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BOUNDARY EFFECTS IN STRING THEORY *

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ABSTRACT

Some of the properties of string theory defined on world-sheets with boundaries are reviewed. Particular emphasis is put on the possibility of identifying string configurations ('D-instantons' and 'D-branes') that give rise to stringy non-perturbative effects.

The perturbative expansion of closed-string theories is defined as a sum over closed orientable Riemann surfaces. Type 2a, 2b and heterotic superstring theories are of this type while type 1 superstring theory has a perturbation expansion that involves a sum over world-sheets with boundaries. The presence of boundaries leads to qualitatively new effects. With conventional Neumann boundary conditions there are open-string states as well as closed-string states in the spectrum – the boundaries represent the trajectories of the string end-points. Upon compactification to lower dimensions T-duality changes this to a theory with Dirichlet boundary conditions in the compactified dimensions[1,2]. The most drastic modification arises in the theory defined by summing over world-sheets with boundaries on which the string space-time coordinates satisfy constant Dirichlet conditions in *all* space-time dimensions – in that case the entire boundary is mapped to a point in the target space-time and the position of that point is then integrated, which restores target-space translation invariance (ref.2 and references therein). The result is a theory that describes closed strings which possess dynamical point-like substructure as is indicated by the fact that fixed-angle scattering is power behaved as a function of energy. Recently an interesting variation of this scheme has been suggested[3] (based on ref.1), involving the idea of 'D-instantons'. As the name suggests, these are world-sheet configurations which correspond to target space-time 'events', giving rise to exponentially suppressed contributions to scattering amplitudes behaving as $e^{-C/\kappa}$ (where κ is the closed-string coupling constant that is determined by the dilaton expectation value and C is a constant). This is in accord with general observations based on studying the convergence of string perturbation theory[4] that suggest that whereas instanton effects in field theory typically behave as $e^{-const./\kappa^2}$, analogous effects in closed-string theory should behave as $e^{-C/\kappa}$. Such effects may be of significance in the study of

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non-perturbative phenomena in superstring theory, particularly in connection with the rôle of BPS exact soliton states in U duality (see the talks by C.M. Hull and E. Witten at this conference).

A novel feature of the sum over world-sheets with fixed Dirichlet boundaries is the presence of momentum-independent divergences arising from boundaries of moduli space at which open-string strips degenerate[2]. It was argued[3] that these divergences cancel when the sum over world-sheets is arranged with the combinatorics appropriate to a D-instanton. I will demonstrate how a single D-instanton contribution can be expressed as an exponential of an instanton 'action' and a 'gas' of such instantons can be defined. This will clarify the cancellation of the Dirichlet divergences (details of this argument were published in ref.5 subsequent to the conference). We will see that the leading contribution to the free energy comes from free D-instantons and is of order κ^{-1} but corrections due to long-distance interactions between D-instantons will be seen to be of order κ^0.

A general oriented string world-sheet has an arbitrary number of boundaries and handles. With Dirichlet boundary conditions each boundary is fixed at a space-time point, y^μ_B (where B labels the boundary), which is then integrated. The boundaries of moduli space are of the following types. Firstly, there are the usual degenerations of cylindrical segments that correspond to the propagation of physical closed-string states:

- (a) Degeneration of handles.
- (b) Degeneration of trivial homology cycles that divide a world-sheet into two disconnected pieces.

In addition, in the presence of boundaries there are degenerations of the following kinds:

- (c) A boundary may shrink to zero length giving the singularities associated with closed-string scalar states coupling to the vacuum through the boundary.
- (d) Degeneration of strips forming open-string loops. If both string endpoints are fixed at the same target-space point this gives an infinite contribution.
- (e) Degeneration of trivial open-string channels, in which the world-sheet divides into two disconnected pieces. In the case of Dirichlet conditions the intermediate open string necessarily has both end-points fixed at the same space-time point leading to another infinite contribution.

