DUALITY, ENHANCED SYMMETRY AND 'MASSLESS BLACK HOLES'

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1. Gauge and Duality Symmetries in Superstring Theories

String theory is defined perturbatively through a set of rules for calculating scattering amplitudes. Perhaps the most important outstanding problem is that of finding the non-perturbative theory whose perturbation theory reproduces that of string theory. This seems a daunting task, but recent progress has led to some striking conjectures regarding the non-perturbative structure of the theory that have passed many tests[1−12]. The picture that seems to be emerging is that there is some as yet unknown theory that, when expanded perturbatively, looks like a perturbative string theory, but which has a surprisingly simple structure at the non-perturbative level which includes U or S duality symmetries relating perturbative states to solitons, and weak coupling to strong. Moreover, there are a number of different coupling constants corresponding to the expectation values of various scalars, and the perturbation expansions with respect to some of these define string theories, but different string theories arise for different coupling constants. This leads to unexpected equivalences between string theories that look very different in perturbation theory: they result from different perturbation expansions of the same theory. In many cases, the strong coupling limit of a given theory with respect to a particular coupling constant is described by the weak coupling expansion of a dual theory, which is sometimes another string theory and sometimes a field theory.

An example which illustrates many of these points is the one obtained from the toroidal compactification of the heterotic string to four dimensions on T^6. When the full non-perturbative theory, including solitons, is considered, there is strong evidence that the theory has an $SL(2,\mathbb{Z})$ S-duality symmetry relating strong to weak coupling and interchanging electric and magnetic charges[6,7,8]. The theory is then self-dual: the strong coupling limit is described by the weak-coupling expansion of a dual heterotic string theory, which is of exactly the same form, but with magnetic charges arising in the perturbative spectrum while electric ones arise as solitons. Expanding the same theory in other directions in coupling constant space can give the perturbative expansion of the type IIA string or of the type IIB string compactified on $K_3 \times T^2$, leading to the conjectured equivalence of the type II and heterotic strings[1]. The expansion with respect to other coupling constants of the theory has been considered in ref. 5.

The type II string compactified on $K_3 \times T^2$ and the heterotic string compactified on T^6 have the same low-energy effective actions. Moreover, there is evidence[1] that the spectrum of Bogomolnyi states – states saturating a Bogomolnyi bound – is the same, although perturbative states of one theory can correspond to solitons of the other. It was these two facts that led to the conjecture[1] that the two string theories are the same. However, such an equivalence would have some remarkable consequences. Perhaps the most striking of these is that it predicts the existence of enhanced non-abelian gauge symmetry in the type II string at special points in the compactification moduli space, because this occurs for the toroidally-compactified heterotic string[13]. For the heterotic string at generic points in the torus moduli space, the Yang-Mills symmetry is the abelian group $U(1)^{28}$, but at certain special points it is enhanced to $U(1)^6 \times K$ where K is a non-abelian group of rank 22 (e.g. $SO(44)$ or $U(1)^6 \times E_8 \times E_8$). The perturbative spectrum of the heterotic string includes certain Bogomolnyi multiplets that contain massive gauge bosons and it is these that become massless when the symmetry is enhanced. In the type II string compactified on $K_3 \times T^2$, states with the same quantum numbers as the heterotic string's massive gauge bosons occur in the non-perturbative spectrum and at generic points in moduli space these can be associated with extreme black hole states[1,4]. As will be discussed below, whenever some of the heterotic string's gauge bosons become massless, the corresponding non-perturbative Bogomolnyi vector states of the type II string become massless and the same non-abelian gauge symmetry arises in both cases. Thus there are black hole states of the type II string that become massless at special points in moduli space and become the gauge bosons of the enhanced gauge symmetry. Strictly speaking, these states can be associated with extreme black hole solitons at generic points in moduli space, but as special points are approached it is probable that the semi-classical approximation will break down and the Bogomolnyi states can no longer be reliably associated with classical solutions. Nonetheless, arguments based on supersymmetry imply that the Bogomolnyi states can be extrapolated to all values of the coupling, even though their representation as black hole solitons is only valid in certain regimes, and it is in this sense that the symmetry enhancement is due to 'black holes becoming massless'.

For theories with $N \geq 4$ supersymmetry the masses of Bogomolnyi states are determined entirely by the low-energy effective field theory. Thus, once the existence of a massive Bogomolnyi state has been established at some particular point in the moduli space of vacua, by whatever means, its mass at other points in the moduli space is determined by the effective low-energy supergravity theory. In particular, the mass of certain Bogomolnyi states must vanish at special points in this moduli space purely as a consequence of N=4 supersymmetry. Since these states must fill out vector supermultiplets the proof of symmetry enhancement in

N=4 supersymmetric theories rests on the existence of the relevant massive states at generic points in moduli space.

In the toroidally compactified *bosonic* string there is perturbative symmetry enhancement in the effective four-dimensional field theory but, since the effective non-abelian gauge theory is asymptotically free, the conventional wisdom is that all particles carrying non-abelian charge (which includes the massless non-abelian gauge bosons) are confined. Thus, the extra massless states found in perturbation theory are not likely to be present in the full theory. If it were possible to bypass perturbation theory and deal directly with the full quantum string theory, one might expect to find a transition to a confining phase at special points of the moduli space, rather than the occurrence of additional massless states at these points.

The status of the symmetry enhancement in the toroidally compactified heterotic string is quite different because the effective non-abelian gauge theory at special points of moduli space is not asymptotically free; in fact, the beta function vanishes. Since confinement no longer operates to remove massless particles with non-abelian charge from the spectrum it might be thought that here, in contrast to the bosonic string, the extra massless particles found in perturbation theory indicate the existence of extra massless particles in the full theory. However, the infrared divergences due to unconfined non-abelian gauge fields do not allow a standard interpretation of the Hilbert space in terms of particles with definite charge quantum numbers. Instead, the existence of vector fields whose masses tend to zero as one approaches special points in moduli space signals a transition to a *non-abelian Coulomb phase* at these points in which there is a non-abelian gauge symmetry associated with long-range forces.

