ON THE IMPOSSIBILITY OF A LARGE RADIUS COMPACTIFICATION IN REALISTIC STRING THEORIES ⋆

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ABSTRACT

We show the existence of a $\frac{1}{M_c^4}(\frac{M_s}{M_c})^2 F_{\mu\nu}^4$ term in the effective action when all the compactification radii are taken to be large and equal. The magnitude of this term implies the breakdown of perturbation theory below the Kaluza-Klein scale, when the process is four-dimensional. Our calculations are done both in string theory and field theory.

Several years ago [1] it was shown that in theories with sensible four dimensional amplitudes, the size of the compactification manifold is related to the string tension. It was found that the four dimensional gauge coupling satisfies:

$$\alpha_4{}^{1/6} \leq O(\frac{M_c}{M_s}) \sim O(1) \tag{1}$$

Therefore, the higher dimensional operators resulting from integrating heavy string modes become relevant and we can no longer analyze the physics of spontaneous compactification in terms of ten-dimensional $N = 1$ supergravity.

The same conclusion can be reached using a four dimensional perturbative argument. Every state from the higher-dimensional theory gives rise to an infinite tower of states in four dimensions.These are the Kaluza-Klein states and their masses are quantized in units of $1/R$. Before the orbifold projection, the spectrum is arranged in $N = 4$ multiplets. In background gauge the cancelations due to supersymmetry are transparent. The two and three point functions identically vanish .However,the

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vector four-point function can have a non-zero value. Indeed, if we consider a box diagram with external gauge bosons we find that it generates an $F^4_{\mu\nu}$ term in the effective action :

$$\mathcal{L}_{\rm box} = c\frac{1}{M_{\rm c}^4}(\frac{M_{\rm s}^2}{M_{\rm c}^2})F^4_{\mu\nu} \tag{2}$$

which makes perturbation theory meaningless unless

$$M_{\rm s} \sim M_{\rm c} \tag{3}$$

Recently [2] it was suggested that (3) does not hold in some orbifold models. It is known that only the $N = 2$ and $N = 4$ sectors give radius dependent contribution to the β function and to the threshold corrections. When summing over fermionic spin structures, the contributions from the $N = 4$ sector vanishes [3]. Assuming that this cancelation is complete, *i.e* holds for all one-particle irreducible amplitudes, and considering orbifolds without $N = 2$ sectors, it could be possible to have a large radius of compactification. The lower limits for R would be given only from phenomenological considerations [4], [5]. We argue that the cancelation does not hold for higher order point functions. In [6] we studied the ten-dimensional $N = 1$ supersymmetric theory using second order formalism for fermions [7] and background-Feynman gauge for bosons. We show that the gauge bosons couple to fermions through a scalar-like coupling and the only non-zero graphs are the ones with second order gauginos running around the loop. This yields:

$$\mathcal{L} = q_4\ ({F_{\mu\nu}}^a + i\widetilde{F}^a_{\mu\nu})^2({F_{\mu\nu}}^b - i\widetilde{F}^b_{\mu\nu})^2 \tag{4}$$

where q_4 is,

$$q_4 = c\sum_{\vec{M}=0}^{M_{\rm s}}{\rm Tr}(\frac{1}{\vec{M}^4}) = c\frac{M_{\rm s}^2}{M_{\rm c}^6} \tag{5}$$

in agreement with equation (2). Notice that the correct dimensionless expansion parameter is $q_4 g_4^4 E^4$ in four dimensions and $q_{10} g_{10}^4 E^{10}$ in ten dimensions. This fact implies that for energies well below the Kaluza-Klein scale $\left(E^4 > \frac{M_{\rm c}^4}{g_4^4}(\frac{M_{\rm c}^2}{M_{\rm s}^2})\right)$ the perturbative approach becomes meaningless. Unfortunately, the above argument is not rigorous since it disregards the possibility of having ten-dimensional terms such that:

$$q_4 = \frac{q_{10}}{M_{\rm c}^6} + c\frac{M_{\rm s}^2}{M_{\rm c}^6} \tag{6}$$

The natural magnitude of q_{10} cannot be constrained by four dimensional considerations and we would have to resort to the ten-dimensional theory . However, no ten dimensional argument can address the issue of how well defined a field theory is in the presence of singularities. Instead we evaluate q_4 directly from string theory. The details of our calculations are given in [6]. We consider a four-point one-loop heterotic string amplitude with external neutral gauge bosons. Using holomorphicity and modular invariance we show that, after subtracting the graviton amplitude, the large radii behavior of the amplitude is:

$$\mathcal{A}_4 \approx M_{\mathrm{s}}^2 R^6 + O(R^4) + \dots \tag{7}$$

which is in complete agreement with the field theory result.

The situation with four or less large radii requires analysis of perturbative consistency above the Kaluza-Klein scale. In the light of [1] and our results, we consider this scenario to be unlikely.

REFERENCES

1. V. Kaplunovsky, *Phys. Rev. Lett.* **55** (1985), 1036.
2. I. Antoniadis, *Phys. Lett.* **246B** (1990), 377.
3. V. Kaplunovsky, *Nucl. Phys.* **307B** (1988), 145.
4. I. Antoniadis, K. Benakli HEP-PH 9403290.
5. I. Antoniadis, C. Muñoz, M. Quirós, *Nucl. Phys.* **397B** (1993), 515.
6. E. Cáceres, V. Kaplunovsky, M. Mandelberg, to appear.
7. J.L. Cortes, J. Gamboa, L. Velazquez, *Phys.Lett.* **313B** (1993), 108.