ON $N=4$ STRINGS

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ABSTRACT

The $N=2$ supersymmetric Kazama-Suzuki coset construction is generalized to the $N=4$ case by requiring the most general non-linear (Goddard-Schwimmer) $N=4$ quasi-superconformal symmetry. The $N=4$ constraints on the allowed cosets have very simple geometrical interpretation, and their natural (symmetric) solutions are quaternionic Wolf's spaces. A quantum BRST charge for the $N=4$ string propagating on a Wolf space is constructed. Surprisingly, the BRST charge nilpotency conditions rule out the non-trivial Wolf spaces as the consistent string backgrounds. The critical dimensions for a flat background and all known $N=4$ algebras are summarized.

The critical (non-topological) $N=4$ strings are known since 1976, [1] but they received little attention in the literature because of their apparently 'negative' critical dimension. A closer inspection of the argument shows that (i) it was implicit that the $N=4$ string constraints form the '*small*' linear $N=4$ superconformal algebra (SCA) having the $\widehat{su(2)}$ affine Lie subalgebra, and (ii) the background space in which an $N=4$ string was supposed to propagate is *flat*. We are going to challenge both assumptions [2], first, by replacing the 'small' linear $N=4$ SCA by a more general non-linear $N=4$ algebra found by Goddard and Schwimmer (GS) [3] and having two affine $\widehat{su(2)}$ subalgebras, and, second, by choosing a *coset* G/H as the embedding space.

The GS $N=4$ algebra comprises stress tensor $T(z)$, four dimension-3/2 supercurrents $G^{\mu}(z)$, and six dimension-1 currents $J^{\mu\nu}(z)$ in the adjoint of $SO(4)\cong SU(2)_{+}\otimes SU(2)_{-}$. The $N=4$ supersymmetry part of the algebra takes the form

$$G^{\mu}(z)G^{\nu}(w)\sim\frac{4k^{+}k^{-}}{(k^{+}+k^{-}+2)}\frac{\delta^{\mu\nu}}{(z-w)^{3}}+\frac{2T(w)\delta^{\mu\nu}}{z-w}-\frac{k^{+}+k^{-}}{k^{+}+k^{-}+2}\left[\frac{2J^{\mu\nu}(w)}{(z-w)^{2}}\right.$$

$$\left.+\frac{\partial J^{\mu\nu}(w)}{z-w}\right]+\frac{(k^{+}-k^{-})\varepsilon^{\mu\nu\rho\lambda}}{k^{+}+k^{-}+2}\left[\frac{J^{\rho\lambda}(w)}{(z-w)^{2}}+\frac{\partial J^{\rho\lambda}(w)}{2(z-w)}\right]-\frac{\varepsilon^{\mu\rho\lambda\zeta}\varepsilon^{\nu\rho\eta\omega}}{2(k^{+}+k^{-}+2)}\frac{:J^{\lambda\zeta}J^{\eta\omega}:(w)}{(z-w)}\ ,$$

where k^{+} and k^{-} are levels of affine Lie algebras associated with $SU(2)_{+}$ and $SU(2)_{-}$.

Requiring the $N=4$ supersymmetry, we expect the standard $N=2$ conditions [4] to be satisfied for each supersymmetry. The general ansatz for the supercurrents is

$$G^{\mu}(z)=\frac{i}{\sqrt{2k}}\left[h^{\mu}_{\bar{a}\bar{b}}\psi^{\bar{a}}(z)\hat{J}^{\bar{b}}(z)+\frac{i}{3}S^{\mu}_{\bar{a}\bar{b}\bar{c}}\psi^{\bar{a}}(z)\psi^{\bar{b}}(z)\psi^{\bar{c}}(z)\right]\ ,$$

