string spectrum consists of a collection of infinite towers of states: each tower corresponds to a different *sector* of the underlying string worldsheet theory, and consists of a ground state with a certain vacuum energy H_i and infinitely many higher excited states with worldsheet energies $n = H_i + \ell$ where $\ell \in \mathbb{Z}$. The crucial observation, however, is that the different sectors in the theory will in general be *misaligned* relative to each other, and start out with different vacuum energies H_i (modulo 1). For example, while one sector may contain states with integer energies n, another sector may contain states with $n \in \mathbb{Z} + 1/2$, and another contain states with $n \in \mathbb{Z} + 1/4$. Thus each sector essentially contributes a separate set of states to the total string spectrum, and we can denote the net degeneracies of these states from the $i^{\rm th}$ individual sector as $\{a_{nn}^{(i)}\}$, where $n \in \mathbb{Z} + H_i$. Thus, $a_{nn}^{(i)}$ represents the number of spacetime bosons minus fermions in the $i^{\rm th}$ sector of the theory having spacetime $(\mathrm{mass})^2 = n \in \mathbb{Z} + H_i$.

For each sector i, let us now take the next step and imagine analytically continuing the set of numbers $\{a_{nn}^{(i)}\}$ to form a smooth function $\Phi^{(i)}(n)$ which not only reproduces $\{a_{nn}^{(i)}\}$ for the appropriate values $n \in \mathbb{Z} + H_i$, but which is continuous as a function of n. These functions $\Phi^{(i)}(n)$ clearly must not only exhibit the leading Hagedorn exponential dependence $e^{C\sqrt{n}}$, but must also contain all of the subleading behavior as well so that exact results can be obtained for the relevant values of n. Such continuations are unique and well-defined, and may be easily generated [3].

Given that such functions $\Phi^{(i)}(n)$ exist, misaligned supersymmetry is then characterized by the cancellation of the sum of these functional forms over all sectors in the theory:

$$\sum_i \Phi^{(i)}(n) ~=~ 0 ~. \tag{1}$$

Note that this is a cancellation in the *functional forms* $\Phi^{(i)}(n)$, and *not* a cancellation in the actual numbers of states $\{a_{nn}^{(i)}\}$.

In order to see the effect of this cancellation on the actual numbers of states $\{a_{nn}^{(i)}\}$, let us examine a simple hypothetical example, a toy string model containing only two sectors A and B. For the sake of concreteness, let us assume that these two sectors have different vacuum energies, with $H_A = 0$ (modulo 1) and $H_B = 1/2$ (modulo 1). We thus have two separate towers of states in this theory, with degeneracies $\{a_{nn}^{(A)}\}$ situated at energy levels $n \in \mathbb{Z}$, and degeneracies $\{a_{nn}^{(B)}\}$ situated at energy levels $n \in \mathbb{Z} + 1/2$ (in units of the Planck mass M_0). Then if the corresponding functional forms that describe these degeneracies are $\Phi^{(A)}(n)$ and $\Phi^{(B)}(n)$ respectively, then misaligned supersymmetry implies that $\Phi^{(B)}(n) = -\Phi^{(A)}(n)$. However, due to the misalignment between the two sectors in this hypothetical example, the actual value of each individual a_{nn} will be $\Phi^{(A)}(n)$ if $n \in \mathbb{Z}$, *or* $\Phi^{(B)}(n)$ if $n \in \mathbb{Z} + 1/2$. This behavior is sketched in Fig. 1. Thus, we see that misaligned supersymmetry leads to an *oscillation* in which any given boson surplus at a certain energy level implies a larger fermion surplus at a higher energy level, followed by an even larger boson surplus at an even higher level, and so forth throughout the string spectrum.

Such oscillatory behavior appears in any modular-invariant tachyon-free theory *regardless* of the number of sectors present, with the simple sketch in Fig. 1 becoming more complicated for more complex string models. In any case, however, the cancellation in Eq. (1) is always preserved, with delicately balanced boson/fermion oscillations persisting throughout the infinite string spectrum.

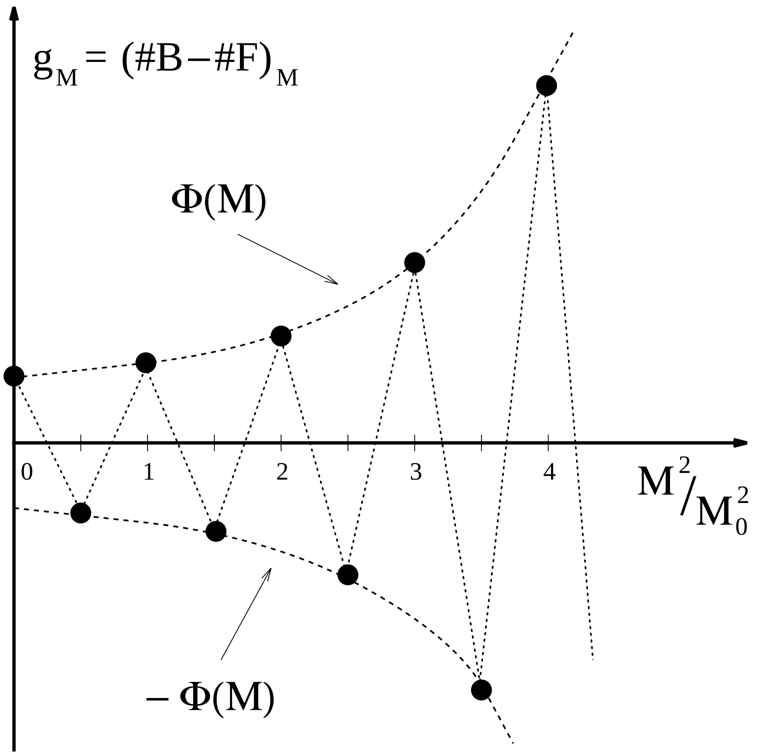

Figure 1: The net number of physical states a_{nn} for the two-sector model discussed in the text, plotted versus energy n [equivalently the spacetime (mass)^2].

It is clear that spacetime supersymmetry is a special case of this generic behavior, for in this case we have $a_{nn} = 0$ level-by-level, and the "amplitude" of this oscillation is zero. Thus, if spacetime supersymmetry is to be broken in such a way that no physical tachyons are introduced and modular invariance is be maintained (as required for a self-consistent string theory), then we can at most "misalign" this bosonic and fermionic cancellation, introducing a mismatch between the bosonic and fermionic state degeneracies at each level in such a way that a carefully balanced "misaligned supersymmetry" survives. It is interesting to see which classes of physical supersymmetry-breaking scenarios do not lead to such behavior, and are thereby precluded. For example, we can already rule out any scenario in which the energies of bosonic and fermionic states are merely shifted relative to each other by some amount Δn. Instead, we would need to simultaneously create a certain number $\Phi(n+\Delta n)-\Phi(n)$ of additional states (presumably winding-mode states coming down from higher mass levels) so that the cancellation of the functional forms Φ is still preserved. Such supersymmetry-breaking scenarios are currently being investigated.

There are also a number of other potential applications and extensions to misaligned supersymmetry. For example, misaligned supersymmetry should be particularly relevant to any system in which the asymptotic numbers of high-energy states plays a crucial role, such as in string thermodynamics and the possible string phase transition. In addition, we would also like to understand the role that misaligned supersymmetry plays in ensuring finiteness to *all* orders (not just one-loop), and also for all n-point functions. Clearly, this requires extending our results to include the *unphysical* string states, as well as string interactions. Such work is in progress.

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