As long as we are away from special points in moduli space there is no problem in providing a standard particle interpretation for the spectrum. At first sight there would appear to be no difficulty in extrapolating this spectrum to those special points at which some massive Bogomolnyi states become massless. Symmetry enhancement in the toroidally compactified heterotic string has so far only been analysed in string perturbation theory, with the result that electrically charged Bogomolnyi perturbative string states become massless at special points in moduli space. However, the heterotic string has non-perturbative magnetically charged Bogomolnyi states arising from BPS monopole and dyonic solutions of the low-energy theory[8,16,17]. For every electrically charged state whose mass tends to zero as one approaches a special point in moduli space, there is also a magnetically charged state and an infinite set of dyonic ones whose masses also approach zero. The interpretation of this is not clear, but possibly signals a non-abelian Coulomb phase of a type rather different from those discussed previously[18]. However, it does show how symmetry enhancement might be consistent with the conjectured S-duality of the heterotic string: whenever the mass of a perturbative string state tends to zero, so do the masses of its magnetically charged $SL(2,\mathbb{Z})$ partners. Thus, even for the toroidally compactified heterotic string we learn something more about the symmetry enhancement mechanism from the analysis based on $N=4$ supersymmetry than we learn from the perturbative theory. A similar picture emerges for the type II string on $K_3 \times T^2$. In fact, since this feature of the enhanced symmetry phase depends only on $N=4$ supersymmetry it applies equally to $N=4$ super Yang-Mills theory, i.e. as the Higgs expectation value tends to zero, *all* charged massive states, including the magnetic monopoles and dyons, become massless together. Whatever the correct interpretation of this may be, we wish to stress that it is a general feature. In particular, if the Bogomolnyi spectrum of the $K_3 \times T^2$ compactified type II string is the same as that of the toroidally compactified heterotic string at some generic points of their respective moduli spaces[1], then whatever happens to the heterotic string at special points also happens to the type II superstring.

Symmetry enhancement can be established in superstring compactifications to four dimensions preserving $N=4$ supersymmetry merely by an analysis of the low-energy effective field theory. The mass of a Bogomolnyi state with a given charge vector is determined entirely by this effective field theory, so that symmetry enhancement is the consequence of the mere existence in the spectrum of certain states. The perturbative symmetry enhancement for the toroidally compactified heterotic string[13] can be extended to the full *non-perturbative* string theory, with the result that magnetically charged states as well as the perturbative electrically charged ones become massless[1]. The existence of the states relevant to symmetry enhancement for K_3 compactifications can be deduced by consideration of the p-brane soliton solutions of the effective supergravity theory in a limit in which semi-classical methods are reliable, because $N=4$ supersymmetry tells us what happens to these states in all other regions of parameter space[4]. These arguments are not specific to string theory and apply in field theories as well, provided only that the theory has the requisite massive states (e.g. arising as solitons).

The new results announced in this talk were obtained in collaboration with Townsend[4], and related results on symmetry enhancement in compactified type II theories were announced in Witten's talk[3].

2. S,T and U Dualities

Consider type II or heterotic strings toroidally compactified to d dimensions. The effective low-energy field theory describing the massless fields of the compactified string is a d-dimensional supergravity theory which has a rigid 'duality' group G, which is a symmetry of the equations of motion, and in odd dimensions is in fact a symmetry of the action. In each case the massless scalar fields of the theory take values in G/H, where H is the maximal compact subgroup of G. G has an $O(10-d, 10-d)$ subgroup for the type II string, and an $O(10-d, 26-d)$ subgroup for the heterotic string. In either string theory, it is known that this subgroup is broken down to the discrete T-duality group, $SO(10-d, 10-d; \mathbb{Z})$ or $O(10-d, 26-d; \mathbb{Z})$, which is an exact symmetry of the perturbative string theory. It is natural to conjecture that the whole supergravity duality group G is broken down to a discrete subgroup $G(\mathbb{Z})$ (defined below) in the d-dimensional string theory. Indeed, the Bogomolnyi solitons (including p-brane solitons) have charges that transform under the duality group G and charge quantization implies that these charges lie on a lattice. Then G is broken to at most the discrete subgroup $G(\mathbb{Z})$ of G that preserves the charge lattice[1]. In tables 1 and 2, we list these groups for toroidally compactified superstring theories (at a generic point in the moduli space so that the gauge group is abelian). In some cases, the duality group is a direct product of the T-duality group with another factor, in which case we refer to the other factor as the S-duality group. We refer to $G(\mathbb{Z})$ as the U-duality group.

Space-time Dimension d	Supergravity Duality Group G	String T-duality	Conjectured Full String Duality
$10A$	$SO(1,1)/\mathbb{Z}_2$	$\mathbb{1}$	$\mathbb{1}$
$10B$	$SL(2,\mathbb{R})$	$\mathbb{1}$	$SL(2,\mathbb{Z})$
9	$SL(2,\mathbb{R}) \times SO(1,1)$	$\mathbb{1}$	$SL(2,\mathbb{Z})$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$O(2,2;\mathbb{Z})$	$SL(3,\mathbb{Z}) \times SL(2,\mathbb{Z})$
7	$SL(5,\mathbb{R})$	$SO(3,3;\mathbb{Z})$	$SL(5,\mathbb{Z})$
6	$O(5,5)$	$SO(4,4;\mathbb{Z})$	$SO(5,5;\mathbb{Z})$
5	$E_{6(6)}$	$SO(5,5;\mathbb{Z})$	$E_{6(6)}(\mathbb{Z})$
4	$E_{7(7)}$	$SO(6,6;\mathbb{Z})$	$E_{7(7)}(\mathbb{Z})$
3	$E_{8(8)}$	$SO(7,7;\mathbb{Z})$	$E_{8(8)}(\mathbb{Z})$
2	$E_{9(9)}$	$SO(8,8;\mathbb{Z})$	$E_{9(9)}(\mathbb{Z})$
1	$E_{10(10)}$	$SO(9,9;\mathbb{Z})$	$E_{10(10)}(\mathbb{Z})$

Table 1 Duality symmetries for type II string compactified to d dimensions.