where $h^{\mu}_{\bar{a}\bar{b}}$ and $S^{\mu}_{\bar{a}\bar{b}\bar{c}}$ are constants, $\mu = 0,1,2,3$. $\hat{J}$ are affine G currents, ψ are free fermions, and $k = k_G + \tilde{h}_G$, where $\tilde{h}_G$ is the dual Coxeter number of G. *Early* lower case Latin indices are used for G-indices, *middle* lower case Latin indices for H-indices, and early lower case Latin indices *with bars* for G/H-indices. Since our $N=4$ coset construction generalizes that of $N=1$, we have $h^{0}_{\bar{a}\bar{b}} = \delta_{\bar{a}\bar{b}}$ and $S^{0}_{\bar{a}\bar{b}\bar{c}} = f_{\bar{a}\bar{b}\bar{c}}$. The OPEs of the $N=4$ GS algebra are satisfied if and only if [2]

$$h^{\mu}_{\bar{a}\bar{b}}h^{\nu}_{\bar{a}\bar{c}} + h^{\nu}_{\bar{a}\bar{b}}h^{\mu}_{\bar{a}\bar{c}} = 2\delta^{\mu\nu}\delta_{\bar{b}\bar{c}} \ , \quad h^{\mu\bar{b}}_{[\bar{a}}h^{\nu\bar{g}}_{\bar{c}}f_{d]\bar{b}\bar{g}} + h^{\nu\bar{b}}_{[\bar{a}}h^{\mu\bar{g}}_{\bar{c}}f_{d]\bar{b}\bar{g}} = 2\delta^{\mu\nu}f_{\bar{a}\bar{c}d} \ , \quad h^{\mu}_{\bar{a}\bar{b}}f_{\bar{b}\bar{c}d} = h^{\mu}_{\bar{b}\bar{c}}f_{\bar{a}\bar{b}d} \ ,$$

so that $S^{\mu}_{\bar{a}\bar{b}\bar{c}} = h^{\mu\bar{g}}_{[\bar{a}}f_{\bar{b}\bar{c}]\bar{g}}$. It just means that the coset must be *quaternionic.* Given a simple Lie group G, there is the *unique* associated quaternionic symmetric space, called the *Wolf* space. [5] The $\widehat{SU(2)}_{\pm}$ currents are simultaneously fixed, [2] while the levels of the two $\widehat{SU(2)}_{\pm}$ affine subalgebras are just $k^{+} = k_G$, and $k^{-} = \tilde{h}_G - 2$. [2] The $N=4$ GS algebra central charge is $c = 6(k_G+1)(\tilde{h}_G-1)/(k_G+\tilde{h}_G) - 3$. No additional consistency conditions arise.

The BRST quantization of an $N=4$ string introduces the conformal ghosts, the $N=4$ superconformal ghosts and the internal symmetry ghosts, as usual. Each ghost pair contributes -26, $+11$ and -2, respectively, to the (chiral) central charge. As to the non-linear $N=4$ GS algebra, a quantum BRST charge is known, [6] and it becomes nilpotent only if $k^{+} = k^{-} = -2$, which implies $\tilde{h}_G = 0$ and, hence, an abelian G. This essentially rules out Wolf's spaces for the $N=4$ string propagation, except for a flat space. As far as the flat background is concerned, the vanishing central charge (conformal anomaly) condition gives for the (quaternionic) critical dimension $D_{\rm c}$ the following equations:

$$\begin{aligned} \text{small N} = 4 \text{ SCA}: &\quad -26 + 4\cdot 11 + 3\cdot(-2) + 4D_{\rm c}(1+\tfrac{1}{2}) = 0 \ , \\ \text{N} = 4 \text{ GS algebra}: &\quad -26 + 4\cdot 11 + 6\cdot(-2) + 4D_{\rm c}(1+\tfrac{1}{2}) = 0 \ , \\ \text{large N} = 4 \text{ SCA}: &\quad -26 + 4\cdot 11 + 7\cdot(-2) + 4\cdot(-1) + 4D_{\rm c}(1+\tfrac{1}{2}) = 0 \ , \end{aligned}$$

where we have used the fact that the 'large' (linear) $N=4$ SCA has an additional $U(1)$ affine symmetry and four free (auxiliary) fermions beyond the content of the GS algebra. This gives $D_{\rm c} = -2, -1, 0$, respectively.

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