The BRST cohomology of the states of the open-string sector where the string end-points are fixed at y^μ_1 and y^μ_2 is isomorphic to that of the usual Neumann open string with momentum $p^\mu = \Delta^\mu \equiv y^\mu_2 - y^\mu_1$. This may be viewed as a simple consequence of target-space duality and it means that the arbitrary diagram possesses a rich spectrum of space-time singularities, just as the usual loop diagrams possess a rich momentum-space singularity structure. However, it is important that the wave functions of these states depend on the mean position, $y^\mu = (y_1^\mu + y_2^\mu)/2$, in addition to Δ^μ – this extra variable has no analogue for the usual open strings. The intermediate open string in the trivial degeneration (e) has $\Delta = 0$ so its cohomology is isomorphic to that of the usual Neumann open-string theory when $p^\mu = 0$. There is only one physical state in this case, which is the level-one vector. This is the isolated zero-momentum physical state with a constant wave function in the usual theory. However, in the Dirichlet theory its wave function $\zeta^\mu(y)$ is an arbitrary function that is physical without the need to impose any constraints on it – it is a target-space Lagrange multiplier field. The presence of this as an intermediate state in a string diagram leads to a divergence (the propagator for the level-one state is singular since a Lagrange multiplier field has no kinetic terms). The vertex operator that describes the coupling of this level-one vector state to a boundary is given by

$$g \oint d\sigma_B \zeta \cdot \partial_n X(\sigma_B, \tau_B) = ig\zeta^\mu \frac{\partial}{\partial y_B^\mu} \tag{1}$$

where n denotes the derivative normal to the boundary that is fixed at the target-space position y_B and τ_B is its world-sheet position (g is the open-string coupling constant that is proportional to $\sqrt{\kappa}$).

There are two schemes for dealing with this level-one divergence. In one of these the Lagrange multiplier field is eliminated by integrating it, thereby imposing a constraint before the perturbation expansion of the theory is considered. Some consequences of the presence of this constraint were discussed in ref.6. The other scheme[3] uses different combinatorics for the sum over boundaries, giving rise to D-instantons. In this case the divergences due to the level-one open-string field are supposed to cancel between an infinite number of diagrams as will be demonstrated below.

The construction of the partition function for a D-instanton gas will involve the exponentiation of a particular combination of contributions of connected world-sheets. Expanding this exponential therefore gives contributions from world-sheets that are disconnected (a concept that is not familiar in conventional discussions of string perturbation theory). We begin by obtaining an equation for the contribution of the particular sum of connected (orientable) world-sheets with Dirichlet boundaries that will later be exponentiated.

Consider a connected orientable world-sheet with p_i boundaries fixed at any one of a finite number of points y_i (where $i = 1, \ldots, n$ and $p_i = 0, \ldots, \infty$). The string free energy, $f_{p_1,p_2,\ldots,p_n}(y_1, y_2, \ldots, y_n)$, is given by the usual multi-dimensional integral over the moduli space of the surface which has a total of $\sum_i p_i$ boundaries. We shall be interested in the sum over surfaces with all possible numbers of boundaries for a given value of n,

$$S^{(n)} = \sum_{p_1,p_2,\ldots,p_n=0}^{\infty} \frac{1}{p_1!p_2!\ldots p_n!} f_{p_1,p_2,\ldots,p_n}(y_1,y_2,\ldots,y_n), \tag{2}$$

where the explicit combinatorial factor accounts for the symmetry under the interchange of identical boundaries – boundaries that are fixed at the same space-time point. The definition of $f_{p_1,\ldots,p_n}$ implicitly contains a sum over handles and the term with all $p_i = 0$ in Eq. (2) is just the usual closed-string free energy, $S^{(0)}$. Both types of open-string degenerations, (d) and (e), described earlier lead to divergences in Eq. (2) due to the intermediate level-one open-string states and in each case the coefficient of the divergent term is proportional to the product of level-one vertex operators attached to the boundary at either end of the degenerating strip. It is sufficient to consider single degenerations since the multiple degenerations are a subspace of these. The divergences of interest have the form $\int_\epsilon^1 dq/q = \ln\epsilon$ where ϵ is a world-sheet regulator. Degenerations of type (e) divide the world-sheet into two factors, so that if the degenerating boundary is fixed at y_1^μ the singular term has the form,

$$\begin{aligned} &\sum_{p_1,p_2,\ldots,p_n;q_1,q_2,\ldots,q_n} \frac{f_{p_1+q_1+1,p_2+q_2,\ldots,p_n+q_n}}{(p_1+q_1+1)!(p_2+q_2)!\ldots(p_n+q_n)!} \sim \\ &\ln\epsilon \left(\frac{\partial}{\partial y_1^\mu} \sum_{p_1,p_2,\ldots,p_n} \frac{f_{p_1,p_2,\ldots,p_n}}{p_1!p_2!\ldots p_n!}\right)\left(\frac{\partial}{\partial y_{1\mu}} \sum_{q_1,q_2,\ldots,q_n} \frac{f_{q_1,q_2,\ldots,q_n}}{q_1!q_2!\ldots q_n!}\right), \end{aligned} \tag{3}$$

