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BPS Saturated and Non-Extreme States in Abelian Kaluza-Klein Theory and Effective $N = 4$ Supersymmetric String Vacua *

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Abstract

We summarize results for all four-dimensional Bogomol'nyi-Sommerfield-Prasat (BPS) saturated and non-extreme solutions of the $(4+n)$-dimensional Abelian Kaluza-Klein theory. Within effective $N = 4$ supersymmetric string vacua, parameterized in terms of fields of the heterotic string on a six-torus, we then present a class of BPS saturated states and the corresponding non-extreme solutions, specified by $O(6, 22, Z)$ and $SL(2, Z)$ orbits of general dyonic charge configurations with zero axion. The BPS saturated states with non-negative $O(6, 22, Z)$ norms for electric and magnetic charge vectors, along with the corresponding set of non-extreme solutions, are regular with non-zero masses. BPS saturated states with the negative charge norms are singular, unaccompanied by non-extreme solutions and become massless at particular points of the moduli space. The role that such massless states may play in the enhancement of non-Abelian gauge symmetry as well as local supersymmetry is addressed.

I. INTRODUCTION

Solitons, *i.e.*, time-independent solutions of classical equations, which saturate the Bogomol'nyi bound for their energy, shed light on non-perturbative phenomena in non-linear

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field theories. Even more intriguing is a recent recognition [1,2] that such configurations, also referred to as Bogomol'nyi-Sommerfield-Prasat (BPS) saturated states, play a crucial role in addressing the full, non-perturbative dynamics of string theory. In particular, at points of moduli space when such configurations become light they can affect the low energy dynamics of the string theory in an important way [3–5]. Thus, the study of BPS saturated states for different string vacua may in turn shed light on the non-perturbative string dynamics, as well as contribute to gathering evidence for string-string duality between certain strongly coupled and the corresponding weakly coupled string vacua. In addition along with the BPS saturated states one would also like to obtain information on the spectrum of the corresponding non-extreme solutions, *i.e.*, configurations in the same topological class, which are compatible with the corresponding Bogomol'nyi bound. Although the latter set of states is in general modified by quantum corrections, they may be relevant in the full string dynamics as well.

In this contribution we report on results for four-dimensional BPS saturated states and the corresponding non-extreme solutions which arise in effective supergravity theories compactified down to four dimensions on tori, *i.e.*, on manifolds with Abelian isometry. In particular, we would like to shed light on the roles that such states play in the effective $N = 4$ superstring vacua. Such string vacua are conjectured to be self-dual, *i.e.*, the string vacua of the heterotic string compactified on a six-torus (T^6) transform into each other under the $SL(2, Z)$ transformations. In addition, the heterotic string compactified on T^6 is conjectured to be dual to the type IIA string compactified on a $T^2 \times K3$ surface, which has its origin in the string-string duality conjecture [6,2,1,7] of the heterotic and the type IIA string theories in six dimensions, as well as to the eleven-dimensional supergravity compactified on $T^3 \times K3$ surface, which has its origin in the duality conjecture [1,8,2] of the type IIA string theory and the eleven-dimensional supergravity in ten dimensions.

At the conference, the results for all the four-dimensional static, spherically symmetric BPS saturated states [9] as well as all the non-extreme solutions [10] in the Kaluza-Klein sector of the $(4+n)$-dimensional (minimally extended) supergravities compactified on n-tori have been presented. The explicit form of these solutions allows for a synthetic classification of all of them. For $n = 7$, those are states in the Kaluza-Klein sector of toroidally compactified eleven-dimensional supergravity [11] with $N = 8$ supersymmetry in four dimensions, which plays an important role as a dual theory [1,2] of the strongly coupled type IIA superstring theory on a six-torus. For $n = 6$, those are states in the Kaluza-Klein sector of the toroidally compactified heterotic sting theory [12], which is dual to the type IIA superstring on a $T^2 \times K3$.