For the type II string, the supergravity duality groups G are given in ref. 15 . The Lie algebra of $E_{9(9)}$ is the $E_{8(8)}$ Kac-Moody algebra, while the algebra corresponding to the E_{10} Dynkin diagram has been discussed in ref. 15. The $d=2$ duality symmetry contains the infinite-dimensional Geroch symmetry group of toroidally compactified general relativity. In $d=9$, a $\mathbb{Z}_2$ T-duality group might have been expected, but the transformation that inverts the radius of the compactifying circle also interchanges the type IIA theory with the type IIB one[19], so that this transformation is not properly speaking a T-duality[20]. In particular, whereas for the bosonic string compactified on a circle one should factor out by the $\mathbb{Z}_2$ T-duality which is a discrete gauge group, in the type II string one should not: the type IIA string compactified on a circle of radius R is not the same as the type IIA string compactified on a circle of radius $1/R$, although it is the same as the type IIB string compactified on a circle of radius $1/R$.

Space-time Dimension d	Supergravity Duality Group G	String T-duality	Conjectured Full String Duality
10	$O(16) \times SO(1,1)$	$O(16;\mathbb{Z})$	$O(16;\mathbb{Z})$
9	$O(1,17) \times SO(1,1)$	$O(1,17;\mathbb{Z})$	$O(1,17;\mathbb{Z})$
8	$O(2,18) \times SO(1,1)$	$O(2,18;\mathbb{Z})$	$O(2,18;\mathbb{Z})$
7	$O(3,19) \times SO(1,1)$	$O(3,19;\mathbb{Z})$	$O(3,19;\mathbb{Z})$
6	$O(4,20) \times SO(1,1)$	$O(4,20;\mathbb{Z})$	$O(4,20;\mathbb{Z})$
5	$O(5,21) \times SO(1,1)$	$O(5,21;\mathbb{Z})$	$O(5,21;\mathbb{Z})$
4	$O(6,22) \times SL(2,\mathbb{R})$	$O(6,22;\mathbb{Z})$	$O(6,22;\mathbb{Z}) \times SL(2,\mathbb{Z})$
3	$O(8,24)$	$O(7,23;\mathbb{Z})$	$O(8,24;\mathbb{Z})$
2	$O(8,24)^{(1)}$	$O(8,24;\mathbb{Z})$	$O(8,24)^{(1)}(\mathbb{Z})$

Table 2 Duality symmetries for heterotic string compactified to d dimensions.

The effective field theory for the $d = 2$ heterotic string should be a $d = 2$ supergravity theory, for which G is given by the affine group $O(8,24)^{(1)}$ symmetry[15]. The heterotic string is conjectured to have an $S \times T$ duality symmetry in $d \geq 4$ and a unified U-duality in $d \leq 3$. Sen conjectured an $O(8,24;\mathbb{Z})$ symmetry of $d = 3$ heterotic strings[21]. The $d = 10$ supergravity theory has an $O(16)$ symmetry acting on the 16 abelian gauge fields which is broken to the finite group $O(16;\mathbb{Z})$; we refer to this as the T-duality symmetry of the ten-dimensional theory.

The discrete duality groups in 4 dimensions have been constructed explicitly[1]; in particular, $E_{7(7)}$ is a subgroup of $Sp(56,\mathbb{R})$ and $E_7(\mathbb{Z})$ is the intersection of $E_{7(7)}$ with $Sp(56,\mathbb{Z})$. The supergravity symmetry group G in d dimensions doesn't act on the d-dimensional space-time and so survives dimensional reduction. Then G is necessarily a subgroup of the symmetry G' in $d' < d$ dimensions and dimensional reduction gives an embedding of G in G', and $G(\mathbb{Z})$ is a subgroup of $G'(\mathbb{Z})$. We use this embedding of G into the duality group in $d' = 4$ dimensions to define the duality group $G(\mathbb{Z})$ in $d > 4$ dimensions as $G \cap E_7(\mathbb{Z})$ for the type II string and as

$G \cap [O(6,22;\mathbb{Z}) \times SL(2,\mathbb{Z})]$ for the heterotic string.

The symmetries in $d < 4$ dimensions can be understood using a type of argument first developed to describe the Geroch symmetry group of general relativity and used by Sen[21] for $d = 3$ heterotic strings. The three-dimensional type II string can be regarded as a four-dimensional theory compactified on a circle and so is expected to have an $E_7(\mathbb{Z})$ symmetry. There would then be seven different $E_7(\mathbb{Z})$ symmetry groups of the three dimensional theory corresponding to each of the seven different ways of first compactifying from ten to four dimensions, and then from four to three. The seven $E_7(\mathbb{Z})$ groups and the $O(7,7;\mathbb{Z})$ T-duality group do not commute with each other and generate a discrete subgroup of E_8 which we define to be $E_8(\mathbb{Z})$. (Note that the corresponding Lie algebras, consisting of seven $E_{7(7)}$ algebras and an $O(7,7)$, generate the whole of the $E_{8(8)}$ Lie algebra.) Similarly, in $d = 2$ dimensions, there are eight $E_8(\mathbb{Z})$ symmetry groups and an $O(8,8;\mathbb{Z})$ T-duality group which generate $E_9(\mathbb{Z})$ as a discrete subgroup of $E_{9(9)}$, and in the heterotic string there are eight $O(8,24;\mathbb{Z})$ symmetry groups from three dimensions and an $O(8,24;\mathbb{Z})$ T-duality group which generate $O(8,24;\mathbb{Z})^{(1)}$ as a discrete subgroup of $O(8,24)^{(1)}$.

