RECENT DEVELOPMENTS IN FERMIONIZATION AND SUPERSTRING MODEL BUILDING

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ABSTRACT

We discuss chiral fermionization as a method for exploring new conformal field theory solutions to string theory. An extension of conventional free fermionic representation theory by twist field realizations of current algebras is described. New exact solutions to heterotic string theory with (4,0) supersymmetry are presented.

1. Introduction

Chiral fermionization [2][3] is a useful technique to discover new classes of conformal field theory (cft) solutions to string theory. There are many overlapping approaches to studying the vacuum configurations of string theory, each with its own particular advantages. The fermionic construction has the capability to sample new moduli spaces at orbifold, or otherwise special, points where the cft is exactly solvable and also has an equivalent fermionic realization. An important benefit for model building is that it enables one to systematically find exact solutions embedding *specified low-energy matter content and couplings* in a calculable framework.

However, the fermionic description only captures isolated pieces of a string moduli space requiring that we infer the structure of the moduli space from the detailed knowledge of the theory at a collection of "points", generally, of higher symmetry. It must therefore be combined with spacetime techniques in studying low-energy string theory. This is also true of other exact (all orders in α') cft descriptions with which the fermionic construction has solutions in common. These include the Gepner construction based on the tensor product of minimal models and the simple current construction of Schellekens and Yankeilowicz. There has been spectacular progress in developing spacetime techniques for studying (2,2) models and also certain classes of $(0,2)$ models. Combining these complementary approaches is likely to give in the future a broader view of low energy string theory than is presently known.

My work on developing fermionization techniques for superstring model building was done in collaboration with Stephen Chung, George Hockney, and Joseph Lykken, further details of which can be found in [3]. New results on $(0,4)$ models presented in the talk have since appeared in the literature[6]. It should be noted that the heterotic light-cone construction described here can be easily modified to find solutions to a theory based on a different world-sheet superconformal algebra.

2. The Fermionic Construction

The fermionic construction of exact conformal field theory solutions is based on current algebras, including both the untwisted and also the twisted affine Lie algebras. The familiar free fermionic construction[1] is restricted to level one current algebra realizations where all currents are realized as Majorana fermion bilinears. Prior to our work, the extension to twist field current algebra realizations was largely unexplored and unfamiliar, even to experts in fermionization. The ability to recognize and combine both classes of fermionic current algebra realizations has enabled us to find many new and exact solutions to heterotic string theory, and in a truly heterotic framework.

The earliest examples of such solutions were obtained in the work of Kawai, Lewellen, Schwartz and Tye[2]. In this work, it was noted that there appeared to be more solutions for modular invariant one-loop partition functions in the fermionic construction, than those that could be accounted for by free field realizations alone. In abstract $(2,0)$ cft constructions a one-loop modular invariant partition function does not in itself define a solution to string theory. It is important to obtain a consistent string vertex operator algebra that is *associative*, so that couplings are well-defined. The issue of which of the solutions in the KLST construction have this property was addressed by us in [3], using techniques from rational conformal field theory. This enables us to use the exactly solvable fermionic description to construct the entire superpotential, including non-renormalizable terms, for any given solution.

To obtain models with generic gauge groups and generic matter content in the fermionic construction we have introduced several new features in the underlying fermionic representation theory. In a modular invariant cft solution to string theory the individual Majorana-Weyl fermions are of three species: paired into right-moving Weyl fermions or left-right Ising fermions, or members of a block of chiral Ising fermions. The choices of spin structure for blocks of chiral Ising fermions consistent with associativity of the cft were analysed in [3]. If all of the chiral Ising fermions in a block are left-moving, this corresponds to a holomorphic cft of central charge c_m=8, 12, 14, 16, 18, 20, or 22. Such holomorphic cfts can be tensored together with holomorphic Weyl fermion cfts to build $(4,0)$ models with moduli spaces of reduced dimension, as we will see in the next section.

The case of N=1 supersymmetry is far richer and more interesting. In an N=1 model, the block of chiral Ising fermions can be split among n_L left-moving and n_R right-moving fermions, such that $n_L+n_R=2c_m$ takes one of the allowed values listed above[3]. This class of $(2,0)$ solutions includes many new possibilities for three generation models, and many new embeddings of the standard model particle content with both higher level and level one realizations, and with varying hypercharge normalization. These models will be explored elsewhere, as will the generic presence of a tree-level anomalous $U(1)$. These models are the first known examples of genuinely heterotic modular invariants based on tensor products of holomorphic cfts which are *not* free fields.