where the derivatives arise from two level-one open-string vertex operators attached to two different boundaries. A divergence also arises when an internal open-string with both ends fixed at the same point degenerates (this is a degeneration of type (d)). In this case the coefficient of the divergence is proportional to two vertex operators of the level-one state attached to the same boundary giving,

$$\begin{aligned} &\sum_{p_1,p_2,\ldots,p_n;q_1,q_2,\ldots,q_n} \frac{f_{p_1+q_1+1,p_2+q_2,\ldots,p_n+q_n}}{(p_1+q_1+1)!(p_2+q_2)!\ldots(p_n+q_n)!} \sim \\ &\ln\epsilon \frac{\partial^2}{\partial y_1^2} \sum_{p_1+q_1,p_2+q_2,\ldots,p_n+q_n} \frac{f_{p_1+q_1,p_2+q_2,\ldots,p_n+q_n}}{(p_1+q_1)!(p_2+q_2)!\ldots(p_n+q_n)!}. \end{aligned} \tag{4}$$

Combining Eqs. (3) and (4) and taking a derivative of the free energy with respect to ϵ extracts the dependence on the divergent degenerations,

$$\epsilon\frac{\partial}{\partial\epsilon} S^{(n)} = \left(\frac{\partial}{\partial y_1^\mu} S^{(n)}\right)\left(\frac{\partial}{\partial y_{1\mu}} S^{(n)}\right) + \frac{\partial^2}{\partial y_1^2} S^{(n)}. \tag{5}$$

The rules for constructing the string partition function in the presence of a single D-instanton[3] were presented as an infinite series of terms as follows. Firstly, sum over world-sheets with insertions of any number of handles and Dirichlet boundaries that are all at the *same point* in the target space, y_1^μ, which is to be integrated over. The sum is now taken to include *disconnected* world-sheets although these do not appear to be disconnected from the point of view of the target space since the boundaries all touch the same point. A suitable factor is to be included to take account of symmetry under the interchange of identical disconnected world-sheets. This series of terms can be expressed as an exponential in terms of the connected world-sheets that were introduced earlier giving the one D-instanton partition function,

$$Z^{(1)} = \int d^D y e^{S^{(1)}(y)}. \tag{6}$$

The exponent $S^{(1)}(y) = S^{(0)} + \sum_{p=1}^{\infty} f_p(y)/p!$ is now interpreted as the one D-instanton 'action' that is given by the $n = 1$ term in Eq. (2). Scattering amplitudes may be generated from this expression if $S^{(1)}$ is taken to be a functional of the background fields.

The requirement of consistent clustering properties in the target space (as well as on the world-sheet) motivates the following generalization that includes the sum over an arbitrary number of D-instantons (and which should presumably be equivalent to the rather schematic generalization motivated by duality[3]). This involves summing over insertions of boundaries at any number of positions, y_i, that are to be integrated. The partition function is given in the language of a conventional instanton gas by the expression,

$$Z = \sum_n \frac{1}{n!} \left(\prod_{i=1}^{n} d^D y_i^\mu \right) e^{S^{(n)}(y_1,\ldots,y_n)}, \tag{7}$$

where $S^{(n)}$ is given by Eq. (2) and is now interpreted as the action for n interacting D-instantons. Recall that $S^{(n)}$ is defined by a functional integral over connected world-sheets and includes a term with no boundaries which is equal to $S^{(0)}$, the usual closed-string free energy. The expression for Z therefore has the form,

$$Z = e^{S^{(0)}} \sum_n \frac{1}{n!} \left(\prod_{i=1}^{n} d^D y_i^\mu \right) e^{S^{(n)\prime}(y_1,\ldots,y_n)}, \tag{8}$$

where $S^{(n)\prime}$ is defined to be $S^{(n)}$ with the zero-boundary term missing. In the general term in the sum any boundary may be located at any one of the n target-space positions, y_i^μ, which are analogous to the collective coordinates describing the positions of instantons in quantum field theory. It is convenient to decompose $S^{(n)\prime}$ into those terms in which all boundaries are fixed at the same point (the free D-instanton terms), those at which the boundaries are fixed at two different points (two-body D-instanton interaction terms), those involving three points (three-body D-instanton interactions), and so on,

$$S^{(n)\prime}(y_1,\ldots,y_n) = \sum_{i=1}^{n} R_1(y_i) + \sum_{i\neq j}^{n} R_2(y_i,y_j) + \sum_{i\neq j\neq k}^{n} R_3(y_i,y_j,y_k) + \ldots. \tag{9}$$