3. Symmetry Enhancement and Bogomolnyi States in Four Dimensions

We now wish to see what can be learned directly from an analysis of the effective four-dimensional theory for any theory which has at least $N = 4$ local supersymmetry in $d = 4$. The bosonic massless fields of an $N \geq 4$ supergravity theory are the four-dimensional space-time metric $g_{\mu\nu}$, scalars ϕ^i taking values in a sigma-model target space $M = G/H$ and vector fields A^I_μ with field strengths $F^I_{\mu\nu}$. The gauge group has rank k and is abelian for generic points in the moduli-space, in which case $I = 1, \ldots, k$. It is a feature of such theories that the mass of any field configuration satisfies a classical bound[22,1,17] of the form

$$M^2 \geq \mathcal{Z}^A \mathcal{R}_{AB}(\bar{\phi}) \mathcal{Z}^B \tag{1}$$

where

$$\mathcal{Z} = \begin{pmatrix} p^I \\ q_I \end{pmatrix} \tag{2}$$

and p and q are the magnetic and Noether electric charges defined by integrals over the two-sphere at spatial infinity[1]. The matrix $\mathcal{R}$ is a function of the asymptotic values $\bar{\phi}^i$ of the scalar fields. In the quantum theory, a similar bound applies to all quantum states, with the numbers $\bar{\phi}^i$ now to be interpreted as the expectation values of the scalar fields ϕ^i, parameterising the possible vacua.

Of particular interest are the 'Bogomolnyi states' saturating this bound, so that their masses satisfy

$$M^2 = \mathcal{Z}^A \mathcal{R}_{AB}(\bar{\phi}) \mathcal{Z}^B \tag{3}$$

These preserve half the supersymmetry and have a number of special properties: (i) Their masses and charges receive no quantum corrections. (ii) The spectrum of Bogomolnyi states is duality invariant. (iii) They fit into ultra-short $N = 4$ or $N = 8$ massive supermultiplets with highest spin h; these have the same spectrum of helicity states as the corresponding massless supermultiplets with highest spin h (apart from the obvious charge doubling) and are the massive multiplets that can become massless without a jump in the number of states. (iv) If the existence of a Bogomolnyi state can be established for some values of the coupling constants, it can be continued to other values and a Bogomolnyi with the given charges will exist for all values of the couplings.

In the weakly coupled theory at generic points in the compactification space moduli space, the Bogomlnyi states can arise either as elementary modes of the string, or as solitons. In some cases, there is a soliton with the same quantum numbers as an elementary string state, in which case the two states should be identified[1]. It remains an open question as to whether singular solutions can be acceptable as solitons and, if so, what types of singularity can be allowed. The form of a solution of an effective theory can only be trusted down to length scales corresponding to the masses of the lightest fields that have been integrated out, and including such massive fields can drastically change the short-distance structure of a solution and in some cases remove the singularity. Thus even if a solution is singular, the singularity might be removed by including extra fields in the effective theory. The solutions of the four-dimensional effective theory of massless fields are extreme 'black holes', and the metrics[23] are often those of naked singularities. For example, if some massive vector fields and scalars are included, the solution can become that of a BPS monopole, while if Kaluza-Klein towers of massive fields corresponding to higher dimensional fields are included, the solution can become a Kaluza-Klein monopole. Many of the Bogomolnyi states arise from higher dimensions as 'wrapping modes', consisting of p-brane solitons[24] of a 10 or 11 dimensional theory wrapped around non-trivial homology cycles of the compactifying space[1] and all have a non-singular higher dimensional interpretation [1], 4. Solitons of the effective theory may be good representations of Bogomolnyi quantum states in certain coupling regimes, in the same way that baryons can be represented as Skyrme solitons of the effective pion theory. Once the existence of Bogomolnyi states has been established at weak coupling and at generic points in moduli space, they can be extrapolated to other values, although the representation as a elementary string state or as a soliton may not be trustworthy.

We now turn to symmetry enhancement. A massive ultrashort supermultiplet with given charge vector $\mathcal{Z}_0$ has a mass $M(\phi)$ given by Eq. (3) as a function of ϕ, and this mass is expected to be exact and to receive no quantum corrections (for $N \geq 4$ supersymmetry). If $M(\phi_0) = 0$ at some point ϕ_0, then the supermultiplet becomes massless in the corresponding vacuum. If the supermultiplet contains a vector field, then there is extra gauge symmetry, and as we shall see, this is usually non-abelian. The heterotic string has charged vectors that become massless at certain values of the moduli, so that if the type II string has the same Bogomolnyi spectrum, it too has vectors that become massless and lead to enhanced gauge symmetry. Thus the question that needs to be addressed is which charge vectors $\mathcal{Z}$ are carried by Bogomolnyi states and which of these can become massless.