In this contribution, we also report on our recent work [13,14] on a class of BPS saturated states of four-dimensional effective $N = 4$ supersymmetric string vacua, which we parameterize in terms of fields of the effective heterotic string theory compactified on a six-torus. We present BPS saturated states corresponding to $O(6, 22, Z)$ and $SL(2, Z)$ orbits of dyonic configurations with zero axion; the $O(6, 22, Z)$ orbits correspond to states with the left-moving and the right-moving electric and magnetic charges orthogonal, *i.e.*, light-like in the $O(6, 22, Z)$ sense. The states with the $O(6, 22, Z)$ norms for the electric and magnetic charges non-negative [13] correspond to regular solutions with non-zero masses everywhere in the moduli space, while states with the charge norms negative [14] are singular solutions that become massless (along with an infinite tower of states, related by $SL(2, Z)$ transformations) for particular charge configurations and at particular points of the moduli space. Potential physical implications of such massless states are also discussed.

We also address non-extreme solutions with the same charge content as the BPS saturated states discussed above. The regular BPS saturated states are accompanied by a set of regular non-extreme solutions with masses compatible with the corresponding Bogomol'nyi bound. On the other hand, singular BPS saturated states, which can become massless at certain points of moduli space, have *no* non-extreme solutions that are compatible with the corresponding Bogomol'nyi bound.

In chapter 2, we give the results for static, spherically symmetric solutions in the Abelian Kaluza-Klein theory with the most general charge configurations, and discuss their thermal properties and singularity structures. In chapter 3, the general BPS saturated states as well as the corresponding non-extreme ones for the effective theory of $N = 4$ superstring vacua are discussed. Conclusions and open problems are relegated to chapter 4.

II. SPHERICALLY SYMMETRIC BLACK HOLES IN ABELIAN KALUZA-KLEIN THEORY

The effective Abelian Kaluza-Klein theory [15] in four dimensions is obtained from $(4+n)$-dimensional pure gravity by compactifying n spatial coordinates on an n-torus by using the following Ansatz for the $(n+4)$-dimensional metric:

$$g^{(4+n)}_{\Lambda\Pi} = \begin{bmatrix} e^{-\frac{1}{\alpha}\varphi} g_{\lambda\pi} + e^{\frac{2\varphi}{n\alpha}}\rho_{ij}A^i_\lambda A^j_\pi & e^{\frac{2\varphi}{n\alpha}}\rho_{ij}A^i_\lambda \\ e^{\frac{2\varphi}{n\alpha}}\rho_{ij}A^j_\pi & e^{\frac{2\varphi}{n\alpha}}\rho_{ij} \end{bmatrix}, \tag{1}$$

where A^i_μ is the Kaluza-Klein n $U(1)$ gauge fields with the field strengths $F^i_{\mu\nu}$, and the internal metric $g_{ij} = e^{\frac{2\varphi}{n\alpha}}\rho_{ij}$ (with ρ_{ij} its unimodular part) and the four-dimensional metric $g_{\mu\nu}$ depend on the four-dimensional space-time coordinates, only. We use the mostly positive signature convention $(+++-+\cdots+)$ for $g^{(4+n)}_{\Lambda\Pi}$ with the time coordinate in the fourth place, and $\alpha = \sqrt{\frac{n+2}{n}}$. The four-dimensional effective action (see for example Ref. [16]) has the global $SO(n)$ target space symmetry:

$$\rho_{ij} \to U_{ik}\rho_{k\ell}(U^T)_{\ell j}, \qquad A^i_\mu \to U_{ij}A^j_\mu, \tag{2}$$

where U is an $SO(n)$ rotation matrix, as well as the rescaling symmetry [16]. In addition, for static or stationary four-dimensional configurations the time-translation can be considered, along with n parameters of the internal isometry, as a part of the $(n+1)$-parameter Abelian isometry group of a $(4+n)$-dimensional space-time manifold $\mathbf{M}$. In this case the projection of the $(4+n)$-dimensional manifold $\mathbf{M}$ onto the set $\mathbf{S}$ of the orbits of the isometry group in $\mathbf{M}$ allows one to express the $(4+n)$-dimensional Einstein gravity action in the following three-dimensional one [17]:

$$\mathcal{L} = -\frac{1}{2}\sqrt{-h}[\mathcal{R}^{(h)} - \frac{1}{4}\mathrm{Tr}(\chi^{-1}\partial_a\chi\chi^{-1}\partial^a\chi)], \tag{3}$$

where $h_{ab} \equiv \tau g^{\perp}_{ab}$ $(a, b = 1, 2, 3)$ is the rescaled metric on $\mathbf{S}$ and