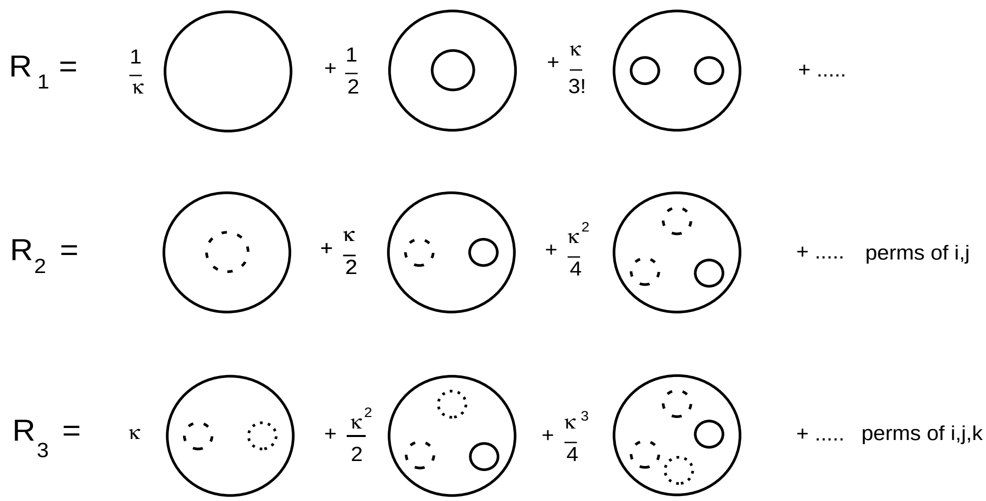

Figure 1: Contributions to the n D-instanton action from the first few powers of κ. a) Diagrams with boundaries fixed at a single space-time point contribute to the free action. b) Diagrams with boundaries fixed at two points (indicated by the full and dashed boundaries) contribute to the two-instanton interaction. c) Diagrams contributing to the three-instanton interaction (at points indicated by full, dashed and dotted boundaries). The coefficients explicitly show the combinatorical factors that arise from symmetry under the interchange of identical boundaries.

It is important that the definition of $S^{(n)\prime}$ includes terms in which any subset of the p_i are zero – these are terms that also contribute to the definition of $S^{(m)\prime}$ with $m < n$.

The free term in Eq. (9), $R_1(y_i) \equiv S^{(1)\prime}(y_i)$, is simply given by the sum over connected orientable world-sheets of arbitrary topology with all boundaries fixed at a single point, y_i, illustrated in fig.1(a). It is independent of y_i (by translational invariance) and has the form,

$$R_1 = -\frac{C}{\kappa} + \ln D + O(\kappa), \tag{10}$$

and thus $\sum_i R_1(y_i) = nR_1$. The term in this series with constant coefficient, C, is determined by functional integration over the disk. Explicit calculations determine $C = 2^8\pi^{25/2}\alpha'^{6}$[7,8]. It is somewhat remarkable that C is a finite (positive) constant with a value that is consistent with the non-vanishing value of the disk with a zero-momentum dilaton attached. Naively C would be expected to vanish since it should be proportional to the inverse of the volume of the conformal Killing group, $SL(2,R)$ (which is infinite) but that would not be consistent with the disk with a zero-momentum dilaton insertion. The ζ function regularization[7] and the proper-distance regularization[8] lead to the subtraction of an infinite constant from the volume of the Killing group, giving the finite positive renormalized value of C. The κ-independent constant D in Eq. (10) comes from the world-sheet annulus with both boundaries at y^μ.

Equation (10) leads to $e^{-C/\kappa}$ contributions to the partition function and to scattering amplitudes. This has the qualitative form expected for non-perturbative effects in string theory on the basis of matrix models and from the analysis of the rate of divergence of closed-string perturbation theory[4]. It is to be contrasted with a characteristic feature of non-perturbative effects (such as instantons and solitons) in field theory, which behave as $e^{-const./\kappa^2}$. This distinction between the non-perturbative behaviour of quantum field theory and that expected in closed-string theory seems likely to be of great significance (some possible consequences are described in ref.9).