The charge vector $\mathcal{Z}$ satisfies the DSZ quantization condition, which implies that q takes values in some lattice Γ and p takes values in the dual lattice $\tilde{\Gamma}$. The matrix $\mathcal{R}$ is a continuous function of $\bar{\phi}$, so that the masses of Bogomolnyi states are also continuous functions of $\bar{\phi}$. Under certain circumstances the matrix $\mathcal{R}_{AB}(\bar{\phi})$ has a (fixed) number of zero eigenvalues, for all values of $\bar{\phi}$. This might make it appear that there should be extra massless particles for all values of the moduli, but this is not the case for two reasons. First, at any given point on moduli space there may be no points in the charge lattice that lie in the Kernel of $\mathcal{R}_{AB}(\bar{\phi})$. Second, as we shall see in more detail later, not all points in the lattice of charges allowed by the DSZ quantization condition actually occur in a given theory. In particular, in string theory only those points in the electric charge lattice that are consistent with the physical state conditions of perturbative string theory can correspond to states in the string spectrum, and there are no such points whose charges are in the kernel of $\mathcal{R}_{AB}$ for generic points in moduli space. For special values of $\bar{\phi}$, however, a finite number of string states have charge vectors in the kernel, so that they become massless. Conversely, the mass of a Bogomolnyi state with given charge vector can vanish only for certain values of $\bar{\phi}$. This type of argument was first used in $N = 2$ theories[10], but in that case there are quantum corrections to the masses that can affect the conclusions. Here we shall restrict ourselves to theories with $N \geq 4$ supersymmetry so that there are no corrections to the Bogomolnyi mass formula and the classical analysis is reliable in the quantum theory. We shall return shortly to consider the circumstances under which all conditions for massive Bogomolnyi states to become massless at special points in moduli space can be satisfied in a string theory, but we shall first examine some general consequences of $N = 4$ or $N = 8$ supersymmetry in the event that this phenomenon occurs.

Thus, as $\bar{\phi}$ is continued to $\bar{\phi}_0$, the ultrashort Bogomolnyi supermultiplets with charge vector $\mathcal{Z}_0$ must continue (at least modulo 16) to massless supermultiplets with the same highest spin. We do not expect the new massless supermultiplets to have highest spin $h \geq 2$, as these would lead to well-known inconsistencies. (These inconsistencies might be avoided if an infinite number of supermultiplets become massless. This occurs in a 'decompactification limit' in which some compact dimensions become non-compact, or in a null string limit. Such phenomena, which we will not consider here, are associated with points on the boundaries of moduli space. These inconsistencies might also be avoided if at the special points there was no low-energy effective field theory description.) Since all $N = 8$ supermultiplets have highest spin of at least two we should not expect any massive supermultiplets to become massless at special points in the moduli space of compactifications that preserve $N = 8$ supersymmetry, e.g. the T^6 compactification of the type II superstring. We shall verify this prediction below. For $N = 4$ there remain two possibilities: $h = 1$ and $h = 3/2$.

Consider first the $h = 3/2$ case. The existence of additional massless spin-3/2 states implies an enhanced $N > 4$ local supersymmetry, but this is possible only if *all* massless states belong to the graviton supermultiplet, since there are no massless matter supermultiplets (with $h \leq 1$) for $N > 4$. Moreover, the total number of massless vectors would increase since the $N = 4$ supermultiplet with $h = 3/2$ contains vector fields. In the cases of most interest to us here, the toroidally compactified heterotic string or type II on $K_3 \times T^2$, the number of massless vector fields at a generic point in the moduli space is already 28, so that we would need an effective $N > 4$ supergravity with more than 28 vector fields. There is no such theory (the $N = 8$ theory has exactly 28). Moreover, the gauge group of the massless vector fields would have to be non-abelian, for reasons explained below, and it is difficult to reconcile this with a vanishing cosmological constant in a pure supergravity theory. For these reasons, we exclude the possibility of additional $h = 3/2$ massless supermultiplets. Since partially shortened supermultiplets saturating a stronger bound must have highest spin $h \geq 3/2$, this exclusion explains why we may restrict our attention to ultrashort multiplets.

This leaves the possibility that massive $N = 4$ vector multiplets become massless at special points in the moduli space of a compactification preserving $N = 4$ supersymmetry. Ultra-short massive vector multiplets come in central charge doublets which couple to the vector field A^0 of the corresponding central charge. Consider the case in which only one such charge doublet with vector fields A^+, A^- (and their superpartners) becomes massless. Since the effective massless theory now contains three vector fields with a trilinear $A^0A^+A^-$ coupling, consistency implies that the original $U(1)^k$ gauge symmetry is enhanced to the non-abelian group $U(1)^{k-1} \times SU(2)$. Note that there are also additional massless scalars, but that these have quartic interactions (as required by $N = 4$ supersymmetry) so that their expectation values do not constitute new moduli. More generally, several charged doublets may become massless simultaneously, leading to an enhanced symmetry group of higher dimension. The rank, however, must remain equal to k (and the maximal rank of the maximal simple subgroup equal to $k-6$). This is because each of the additional massless vector multiplets is charged with respect to one of the k original $U(1)$'s.

Before turning to the conditions under which the matrix $\mathcal{R}$ has zero eigenvalues, we will discuss the Bogomolnyi mass formula and its coupling constant dependence.

4. Bogomolnyi Masses in Four Dimensions

In four dimensions, the toroidally compactified heterotic string at a generic point in its moduli space has gauge group $U(1)^{28}$ so that there are 28 electric charges q_I and 28 magnetic charges[1] p^I. The supergravity field equations are invariant under $G = SL(2;\mathbb{R}) \times O(6,22)$ and this is broken down to the integral subgroup $SL(2;\mathbb{Z}) \times O(6,22;\mathbb{Z})$ of S and T dualities[7,8]. The scalar fields take values in the coset space

$$\frac{G}{H} = \frac{SL(2;\mathbb{R})}{U(1)} \times \frac{O(6,22)}{O(6) \times O(22)}$$