$$\chi = \begin{bmatrix} \tau^{-1} & -\tau^{-1}\omega^T \\ -\tau^{-1}\omega & \breve{\lambda} + \tau^{-1}\omega\omega^T \end{bmatrix} \tag{4}$$

is the $(n+2) \times (n+2)$ symmetric, unimodular matrix of scalar fields on $\mathbf{S}$. Here, $\breve{\lambda}_{ij} \equiv g^{(4+n)}_{\Lambda\Pi}\xi^{\Lambda}_i\xi^{\Pi}_j$, $\tau \equiv \det\breve{\lambda}_{ij}$ and $g^{\perp}_{ab} \equiv g^{(4+n)}_{ab} - \breve{\lambda}^{ij}\xi_{ia}\xi_{jb}$. The "potential" $\omega^T \equiv (\omega_1, ..., \omega_{n+1})$ defined as $\partial_a\omega_i = \omega_{ia} \equiv \hat{\epsilon}_{abc}\xi^{b;c}_i$ ($\hat{\epsilon}_{abc} \equiv \epsilon_{abc4...(4+n)}$) replaces the degrees of freedom of $\xi_{ia} = g^{(4+n)}_{i+3,a}$. The effective three-dimensional Lagrangian density (3) is invariant under the global $SL(2+n, R)$ target space transformations [17]:

$$\chi \to \mathcal{U}\chi\mathcal{U}^T, \qquad h_{ab} \to h_{ab}, \tag{5}$$

where $\mathcal{U} \in SL(2+n, R)$.

For the purpose of obtaining static, spherically symmetric configurations, one chooses the following Ansatz for the four-dimensional metric:

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -\lambda(r)dt^2 + \lambda^{-1}(r)dr^2 + R(r)(d\theta^2 + \sin^2\theta d\phi^2). \tag{6}$$

The corresponding three-dimensional metric is then given by $h_{ab} = \mathrm{diag}(1, \lambda R, \lambda R\sin^2\theta)$ and n $U(1)$ gauge fields solve the equations of motion with spherically symmetric Ansätze for the other fields, while the scalar fields associated with the internal metric depend on the radial coordinate r, only.

A. Supersymmetric Configurations

Among a class of solutions with a given charge configuration the solution that saturates the corresponding Bogomol'nyi bound, referred to as BPS saturated states, corresponds to the minimum energy (or vacuum) configuration in its class. These BPS saturated states satisfy the Killing spinor equations which are obtained by setting the supersymmetric variations of fermionic fields equal to zero, and therefore are bosonic configurations which preserve some of the supersymmetries.

With the Kaluza-Klein Ansatz for the $(4+n)$-dimensional metric (Eq. (1)) one turns off all the other bosonic fields of the corresponding supergravity theory. Then the only non-trivial Killing spinor equations [9] turn out to be those arising from the vanishing of the supersymmetry transformation of (dimensionally reduced) $(4+n)$-dimensional gravitini.

With the spherical Ansätze for the four-dimensional fields, the Killing spinor equations (corresponding to the t, θ and ϕ components of the four-dimensional gravitini) restrict a general supersymmetric configuration in this class to have n electric and n magnetic charges subject to the following orthogonality constraint [9]:

$$\vec{Q} \cdot \vec{P} = 0. \tag{7}$$

All the supersymmetric configurations in this class can therefore be obtained by imposing $SO(n)/SO(n-2)$ rotations (with $2n-3$ parameters) on the supersymmetric solution with $U(1)_M \times U(1)_E$ charge configuration [16]. The latter one, which we refer to as the generating configuration, is parameterized by one magnetic charge P and one electric charge Q, arising from *different* $U(1)$ factors. It turns out that for this charge configuration, among the scalar fields only diagonal components of internal metric g_{mn} are turned on [1]. The generating solutions with only electric [or only magnetic] charge turned on preserve $\frac{1}{2}$ of the original supersymmetry while the dyonic ones preserve $\frac{1}{4}$ of the original supersymmetry [9].

Four-dimensional space-time for such solutions is specified by two parameters $|\vec{P}|$ and $|\vec{Q}|$, with the following ADM mass:

$$M_{\rm BPS} = |\vec{P}| + |\vec{Q}|. \tag{8}$$

The explicit form of these solutions corresponds to a special case of the general class of solutions with the ADM masses compatible with the Bogomol'nyi bound (8) and are discussed in the following subsection.