The two-instanton interactions are given by the series of terms in R_2 shown in fig.1(b). The diagrams contributing to R_2 are those in which at least one boundary is fixed at either of the two space-time points. The leading terms in this series have the form

$$R_2(y_i, y_j) = f_{1,1}(y_i, y_j) + \frac{\kappa}{2}\left(f_{2,1}(y_i, y_j) + f_{1,2}(y_i, y_j)\right) + O(\kappa^2), \tag{11}$$

where $f_{p_i,p_j}(y_i, y_j)$ indicates a term with all $p_r = 0$ apart from p_i and p_j. The two-boundary term is given by the expression

$$f_{1,1}(y_i, y_j) = c\int_0^\infty d\tau e^{-\Delta_{ij}^2/\tau} e^{2\tau} \prod_{n=1}^{\infty} (1 - e^{-2\tau})^{-24}, \tag{12}$$

where c is a constant and $\Delta_{ij} = y_2 - y_1$. This diverges at the endpoint $\tau \to \infty$ due to the presence of a closed-string tachyon state. This is a familiar problem of the bosonic theory which we shall bypass by tranforming to momentum space and declaring that at low momenta (or large distance) only the massless dilaton singularity survives so that in the long-distance limit $\Delta^2 \to \infty$

$$f_{1,1}(y_i, y_j) \sim |y_i - y_j|^{2-D}. \tag{13}$$

This coulomb-like behaviour due to dilaton exchange is analogous to the long-distance force between two classical instantons (magnetic monopoles) in the three-dimensional euclidean Georgi–Glashow model. However, unlike the case of magnetic monopoles the interaction term, Eq. (13) is not of the same order in κ as the leading term, $S^{(1)\prime}$.

It is straightforward to show that the partition function defined by Eq. (8) does not have the divergences arising from the level-one open-string vector state. The term in the partition function coming from n D-instantons has a dependence on ϵ that can be written by expanding the exponent to first order in $\ln\epsilon$ using Eq. (5), giving,

$$\ln\epsilon \int \prod_{i=1}^{n} d^D y_i^\mu \left\{ \left(\frac{\partial}{\partial y_1^\mu} S^{(n)}\right)\left(\frac{\partial}{\partial y_{1\mu}} S^{(n)}\right) + \frac{\partial^2}{\partial y_1^2} S^{(n)} \right\} e^{S^{(n)}} = 0. \tag{14}$$

The fact that the expression vanishes makes use of an integration by parts of the second term.

The cancellation is illustrated in an example in fig. 3. This shows contributions to a particular divergence coming from the sum of (a) a planar connected world-sheet,

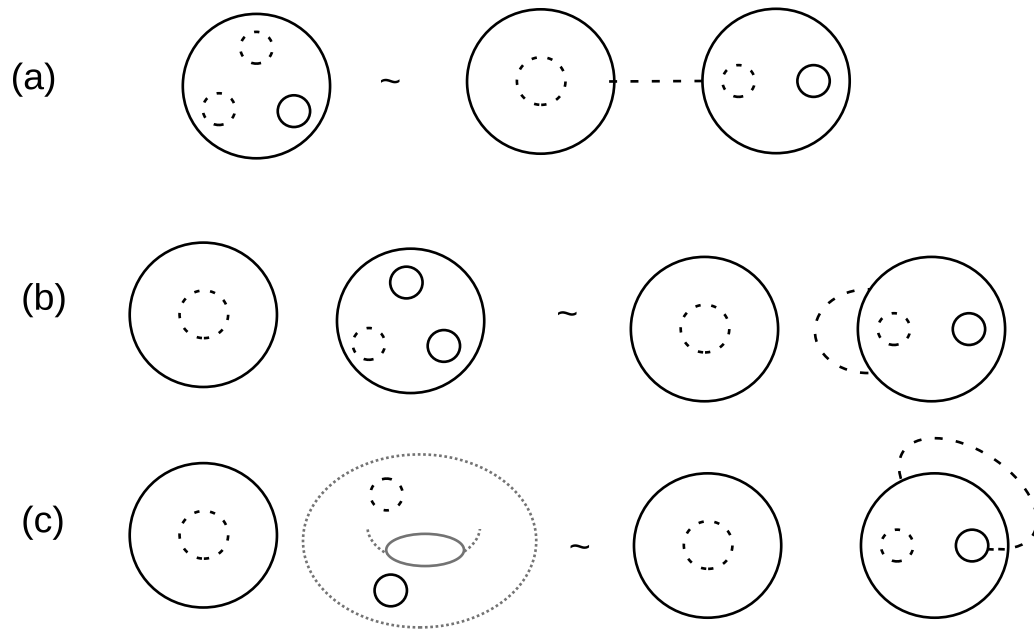

Figure 2: An example of the cancelation of a divergence that requires world-sheets with handles. a) One of the divergent degenerations of a world-sheet with two boundaries at y_1 (full lines) and two at y_2 (dashed lines). b) A degeneration of a disconnected planar world-sheet that, after integration over y_1, contributes to the same divergence as in a). c) A degeneration on a disconnected world-sheet with a handle that gives a divergence that adds to the divergences in b) and c) to give a total y_1 derivative.