and the 56 electric and magnetic charges transform as the irreducible $(\mathbf{2},\mathbf{28})$ representation of $SL(2;\mathbb{R}) \times O(6,22)$. The $N = 4$ supersymmetry algebra has 6 electric and 6 magnetic central charges $\tilde{q}_I, \tilde{p}^I$ given in terms of the charges q_I, p^I by $\tilde{q} = Kq, \tilde{p} = Kp$ where K is a 6×28 matrix function of the moduli that are given by the asymptotic values of the scalar fields taking values in $O(6,22)/O(6) \times O(22)$. The $N = 4$ Bogomolnyi mass formula for BPS saturated states (preserving half the supersymmetry) is[7,8]

$$M^2 = (\,\tilde{p} \quad \tilde{q}\,)\,\mathcal{S} \begin{pmatrix} \tilde{p} \\ \tilde{q} \end{pmatrix} \tag{4}$$

where M is the ADM mass in the Einstein frame and

$$\mathcal{S} = \frac{1}{\lambda_2} \begin{pmatrix} |\lambda|^2 & \lambda_1 \\ \lambda_1 & 1 \end{pmatrix} \tag{5}$$

is an $SL(2,\mathbb{R})$ matrix depending on $\lambda = \langle a + ie^{-\Phi} \rangle = \lambda_1 + i\lambda_2$, where a is the axion and Φ is the dilaton. For vanishing axion expectation value λ_1, the mass is given by

$$M^2 = g^2 \tilde{q}^2 + \frac{1}{g^2}\tilde{p}^2 \tag{6}$$

where $g^2 = 1/\lambda_2 = \langle e^{\Phi} \rangle$ is the string coupling constant. For the mass M_s measured with respect to the stringy metric $\tilde{g}_{\mu\nu}$, which is given in terms of the Einstein metric $g_{\mu\nu}$ by $\tilde{g}_{\mu\nu} = e^{\Phi} g_{\mu\nu}$, the formula Eq. (6) becomes

$$M_s^2 = \tilde{q}^2 + \frac{1}{g^4}\tilde{p}^2 \tag{7}$$

This form was to be expected, since the electric charges are carried by perturbative string states while the magnetic ones arise from solitons, and the mass of a magnetically charged state has the standard $1/g^2$ coupling constant dependence of a soliton.

The $SL(2,\mathbb{R})$ symmetry acts as

$$\mathcal{S} \to \Lambda \mathcal{S} \Lambda^t, \qquad \begin{pmatrix} \tilde{p} \\ \tilde{q} \end{pmatrix} \to \Lambda^{-1} \begin{pmatrix} \tilde{p} \\ \tilde{q} \end{pmatrix} \tag{8}$$

where Λ is a 2×2 matrix in $SL(2,\mathbb{R})$, and (p,q) transforms in the same way as $(\tilde{p},\tilde{q})$. The Einstein metric $g_{\mu\nu}$ is invariant, and the mass formula Eq. (4) is manifestly invariant. The $SL(2,\mathbb{Z})$ transformation given by Eq. (8) with

$$\Lambda = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{9}$$

interchanges electric and magnetic charges while $\lambda \to -1/\lambda$. If $\langle a \rangle = 0$, then the coupling constant is inverted, $g \to 1/g$, and weak and strong coupling regimes are interchanged. Thus the theory is self-dual: the strongly coupled regime can be treated using perturbation theory in the small coupling constant $\hat{g} = 1/g$ and this gives a dual heterotic string theory. In the weakly coupled theory, the electric charges q were carried by g-perturbative states (i.e. ones that arise in the perturbation theory with respect to g) and the magnetic ones p by solitons, while in the dual theory the electric charges q are carried by solitons and the magnetic ones p are carried by states that arise as $\hat{g}$-perturbative states.

For the dual theory, it is appropriate to use the dual stringy metric $\tilde{g}_{\mu\nu}$, which is given in terms of the Einstein metric $g_{\mu\nu}$ by $\tilde{g}_{\mu\nu} = e^{-\Phi} g_{\mu\nu}$. The mass M_d measured with respect to this metric is then, from Eq. (6),

$$M_d^2 = \tilde{p}^2 + \frac{1}{\hat{g}^4} \tilde{q}^2 \tag{10}$$

This is consistent with the fact that p is carried by $\hat{g}$-perturbative states and q by solitons for $g >> 1$ ($\hat{g} << 1$).

Consider now the type II theory toroidally compactified to four dimensions. The gauge group is again $U(1)^{28}$, so that there are again $28+28$ electric and magnetic charges that form a 56-vector $\mathcal{Z}$ which in the quantum theory must take values in a self-dual lattice. The low-energy effective action is that of $N=8$ supergravity[14], which has an $E_{7(7)}$ symmetry of the equations of motion which is broken to the discrete $E_7(\mathbb{Z})$ U-duality symmetry of the string theory[1]. The charge vector $\mathcal{Z}$ transforms according to the irreducible **56** representation of E_7, which has the decomposition

$$\mathbf{56} \to (\mathbf{2},\mathbf{12}) + (\mathbf{1},\mathbf{32}) \tag{11}$$

under the subgroup $SL(2;\mathbb{R}) \times SO(6,6)$. This is to be compared with the heterotic string, for which the charge vector (p,q) has the decomposition

$$(\mathbf{2},\mathbf{28}) \to (\mathbf{2},\mathbf{12}) + 16 \times (\mathbf{2},\mathbf{1}) \tag{12}$$

in terms of representations of $SL(2;\mathbb{R})\times SO(6,6)$. In both cases there is a common sector corresponding to the $(\mathbf{2},\mathbf{12})$ representation of $SL(2;\mathbb{R})\times SO(6,6)$, plus an additional 32-dimensional representation corresponding, for the heterotic string, to the charges for the additional $U(1)^{16}$ gauge group and, for the type II strings, to the charges for the Ramond-Ramond (RR) sector gauge fields. It is remarkable that the 16 electric and 16 magnetic RR charges are singlets of S-duality and fit into the irreducible spinor representation of the T-duality group; they are all carried by solitons as they are not carried by elementary string states[1].