B. General Class of Configurations

General, four-dimensional, static, spherically symmetric solutions are parameterized by the ADM mass $M \geq M_{\rm BPS}$, n electric $\vec{Q} = (Q_1, ..., Q_n)$ and n magnetic $\vec{P} = (P_1, ..., P_n)$ charges. The explicit solution with such configurations was obtained [10] by performing symmetry transformations (of the three-dimensional action (3)) on the Schwarzschield solution.

By using the $SO(n)$ and the rescaling symmetry of the corresponding four-dimensional action, one can bring the asymptotic value of matrix χ (Eq.(4)) into the form $\chi_\infty = {\rm diag}(-1,-1,1,...,1)$. Then, the subset of $SL(2+n,R)$ symmetry transformations of three-dimensional action Eq. (3) that preserves this asymptotic form of χ is $SO(2,n)$.

By performing a set of two $SO(1,1)$ boosts (on the $1st$ and the $(n{+}1)$-th, and the $2nd$ and the $(n+2)$-th indices of χ) with the boost parameters $\delta_{P,Q}$ on the Schwarzschield solution, *i.e.*, $\chi = {\rm diag}(-(1-\frac{m}{r})^{-1}, -(1-\frac{m}{r}), 1, ..., 1)$, one obtains the non-extreme $U(1)_M \times U(1)_E$ black hole solutions parameterized in terms of the magnetic charge $P \equiv m{\rm sinh}\delta_P{\rm cosh}\delta_P$, electric charge $Q = m{\rm sinh}\delta_Q{\rm cosh}\delta_Q$, and the ADM mass $M \equiv m({\rm cosh}^2\delta_Q + {\rm cosh}^2\delta_P)$. The ADM mass is traded for the non-extremality parameter $\beta \equiv \frac{m}{2} > 0$. Additional two $SO(1,1)$ boosts (on the $1st$ and the $(n+2)$-th, and the $2nd$ and the $(n+1)$-th indices of χ) with boost parameters δ_1 and δ_2 generate a solution with the following explicit form [10]:

$$\lambda = \frac{r(r+2\beta)}{(XY)^{1/2}},\ R = (XY)^{1/2},\ e^{\frac{2\varphi}{\alpha}} = \frac{X}{Y},\ \rho_{ij} = \delta_{ij}{\rm e}^{-\frac{2\varphi}{n\alpha}}\ \ (i,j \neq n-1, n),$$
$$\rho_{n-1,n-1} = \frac{We^{\frac{2(n-2)}{n\alpha}\varphi}}{(XY)^{1/2}},\ \ \rho_{n-1,n} = \frac{Ze^{\frac{2(n-2)}{n\alpha}\varphi}}{(XY)^{1/2}},\ \rho_{nn} = \frac{(r+\hat{Q})(r+\hat{P})}{(XY)^{1/2}}e^{\frac{2(n-2)}{n\alpha}\varphi}, \tag{9}$$

where

[1]In general with a diagonal internal metric Ansatz the static, spherically symmetric configurations can have at most one electric and one magnetic charge [16], which can also arise from the same $U(1)$ gauge fields.

$$
\begin{aligned}
X &= r^2 + [(2\beta - \hat{P} + \hat{Q})\cosh^2\delta_2 + \hat{P}]r + 2\beta\hat{Q}\cosh^2\delta_2, \\
Y &= r^2 + [(2\beta + \hat{P} - \hat{Q})\cosh^2\delta_1 + \hat{Q}]r + 2\beta\hat{P}\cosh^2\delta_1, \\
W &= r^2 + [(2\beta + \hat{P} - \hat{Q})\cosh^2\delta_1 + (2\beta - \hat{P} + \hat{Q})\cosh^2\delta_2]r + |P||Q|\cosh\delta_1\cosh\delta_2\sinh\delta_1\sinh\delta_2 \\
&\quad +2[\beta(2\beta - \hat{P} - \hat{Q}) + \hat{P}\hat{Q}]\cosh^2\delta_1\cosh^2\delta_2 + (2\beta - \hat{Q})\hat{P}\cosh^2\delta_1 + (2\beta - \hat{P})\hat{Q}\cosh^2\delta_2, \\
Z &= [|P|\sinh\delta_1\cosh\delta_2 + |Q|\sinh\delta_2\cosh\delta_1]r + |P|\hat{Q}\sinh\delta_1 + \hat{P}|Q|\sinh\delta_2,
\end{aligned} \tag{10}
$$

