HOT STRING SOUP

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ABSTRACT

Above the Hagedorn energy density closed fundamental strings form a long string phase. The dynamics of weakly interacting long strings is described by a simple Boltzmann equation which can be solved explicitly for equilibrium distributions. The average total number of long strings grows logarithmically with total energy in the microcanonical ensemble. This is consistent with calculations of the free single string density of states provided the thermodynamic limit is carefully defined. If the theory contains open strings the long string phase is suppressed.

1. Summary

In this talk we give a brief summary of the work described in more detail in [1]. As is well-known, the exponential growth at high energies of the single string density of states makes the canonical partition function ill-defined above the Hagedorn temperature T_H. Different physical interpretations of this fact have been offered, including that the Hagedorn temperature defines an absolute limiting temperature in physics [2],[3] or that it signals a transition to an unknown high-temperature phase where strings may be replaced by more fundamental degrees of freedom [4].

The microcanonical ensemble, on the other hand, is well-defined above the Hagedorn energy density. Within the context of the non-interacting theory, it has been argued that at high energy density, most of the energy is carried by a *single* long string [5]. This physical picture is suspect because it neglects the effect of interactions. As soon as one includes interactions, one must worry about the Jeans instability which sets in at length scales satisfying $R^2 > 1/(g^2\rho)$, where ρ is the energy density and g is the string coupling constant. The thermal ensemble will only be well-defined in finite volume V, for sufficiently small g.

To include the effect of interactions, we set up a Boltzmann equation to describe the long string phase. The physical picture we have in mind is a gas of very long strings, where each string traverses the volume of the system many times. To leading order, the physics will be independent of the details of the embedding of the string in the target space, and will only depend on intrinsic properties such as its length. We assume the energy of the long string is dominated by the string tension, so is proportional to length. Each string interaction will involve some average over the momenta and relative orientation of the strings – in the long string limit this will give the same factor for each interaction, c.f. [6]. With these assumptions, the Boltzmann equation for long closed strings is

$$\begin{aligned}\frac{\partial n(\ell)}{\partial t} &= \frac{\kappa}{V}\Big\{-\frac{1}{2}\ell^2 n(\ell) - \int_0^\infty d\ell'\ell' n(\ell')\ell n(\ell) + \frac{1}{2}\int_0^\ell d\ell'\ \ell'(\ell-\ell')n(\ell')n(\ell-\ell') \\ &\quad + \int_\ell^\infty d\ell'\ell' n(\ell')\Big\}, \qquad (1)\end{aligned}$$

where κ is some positive constant which depends on the string coupling, and for convenience we have set $\alpha' = 1$. Here $n(\ell)$ is the average number of strings of length ℓ.

As described further in [1], the equilibrium distribution may be obtained which leads to the single string density of states. In the microcanonical ensemble, one then finds a stable distribution of long closed strings, with on average $\log E$ long strings, where E is the total energy. One concludes the single long string phase previously found is unstable when interactions are included. The Boltzmann equation we have constructed is a starting point for the study of the nonequilibrium thermodynamics of fundamental strings at high energies.

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