In this talk, we consider fermionic realizations where all of the right-moving world-sheet fermions are Majorana-Weyl, with periodic or anti-periodic boundary conditions alone. We use the conventional spin-$\frac{3}{2}$ generator of the $(1,0)$ world-sheet supersymmetry

$$T_F(\bar{z}) \;=\; i\sum_{\mu=1}^{2}\psi_\mu\partial_{\bar{z}}X^\mu \;+\; i\sum_{k=1}^{6}\psi_{3k}\psi_{3k+1}\psi_{3k+2} \tag{1}$$

where the index μ=1, 2 sums over the two transverse dimensions in D=4, and we work in light-cone gauge[1]. The N=1 spacetime supersymmetry charges are embedded in the spin structure of eight right-moving fermions, which are paired into four Weyl fermions as follows, $\psi_1 + i\psi_2$, $\psi_{3k} + \psi_{3k+3}$, k=1, 3, 5. The remaining 12 right-movers can be Weyl, Ising, or chiral Ising fermions. Left-moving Weyl fermions are unrestricted by world-sheet supersymmetry and are allowed to satisfy any rational boundary condition.

To enlarge the scope of conventional free fermionization we make two modifications. We allow overlapping embeddings of the current algebra weights into fermionic charges, Q_F^i, where i labels individual Weyl fermions, and G and G' are commuting current algebras,

$$w_G^i + w_{G'}^i \;=\; Q_F^i \tag{2}$$

Thus, in many of our conformal field theory solutions the group weights of the hidden and the visible gauge groups actually overlap! This has no bearing on spacetime physics or equivalently on the conformal field theory, but is simply a trick that allows a free fermionic representation for many new modular invariant partition functions. In addition to conventional fermion bilinear currents we also consider twist field realizations. These are obtained by tensoring together 4, 8, 12, or 16 dimension $\frac{1}{16}$ spin operators with a free field operator so as to give holomorphic operators of dimension $(0,1)$, for example,

$$J_{ijkl}(z) \;=\; j_{free}(z)\left(\sigma_i^+\sigma_j^+\sigma_k^+\sigma_l^+ \;+\; \sigma_i^-\sigma_j^-\sigma_k^-\sigma_l^-\right) \tag{3}$$

where $i{\neq}j{\neq}k{\neq}l$. In fact, it is often the case that the currents in the Cartan subalgebra are not of the conventional Neveu-Schwarz fermion bilinear form. Enlarging the class of allowed embeddings considerably reduces the ad-hoc restrictions on groups/weights obtained in conventional free fermionic solutions [3].

3. N=4 Models of reduced rank

It is helpful to begin a general study of new cft solutions by considering cases with extended spacetime supersymmetry. We therefore consider the possibility of exact solutions to string theory beyond those obtained by dimensional reduction from a ten-dimensional superstring. Toroidal compactification of the ten-dimensional N=1 heterotic string to six (four) dimensions results in a low-energy effective N=2 (N=4) supergravity coupled to 20 (22) abelian vector multiplets, giving a total of 24 (28) abelian vector gauge fields with gauge group $(U(1))^{24}$ ($(U(1))^{28}$), respectively. Four (six) of these abelian multiplets are contained within the N=2 (N=4) supergravity multiplets. At enhanced symmetry points in the moduli space the abelian group $(U(1))^{20}$ ($(U(1))^{22}$) is enlarged to a simply-laced group of rank 20 (22). The low energy field theory limit of such a solution has maximally extended spacetime supersymmetry. Since all of the elementary scalars appear in the adjoint representation of the gauge group, symmetry breaking via the Higgs mechanism is only adequate in describing the moduli space of vacua with a *fixed* number of abelian multiplets.

We will show that there exist maximally supersymmetric vacua with four-dimensional Lorentz invariance that are not obtained by toroidal compactification of a ten-dimensional heterotic string. The total number of abelian vector multiplets in the four-dimensional theory can be reduced to just *six*, namely, those contained within the N=4 supersymmetry algebra. This is consistent with known theorems on the world-sheet realizations of extended spacetime supersymmetry in string theory[4]. In the world-sheet description of an N=4 supersymmetric solution of the heterotic string in four dimensions, the internal right moving superconformal field theory of central charge c_R=9 is required to be composed of nine free bosons. A reduction of the rank of the low-energy gauge group in an N=4 solution implies that the internal left-moving conformal field theory of central charge c_L=22 is not entirely composed of free bosons. This is unlike the 4D toroidal compactifications described by Narain [5] where *both* right and left moving conformal field theories are free boson theories.

