THE TOPOLOGICAL TWIST AND BRST ANTI-BRST GAUGE FIXING
IN $N = 2$ LANDAU-GINZBURG MODELS[*]

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ABSTRACT

We propose an alternative to Vafa's BRST operator, and an anti-BRST operator, for B-twisted topological Landau-Ginzburg models. The advantage of this approach is, that no exotic concepts, like non-integer ghost numbers, are needed for the interpretation of the model as a gauge fixed theory.

Let us first fix the notation and recall the approach of Vafa[2]. The $N = 2$ LG model is a 2-dimensional quantum field theory, with bosons X^i, X^{i*}, 2 sets of fermions with different chirality, viz. ψ^i, ψ^{i*} (spin -1/2) and ξ^i, ξ^{i*} (spin 1/2), and auxiliary fields F^i, F^{i*}. The interaction is characterised by a potential $W(X)$, which is a quasi-homogeneous polynomial of degree d with scaling weights ω^i/d.

The model has an algebra of symmetries that consists of two $N = 2$ supersymmetry algebras. We denote their susy-charges by G^+, G^- for the chiral sector, and $\tilde{G}^+, \tilde{G}^-$ for the anti-chiral sector. The auxiliary fields ensure that the supersymmetry transformation rules close off shell. Furthermore, each sector has an R-symmetry, with charges q_c and q_{ac} (notation: $q_\pm = q_c \pm q_{ac}$). In this basis, q_- is an integer while q_+ takes fractional values, related to the scaling weights of the potential W.

Twisting an $N = 2$ theory is basically a three step procedure [3]. First one redefines the Lorentz spin. Then one defines a BRST operator Q together with a ghost number such that Q has ghost number one. Finally, one shows that the energy momentum tensor is BRST exact, exhibiting the topological nature of the theory.

1. Vafa's scenario

In the twisted model the fermions change spin. The fields ψ^{i*} and ξ^{i*} become spinless, and ψ^i and ξ^i acquire spin -1 and 1 respectively. The BRST charge is $Q_V = G^+ + \tilde{G}^+$. It is easy to show that the energy momentum tensor is BRST exact.

There is a problem with the interpretation of the model as a gauge theory. This is most apparent when one assigns ghost numbers to all the fields, requiring that Q

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has ghost number one. If one requires that all fields have integer ghost numbers, then the action contains a part with ghost number minus two [3], which is puzzling. If one requires that the action has zero ghost number, then the fields have fractional ghostnumbers (equal to q_+). In both cases, it is not possible to interpret the model as a gauge fixed action of an underlying gauge theory. Nevertheless, the topological nature of the model is not in doubt, since the energy momentum tensor is BRST exact. Also, the cohomology of Q_V leads to a sensible physical spectrum and interesting correlation functions.

2. Our proposal

The first step, namely the twist that changes the spins of the fermion fields, is the same as in Vafa's approach. To overcome the problem of the ghost number assignment, we propose to change the BRST operator. Instead of using the two supersymmetries to construct a BRST charge, we will use only one, namely $Q = G^+$. The second supersymmetry we propose to identify with the anti-BRST charge $\bar{Q}$. We require that Q raises the ghost number by one, and $\bar{Q}$ lowers it by one. An immediate consequence is that all ghost numbers are fixed: they are equal to q_-. This makes a conventional separation into classical fields, ghosts and antighosts straightforward. All fields have integer ghost number and the action has zero ghost number. The interpretation of the model as coming from gauge fixing a zero action[1] is also immediate.

To check the topological nature, we investigate the energy-momentum tensor. The $(++)$ component is anti-BRST exact, while the $(--)$ component is BRST exact. This means that BRST invariance for the physical operators is not sufficient to prove the metric independence: one also needs the Ward identity for the anti-BRST operator. This implies that observables are subjected to two conditions, namely they should be BRST invariant and their anti-BRST transformation should be BRST exact. The physical spectrum then corresponds to the elements of the anti-BRST cohomology defined in the BRST cohomology. We have computed the spectrum in this way: it leads to the same topological observables as in Vafa's approach.

For details we refer to the paper[1].

3. References

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3. M. Billó and P. Frè, Class. Quantum Grav. **4** (1994) 785.