with the non-zero electric and magnetic charges and the ADM mass given by

$$
\begin{aligned}
P_{n-1} &= |P|\cosh\delta_1\cosh\delta_2 + |Q|\sinh\delta_1\sinh\delta_2, \quad P_n = -(\hat{P} - \hat{Q} + 2\beta)\cosh\delta_1\sinh\delta_1, \\
Q_{n-1} &= -(\hat{P} - \hat{Q} - 2\beta)\cosh\delta_2\sinh\delta_2, \quad Q_n = |Q|\cosh\delta_1\cosh\delta_2 + |P|\sinh\delta_1\sinh\delta_2, \\
M &= (2\beta + \hat{P} - \hat{Q})\cosh^2\delta_1 + (2\beta + \hat{Q} - \hat{P})\cosh^2\delta_2 + \hat{P} + \hat{Q} - 4\beta,
\end{aligned} \tag{11}
$$

where $\hat{P} \equiv \beta + \sqrt{P^2 + \beta^2}$ and $\hat{Q} \equiv \beta + \sqrt{Q^2 + \beta^2}$. The requirement of zero Taub-NUT charge relates the two boost parameters $\delta_{1,2}$ in the following way:

$$
|P|\tanh\delta_2 + |Q|\tanh\delta_1 = 0. \tag{12}
$$

The most general solution in this class is finally obtained by performing $SO(n)/SO(n-2)$ rotations on (9), thus providing the remaining $2n - 3$ degrees of freedom.

Thus, the general class of solutions is parameterized in terms of the following $2n + 1$ parameters: the non-extremality parameter $\beta \geq 0$, magnetic P and electric Q charges of the $U(1)_M \times U(1)_E$ black hole solution, two $SO(1,1)$ boost parameters $\delta_{1,2}$, which are subject to the zero Taub-Nut constraint (12), and $2n-3$ parameters of $SO(n)/SO(n-2) \subset SO(n)$ symmetry transformations (2) of the four-dimensional action.

One can show that with the zero Taub-NUT constraint (12) $M \geq M_{\mathrm{BPS}} \equiv |\vec{P}| + |\vec{Q}|$ for $\beta \geq 0$. For this case $\vec{P} \cdot \vec{Q} \propto \beta$. Thus the solutions with $\beta = 0$ and other parameters finite correspond to the supersymmetric solutions discussed in the previous subsection. On the other hand for $\delta_{1,2} \to \infty$ and $|Q| - |P| \to 0$, as $\beta \to 0$, in such a way that $\beta e^{2|\delta_{1,2}|} \equiv 2|q|$ and $||Q| - |P||e^{2|\delta_{1,2}|} \equiv 4|\Delta|$ remain finite, one obtains non-supersymmetric extreme solutions, *i.e.*, those with $\beta = 0$, however, $\vec{P} \cdot \vec{Q} \neq 0$.

Since the $SO(n)/SO(n-2) \subset SO(n)$ symmetry transformations with $2n-3$ parameters do not affect the four-dimensional space-time metric, the four-dimensional properties of the solution are fully determined by four parameters: β, P, Q and δ_1 [or δ_2]. Without loss of generality we assume that $|Q| \geq |P|$; for solutions with $|Q| \leq |P|$, the roles of δ_1 and δ_2 are interchanged. The Hawking temperature $T_H = \partial_r\lambda|_{r=0}/(2\pi)$ and the entropy $S =$ (1/4 of the area of the event horizon) are of the following form:

$$
T_H = \frac{[|Q|^2\cosh^2\delta_2 - |P|^2\sinh^2\delta_2]^{1/2}}{4\pi\left(\hat{P}\hat{Q}\right)^{1/2}|Q|\cosh^2\delta_2}, \quad S = \frac{2\pi\beta\left(\hat{P}\hat{Q}\right)^{1/2}|Q|\cosh^2\delta_2}{[|Q|^2\cosh^2\delta_2 - |P|^2\sinh^2\delta_2]^{1/2}}. \tag{13}
$$

The thermal properties and the singularity structure of the whole class of the solutions can be summarized according to the values of parameters δ_2, P and β as [10]:

- Non-extreme black holes with $\delta_2 \neq 0$ and $P \neq 0$:
  The global space-time is that of non-extreme Reissner-Nordström black holes, *i.e.*, the time-like singularity is hidden behind the inner horizon. T_H [S] is finite, and decreases [increases] as δ_2 or β increases, approaching zero [infinity].