(b) a planar disconnected world-sheet and (c) a non-planar disconnected world-sheet. The sum of these contributions may be written symbolically as

$$(a) + (b) + (c) \;=\; \ln\epsilon \int d^D y_1^\mu \frac{\partial^2}{\partial y_1^2} \left(f_{1,1}(y_1, y_2) f_{2,1}(y_1, y_2) \right), \tag{15}$$

which vanishes after integration over y_1 (assuming suitable boundary conditions). The cancellation of divergences evidently involves a conspiracy between terms with different numbers of boundaries and handles. Therefore it is only possible when the boundary weight has a specific value – it is not possible to add Chan–Paton factors to the boundaries as is usually the case in open string theories. It is disturbing that the cancellation of the divergences requires an integration by parts which looks nonlocal since the D-instanton gas is supposed to satisfy clustering properties that express the locality of the theory. However, (at least in flat space) the potentially dangerous surface terms that arise are suppressed since they involve interactions between boundaries fixed at widely separated points.

On-shell scattering amplitudes may be defined in the usual manner by considering fluctuations in the background fields, resulting in closed-string vertex operators coupled to the world-sheets. The connected scattering amplitudes are obtained from $\ln Z$, where the word ‘connected’ here refers to the target space. It is a important that the expression for $\ln Z$ only generates diagrams that are connected in the target space – in other words that disconnected world-sheets only arise when the disconnected components have at least one boundary fixed at a common target-space point. This follows from combinatorics that are similar to the usual field theory combinatoric arguments that show that the perturbative expansion of $\ln Z$ generates connected Feynman diagrams. The terms that are disconnected in the target space (terms in which vertex operators are attached to disconnected world-sheet components that do not have any boundary fixed at a common point) can be shown to cancel out of the expansion of $\ln Z$. The presence of Dirichlet boundaries gives terms of order $e^{-C/\kappa}$ in scattering amplitudes that have qualitatively different behaviour from the usual behaviour. For example, there is a contribution to the four-tachyon amplitude with each vertex operator attached to a separate component of the world-sheet – this is a ϕ^4 contact interaction. The lowest-order contribution to the scattering amplitude with four physical gravitons (which comes from a diagram with a pair of vertex operators attached to two disks with boundaries at the same point in the target space) falls as a power of the energy[5].

The inconsistencies of the critical bosonic string make it difficult to interpret this theory in more detail. In particular, it is not at all clear in what sense these ideas make contact with more conventional instanton ideas, such as those that arise in matrix models. It would be of interest to study similar boundary effects in two-dimensional bosonic string theories and compare them with other descriptions of instantons in such theories (such as ref.10). One peculiarity, at least in the bosonic theory, is that there are no anti D-instantons.

The absence of tachyons in superstring theories suggests that this may be an arena where a more consistent discussion could be given. The very recent interest in the possible connection between type 1 superstrings and the heterotic string (E. Witten at this conference and refs.11, 12 subsequently) gives added impetus to the study of the effect of unusual boundary conditions in superstring theory (previous work on the construction of type 1 theories in various dimensions is described in ref.13 and references therein). In the type 2b theory there are two types of Dirichlet boundaries, each preserving one half of the space-time supersymmetry[14] – these may be thought of as self-dual and anti self-dual. A gas of self-dual (or anti self-dual) super D-instantons is non-interacting just as in the case of self-dual monopoles. This follows because the multi self-dual D-instanton action is given by a sum over connected world-sheets with any number of boundaries of the same type that are fixed at any of n points. This configuration has world-sheet supersymmetry and the integration over fermion zero modes kills all diagrams with more than one boundary. Therefore for n self-dual super D-instantons the free energy is simply n times the free energy of a single free D-instanton – n times the disk diagram, which is again a finite constant, given by the non-zero coupling of the dilaton to the disk with one particular spin structure of the world-sheet fermions.

There are also soliton-like 'D-brane' configurations[3] whose rôle in the context of superstrings has not yet been illuminated. However these might provide solitonic states that are needed if the suggested non-perturbative equivalence of the type 1 and heterotic theories is correct.

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