The scalar fields take values in the coset space $E_7/SU(8)$ and can be represented by a 56×56 matrix $\mathcal{V}$ that transforms under E_7 from the right and under local $SU(8)$ transformations from the left[14]. The charge vector $\mathcal{Z}$ enters the Bogomolnyi mass formula through the E_7-invariant combination $\bar{\mathcal{Z}}=\overline{\mathcal{V}}\mathcal{Z}$, where $\overline{\mathcal{V}}$ is the asymptotic value of $\mathcal{V}$. The Lie algebra of E_7 can be decomposed into that of $SL(2;\mathbb{R})\times O(6,6)$ and its orthogonal complement X, so that $\overline{\mathcal{V}}$ can be written as $\overline{V}=STR$ where $S\in SL(2;\mathbb{R})$, $T\in O(6,6)$ and R is the exponential of an element of X. Then the dressed charge vector $\tilde{\mathcal{Z}}=TR\mathcal{Z}$ decomposes into 12 doublets of $SL(2,\mathbb{R})$, consisting of $12+12$ 'dressed' electric and magnetic charges $(\tilde{p}^I,\tilde{q}_I)$, together with 32 singlets of $SL(2;\mathbb{R})$, the 'dressed' RR charges $\tilde{r}_a$. The ADM mass formula for Bogomolnyi states in the Einstein-frame is then

$$M^2=(\,\tilde{p}\quad\tilde{q}\,)\,\mathcal{S}\begin{pmatrix}\tilde{p}\\ \tilde{q}\end{pmatrix}+\tilde{r}^2 \tag{13}$$

where $\mathcal{S}$ is given in terms of $\lambda=\langle a+ie^{-\Phi}\rangle$ by Eq. (5). The dependence on λ can be understood from group theory, and in particular the fact that the RR charges $\tilde{r}$ occur without any dependence on λ follows from the fact that they are $SL(2,\mathbb{R})$ singlets. For vanishing λ_1, the mass is given by

$$M^2=g^2\tilde{q}^2+\frac{1}{g^2}\tilde{p}^2+\tilde{r}^2 \tag{14}$$

For the mass M_s measured with respect to the stringy metric $e^{\Phi}g_{\mu\nu}$, this becomes

$$M_s^2=\tilde{q}^2+\frac{1}{g^2}\tilde{r}^2+\frac{1}{g^4}\tilde{p}^2 \tag{15}$$

while for mass M_d corresponding to the dual stringy metric $e^{-\Phi}g_{\mu\nu}$ the Bogomolnyi mass formula is

$$M_d^2=\tilde{p}^2+\frac{1}{\hat{g}^2}\tilde{r}^2+\frac{1}{\hat{g}^4}\tilde{q}^2 \tag{16}$$

Thus, as in the $N=4$ case, NS-NS electric and magnetic fields are interchanged under strong/weak duality, but the states carrying RR charges are non-perturbative at both weak and strong coupling. Whereas magnetic charges are associated with effects with the usual non-perturbative coupling dependence of e^{-1/g^2}, the RR charges are associated with ones with the stringy dependence $e^{-1/g}$, similar to that found in matrix models[25].

5. Symmetry Enhancement

We now turn to the conditions under which the matrix $\mathcal{R}$ has zero eigenvalues. $N = 8$ supergravity, the effective theory for the toroidally compactified type II superstring, has a non-singular $\mathcal{R}$-matrix so that, as predicted, there can be no points at which $\mathcal{R}$ has zero eigenvalues[4]. $N = 4$ supergravity coupled to m vector multiplets has $k = 6+m$ vector fields and the scalars take values in the coset space G/H where $G = SL(2;\mathbb{R}) \times O(6,m)$ and $H = U(1) \times O(6) \times O(m)$. In this case, $\mathcal{R}$ has precisely $2m$ eigenvectors with zero eigenvalue[4] for all values of ϕ. If an electric Bogomolnyi state with charge given by $(p,q) = (0,V)$ (for some lattice vector V) becomes massless at some special modulus φ_0, then the $O(6,22)$ norm V^2 of V must be negative[4] and any state with charge vector $(p,q) = (mV, nV)$ with m,n integers will have a mass that also tends to zero as $\phi \to \phi_0$[4].

Consider the T^6 compactified heterotic string. In this case the parameters are the T^6 moduli together with the real and imaginary parts of the complex variable λ, which are the constant values of the four-dimensional axion and dilaton fields a_4 and Φ_4[7]. For weak coupling, i.e. $g_4 << 1$, where $g_4 \equiv \langle e^{\Phi_4} \rangle$ and is the same as the ten-dimensional string coupling $\langle e^{\Phi_{10}} \rangle$ for the heterotic string on T^6, the electrically charged Bogomolnyi states arise as modes of the fundamental string and have charges q for which q^2 is even and satisfies $q^2 \geq -2$. As noted above, states with $q^2 \geq 0$ do not become massless, which is just as well since that would have meant higher-spin supermultiplets becoming massless. Also, the $q^2 = -2$ states indeed fit into vector supermultiplets, as expected. Thus, only Bogomolnyi states with $q^2 = -2$ can become massless and for a given value of ϕ, the extra massless states are the ones satisfying $q^2 = -2$ and such that $\hat{q} \in \ker(\mathbb{1} + L)$. For generic values of ϕ, there will be no such massless states, but for special values there will a finite number of vectors in the charge lattice satisfying these conditions, and these can be identified with the root vectors of the enhanced gauge algebra [13]. Conversely, for a given charge vector satisfying these conditions there is always a vacuum for which this happens[4].