As a consequence, it will also be possible to realize non-simply-laced gauge symmetry consistent with the maximally extended supersymmetries. We will construct such solutions using the fermionization methods described above. They are examples of $(4,0)$ rational superconformal field theories, where the underlying chiral algebras also have a world-sheet fermionic realization. In order to have an unambiguous identification of the vertex operator algebra in this construction, it is essential to have explicit knowledge of the correlators of the real fermion conformal field theories [3].

Eliminating longitudinal and time-like modes, the number of transverse degrees of freedom describing a vacuum with D-dimensional Lorentz invariance is (c_R, c_L)=$(\frac{3}{2}\cdot(D-2), D-2)$+ (c_R^{int}, c_L^{int}). In this class of exact solutions, the internal degrees of freedom have an *equivalent* world-sheet fermionic realization with $(3\cdot(10-D), 2\cdot(26-D))$ Majorana-Weyl fermions. string theory which embeds *a specified low-energy matter content.*

We restrict ourselves to fermionic realizations where the world-sheet fermions are Majorana-Weyl, with periodic or anti-periodic boundary conditions only. All of the right-moving world-sheet fermions will be paired into Weyl fermions, or equivalently free bosons, as required by the extended spacetime supersymmetry. A free boson conformal field theory implies, with no loss of generality, the existence of an abelian current in the right-moving superconformal field theory. In maximally supersymmetric solutions the allowed right-moving chiral algebras are, therefore, restricted to level one simply-laced affine Lie algebras [5] [4]. This follows from the fact that for a Lie algebra with roots of equal length, the central charge of the level one realization also equals the rank of the algebra, i.e., the number of abelian currents.

A free fermionic realization with n Weyl (complex) fermions exists for any of the following affine Lie algebras: $SO(2n)$, $U(n)$, and E_8 (for n=8), in addition to the abelian algebra $(U(1))^n$. In toroidal compactifications that have an equivalent free fermionic realization these properties also extend to the allowed left-moving chiral algebras and, hence, to the observed non-abelian gauge symmetry in these solutions.

Incorporating *chiral* Majorana fermion world-sheet fields in the left-moving internal conformal field theory will enable us to construct maximally supersymmetric solutions that embed non-simply-laced gauge symmetry, i.e., gauge groups with roots of unequal length. Such solutions necessarily lie in a moduli space where the gauge group has rank < 28. This is evident from the formula for the central charge of an affine Lie algebra:

$$c \;=\; \frac{k\ Dim(G)}{k+\tilde{h}} \tag{4}$$

where the dual Coxeter number, $\tilde{h}$, of the non-simply-laced algebras, $SO(2n+1)$, $Sp(2n)$, G_2 and F_4 are, respectively, $2n-1$, $n+1$, 4, and 9. Note that the dimension of the dual algebras $SO(2n{+}1)$ and $Sp(2n)$ are identical, given by $Dim(G){=}n(2n{+}1)$. However, unlike the simply-laced algebras, the central charge does not equal the rank of the group even at level k=1, and does not, in fact, coincide for the algebra and its dual. Chiral Majorana fermion realizations exist for all of the non-simply-laced affine algebras. Extending a world-sheet fermionic realization of the generators of the affine algebra to a $(4,0)$ superconformal field theory that is an exact solution to heterotic string theory, however, requires consistency with modular invariance of the one-loop vacuum amplitude and with world-sheet supersymmetry [3]. These conditions can be quite restrictive and, in fact, preclude N=1 supersymmetric solutions in ten spacetime dimensions with non-simply-laced gauge symmetry.

Now consider the possibility of non-simply-laced gauge symmetry in D=4. For example, an affine realization of the rank ten algebra $Sp(20)$ at level one requires central charge $c{=}\frac{35}{2}$. Appending nine real fermions with $c{=}\frac{1}{2}$, which form a realization of the non-simply-laced algebra $SO(9)$, gives c=22, making this a plausible candidate for the gauge group of an N=4 spacetime supersymmetric solution in D=4. It is not difficult to verify the existence of such a solution using its fermionic realization.

We will adopt the notation of [2][3]. The tree level spectrum is described by the one-loop vacuum amplitude, which sums over sectors labelled by the associated spin structure of the world-sheet fermions. The N=4 spacetime supersymmetry charges are embedded in the spin-structure of eight right-moving Majorana-Weyl fermions, which we will label ψ_μ, μ=1, 2, and ψ^I, I=3k, k=1, $\cdots$, 6. The first two right-movers carry a (transverse) spacetime index. In sectors contributing spacetime bosonic and fermionic components of an N=4 supermultiplet, these eight fermions are, respectively, Neveu-Schwarz and Ramond. In particular the *untwisted* sector, $\mathcal{U}$, in which all of the world-sheet fermions are Neveu-Schwarz, contributes the bosonic components of the N=4 supergravity multiplet in four dimensions. It also contributes six massless abelian multiplets, each associated with an internal right-moving Weyl fermion: $\psi_{3k+1} + i\psi_{3k+4}$, $\psi_{3k+2} + i\psi_{3k+5}$, with $k1, 3, 5$.

The remaining massless spectrum is arranged into D=4 N=4 Yang-Mills supermultiplets, each containing 6 spacetime vector components, 8 spinor components, and 2 scalar components. All of the gauge bosons of $SO(9)$ are contributed by the untwisted sector and correspond to fermion bilinear pairs. The sector-wise decomposition of the 210 states in the adjoint representation of $Sp(20)$ is most easily described by the regular embedding:

$$Sp(20) \ \supset \ (SO(4))^5 \ \sim \ (SU(2))^{10} \tag{5}$$

The untwisted sector, $\mathcal{U}$, contributes states corresponding to all 30 long roots, and a subset (20) of the short roots of $Sp(20)$. These states transform, respectively, in the adjoint (10 copies of a $\mathbf{3}$) and the spinor (10 copies of a doublet) representation of its $(SU(2))^{10}$ sub-group. The states are identified by fermionic charge: the roots and weights of the rank ten sub-group are embedded in the fermionic charge of ten *Weyl* fermions. In the fermionic construction these are obtained by pairing 20 Majorana-Weyl left-movers, $\psi_{2l+1}(z) + i\psi_{2l+2}(z)$=$\lambda_l(z)$, l=0, $\cdots$ 9.

The remaining 16 left-moving Majorana-Weyl fermions are unpaired fermions. The vertex operator construction for an $SO(2n+1)$ algebra requires a single unpaired Majorana fermion, in addition to n Weyl fermions. The long-root lattice of $SO(2n+1)$ coincides with the root-lattice of $SO(2n)$, $\mathbf{\Lambda_L}(B_n)$=D_n. Thus the $n{\cdot}(2n{-}1)$ Majorana-Weyl fermion bilinears are the currents corresponding to long roots, while those corresponding to the short roots are the $2n$ bilinears containing the single real fermion. In this example, of the $\frac{20\cdot 19}{2}$ Neveu-Schwarz fermion bilinear currents contributed by the untwisted sector only $\frac{5\cdot 5\cdot 4}{2}$ remain after GSO projection from four *twisted* sectors, $\mathcal{T}_1 \cdots \mathcal{T}_4$, in which some of the fermions are Ramond. The untwisted sector therefore contributes a total of 400 states: the eight bosonic components of an N=1 supermultiplet transforming in the adjoint representation of the non-simply-laced group $(SO(5))^5$.

Extension of this vertex operator construction to a symplectic current algebra requires conformal dimension (h_R, h_L)=$(0, 1)$ operators corresponding to the additional short roots. These are contributed by the twisted sectors. The currents are composite operators constructed out of sixteen *twist* fields, i.e., dimension $(0, \frac{1}{16})$ operators in the Majorana-Weyl fermion field theory.

The twisted sectors, $\mathcal{T}_i$, were chosen so as to generate the necessary projection on the untwisted sector. They will simultaenously determine the internal right-moving chiral algebra: in this solution, the twelve (internal) Majorana fermions are divided into blocks of four, either all Neveu-Schwarz, or all Ramond, in every sector of the Hilbert space. Thus the underlying right-moving chiral algebra is $(SO(4))^3$. Possible twists are, of course, subject to constraints from modular invariance and world-sheet supersymmetry. Given a set of valid $\mathcal{T}_i$, modular invariance of the one-loop vacuum amplitude automatically generates additional twisted sectors in the Hilbert space. Thus, in this example, the $\mathcal{T}_i+\mathcal{T}_j$, $i\neq j$, also contribute massless states in the spectrum. Each of the ten twisted sectors contributes 128 states: 8 bosonic components of an N=4 supermultiplet transforming in the 16 dimensional spinor representation of an $(SU(2))^4$ sub-group. $Sp(20)$ has ten distinct $(SU(2))^4$ sub-groups, each corresponding to a different twisted sector. Combining the 400 untwisted sector states with these 1280 states gives all 8·210 bosonic components of an N=4 supermultiplet transforming in the adjoint representation of $Sp(20)$.