Given any Bogomolnyi vector supermultiplet with $(p,q) = (0,V)$ such that $V^2 = -2$, there are also supermultiplets with $(p,q) = (V, nV)$ and $(p,q) = (2V, nV)$ which are represented for weak coupling and at generic points in the moduli space by BPS monopoles and BPS dyons[8,16,17]. It has been shown[7,8] that the conjectured

$SL(2,\mathbb{Z})$ symmetry of the heterotic string spectrum implies Bogomolnyi states with $(p,q) = (mV, nV)$ for all co-prime integers m, n. All these states must become massless together[4], so that S-duality and N=4 supersymmetry imply that as an enhanced symmetry point of the heterotic string is approached there is an infinite set of dyon states whose masses tend to zero, in addition to the purely electric and purely magnetic states. The interpretation of the magnetic and dyon states as due to quantization of solitons presumably fails at points of enhanced symmetry because the sizes of the monopole and dyons approach infinity as their mass approaches zero. This phenomenon is presumably another indication of a phase transition to a special type of non-abelian Coulomb phase.

It is instructive to explore further the consistency of the conjectured S-duality[6,7,8] of the four-dimensional heterotic string with the phenomenon of symmetry enhancement. At strong coupling, $g_4 >> 1$, the four-dimensional heterotic string is conjectured to be related to the weakly coupled theory by S-duality, with the roles of electric and magnetic charges interchanged. If so, then at generic points in moduli space, electric states with $(p,q) = (0,V)$ are perturbative string states for weak coupling and represented at strong coupling by solitons of the dual theory, while magnetic states with $(p,q) = (V,0)$ are solitons of the weakly coupled theory but are perturbative states of the dual theory. It is expected that the theory can be smoothly continued in g_4 without encountering any phase transition, in which case S-duality implies that the electric and magnetic charges should be on exactly the same footing. This would mean, in particular, that magnetically charged vector states become massless at strong coupling through in the perturbative dual theory. S-duality implies that magnetically charged states occur as perturbative states of the dual strong-coupling theory, and if, as we are assuming, these can be continued in g_4 back to magnetically charged states at weak coupling, it is clear that magnetic states with masses tending to zero at the special points must be present in the weakly-coupled theory, even though their perturbative description as solitons in the weak coupling theory breaks down.

The string theory coupling constant is $g_{10} = \langle e^{\Phi_{10}} \rangle$, but the dilaton Φ_{10} appears differently in the two theories, as we shall now argue. In particular, Φ_{10} occurs in the $SL(2;\mathbb{R})/U(1)$ coset space for the heterotic string (and so can identified with Φ_4) while the type II dilaton lies in the $O(6,22)/O(6) \times O(22)$ coset space after compactification on $K_3 \times T^2$. This means that perturbative effects in the heterotic string can be non-perturbative in the type II string, and vice versa. To see this, we shall focus on the subgroup $O(4,20) \times SL(2;\mathbb{R})_S \times SL(2;\mathbb{R})_T \times SL(2;\mathbb{R})_U$ of the duality group $SL(2;\mathbb{R}) \times O(6,22)$. For both superstring compactifications, $SO(2,2) \sim SL(2;\mathbb{R})_T \times SL(2;\mathbb{R})_U$ acts on the moduli space of T^2 and $SL(2;\mathbb{R})_S$ acts on the dilaton and axion fields arising in the usual way from the $D = 10$ dilaton Φ_{10} and the antisymmetric tensor gauge field. The space $O(4,20)/[O(4) \times$

$O(20)]$, modulo the discrete group $O(4,20;\mathbb{Z})$, is the moduli space for K_3 and for the Narain construction of heterotic strings in six dimensions. For the heterotic string, $SL(2,\mathbb{R})_T \times SL(2,\mathbb{R})_U \subset O(6,22)$. For the type II string, however, the RR charges are inert under S-duality[1] and do not couple to the ten-dimensional dilaton Φ_{10} [20,3], so that Φ_{10} cannot be identified with Φ_4, which does couple to all charges. Thus it must be the case that $SL(2,\mathbb{R})_S \subset O(6,22)$, and in fact $SL(2,\mathbb{R})_S \times SL(2,\mathbb{R})_U \subset O(6,22)$, so that the $SL(2,\mathbb{R})_S$ and $SL(2,\mathbb{R})_T$ factors are interchanged compared with the heterotic string, as in ref. 9. In particular, for the heterotic string, Φ_4 coincides with the $D=10$ dilaton Φ_{10} while for the type II string they are distinct: Φ_{10} lies in $O(6,22)/[O(6)\times O(22)]$ while Φ_4 is one of the T^2 moduli. It is interesting to note that the equivalence of the heterotic and type II superstring compactifications together with T-duality in each theory implies invariance of the spectrum under the full group $O(4,20)\times SL(2;\mathbb{R})_S \times SL(2;\mathbb{R})_T \times SL(2;\mathbb{R})_U$ since what is non-perturbative in one is perturbative in the other.

For the $K_3 \times T^2$ compactification of the type II superstring is that symmetry enhancement will occur if the Bogomolnyi spectrum includes electrically charged states with $q^2 < 0$. These states do not occur in perturbation theory but they may appear in the non-perturbative spectrum as Ramond-Ramond (RR) solitons[1]. Note, however, that if the conjectured equivalence to the heterotic string is correct the type II string studied using perturbation theory in the T^2 modulus $g_4 \equiv \langle e^{\Phi_4} \rangle$ should give the same results as the heterotic string expanded in the usual way order by order in the string coupling g. For the type II string on $K_3 \times T^2$, the usual string perturbation theory in g is not useful as the RR soliton effects are non-perturbative in g, whereas an expansion in g_4 should yield results equivalent to those of the usual perturbative heterotic string.

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