It is straightforward to construct the twisted sector vertex operator corresponding to a given weight. We will use the bosonic realization for the corresponding free field vertex operator. A state transforming as a spinor weight, α, of $(SU(2))^4$ corresponds to a dimension $(0,\frac{1}{2})$ operator, $j_{\rm free}(z)$, obtained by bosonization:

$$\lambda_l^\dagger\lambda_l \ \leftrightarrow\ \partial\phi_l \qquad j_{\rm free}(z)\ =\ \hat{C}(\alpha)\ e^{i\alpha\cdot\phi} \tag{6}$$

where $\alpha\cdot\alpha{=}1$, $l{=}0,\ \cdots\ 9$, and the $\hat{C}(\alpha)$ are suitable cocycle operators. This free field vertex operator must be dressed by four pseudo-Weyl fermion spin fields, $\sigma_l^{\pm}$, $l\,=\,1,\ \cdots\ 4$, so as to give a current. These spin fields are identified by pseudo-complexifying, i.e., pairing, the real fermions in a twisted sector [3]. Thus:

$$J_{ijkl}(z)=j_{free}(z)\ \left(\sigma_i^+\sigma_j^+\sigma_k^+\sigma_l^+\ +\ \sigma_i^-\sigma_j^-\sigma_k^-\sigma_l^-\right) \tag{7}$$

where $i\neq j\neq k\neq l$, giving a dimension $(0,1)$ twisted sector current. Verification of the vertex operator algebra for $Sp(20)$ is now straightforward.

This completes the discussion of the massless spectrum of an exactly solvable heterotic N=4 enhanced symmetry point with gauge group $Sp(20){\times}SO(9)$ in four dimensions. The string theory it belongs to has only *twenty* abelian vector multiplets, or rank $(6,14)$, at generic points in the moduli space. We have constructed fermionic realizations of a large range of four-dimensional N=4 supersymmetric solutions to the heterotic string with semi-simple groups of varying rank, containing both simply-laced and non-simply-laced factors, and with part or all of the gauge symmetry realized at higher level. It should be stressed that four-dimensional N=4 supersymmetry need not arise via toroidal compactification from a higher dimensional theory. The clearest evidence for this is the existence of an N=4 four-dimensional solution where the gauge symmetry is reduced to the minimum consistent with the world-sheet supersymmetry constraints. Its fermionic realization uses a spin structure block of 44 left-moving real fermions. The number of abelian vector multiplets in this N=4 theory is just *six*.

4. Conclusions

The development of fermionization techniques has enabled the systematic sampling of new classes of exact solutions to string theory. It is important to focus on those aspects of the solutions that have generic implications for our understanding of string theory. Consider, for example, our experience with $(4,0)$ solutions as discussed above. The particular choices of affine Lie group, rank, or Kac-Moody level, obtained in the fermionic solutions should not be emphasized. On the other hand the existence of maximally supersymmetric theories with distinct target space duality groups, and the fact that non-simply-laced and simply-laced gauge groups enter on an equal footing, are generic observations relevant for further study.

Similar considerations will apply when we begin a systematic exploration of $(2,0)$ solutions. Recent interest in $(2,0)$ constructions is in part due to the difficulty in obtaining low numbers of generation-anti-generation pairs in the simpler class of $(2,2)$ compactifications. Despite technical developments, the few examples of semi-realistic models available to the string phenomenologist have remained those obtained a while ago in the Z_3 orbifold and free fermionic constructions. A generic feature of such models is the presence of extra low-energy matter, at least some of which survives to accelerator energies. It is intriguing that every semi-realistic example known to date also has a tree-level anomalous $U(1)$. However, it is not known whether these are essential features of a semi-realistic heterotic vacuum in which all of the MSSM particle content is assumed to come from weak-coupling string states.

For meaningful string phenomenology, it is essential therefore to begin with a broad sample of semi-realistic three generation $(0,2)$ models. Although a study of orbifold, or otherwise special, points of $(0,2)$ theories is of limited scope in addressing many fundamental questions in superstring phenomenology, such as vacuum selection and the vanishing of the cosmological constant, it appears worthwhile to enlarge the sample of three generation models. Such examples continue to serve as useful pedagogical models for weak coupling string phenomenology. We will address this issue in forthcoming work.

5. References

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