- Non-extreme black holes with $\delta_2 = 0$ or $P = 0$:
  The singularity structure is that of the Schwarzschield black holes, *i.e.*, the space-like singularity is hidden behind the (outer) horizon. T_H [S] is finite and decreases [increases] as β increases, approaching zero [infinity].

- Supersymmetric extreme black holes, *i.e.*, δ_2 finite:
  For $P \neq 0$, the solution has a null singularity, which becomes naked when $P = 0$. T_H [S] is finite and becomes infinite [zero] when $P = 0$.

- Non-supersymmetric extreme black holes, *i.e.*, $|\delta_2| \to \infty$ with (q,Δ) finite:
  The global space-time is that of extreme Reissner-Nordström black holes with zero T_H and finite S.

III. SPHERICALLY SYMMETRIC BLACK HOLES OF EFFECTIVE FOUR-DIMENSIONAL $N = 4$ SUPERSYMMETRIC STRING VACUA

In the following we shall summarize the results for a class of BPS saturated solutions [13,14] and the corresponding non-extreme configurations parameterized in terms of the fields of the heterotic string compactified on a six-torus.

The effective field theory of massless bosonic fields for the heterotic string on a Narain torus [18] at a generic point of moduli space is obtained by compactifying the ten-dimensional $N = 1$ Maxwell/Einstein supergravity theory on a six-torus [19,20]. The effective four-dimensional action [2] for massless bosonic fields consists of the graviton $g_{\mu\nu}$, 28 $U(1)$ gauge fields $\mathcal{A}^i_\mu \equiv (A^{(1)\,m}_\mu, A^{(2)}_{\mu\, m}, A^{(3)\,I}_\mu)$, corresponding to the $U(1)$ gauge fields of dimensionally reduced ten-dimensional metric (Kaluza-Klein sector), two-form fields, and Yang-Mills fields, respectively, and 134 scalar fields. The scalar fields consist of the dilaton ϕ (which parameterizes the strength of the string coupling), the axion field Ψ (which is obtained from the two-form field $B_{\mu\nu}$ through the duality transformation), and a symmetric $O(6,22)$ matrix M of 132 scalar fields (moduli fields whose vacuum expectation values parameterize the string vacua). The matrix M consists of 21 internal metric g_{mn} components, 15 pseudo-scalar fields B_{mn}, and 96 scalar fields a^I_m, which arise from the dimensionally reduced ten-dimensional metric, two-form field and Yang Mills fields, respectively. Here, $(\mu,\nu) = 0,\cdots,3$, $(m,n) = 1,\cdots,6$ and $I = 1,\cdots,16$.

The four-dimensional effective action is invariant under the $O(6,22,R)$ transformations [19,20]:

[2]See Refs. [19,20] for notational conventions and the relationship of four-dimensional fields to the corresponding ten-dimensional ones. Also, we are not addressing α' corrections.

$$M \to \Omega M \Omega^T, \quad \mathcal{A}^i_\mu \to \Omega_{ij}\mathcal{A}^j_\mu, \quad g_{\mu\nu} \to g_{\mu\nu}, \quad \phi \to \phi. \tag{14}$$

Here, $\Omega \in O(6,22)$, *i.e.*, $\Omega^T L \Omega = L$, where L is an $O(6,22)$ invariant matrix. The world-sheet instanton effects break $O(6,22,R)$ invariance of the effective action down to its discrete subgroup $O(6,22,Z)$ referred to as T-duality. T-duality is an exact string symmetry to all orders in string perturbation and is assumed to survive non-perturbative corrections.

In addition, the equations of motion and Bianchi identities are invariant under the $SL(2,R)$ transformations [21,20]:

$$S \to S' = \frac{aS+b}{cS+d}, \; M \to M, \; g_{\mu\nu} \to g_{\mu\nu}, \; \mathcal{F}^i_{\mu\nu} \to \mathcal{F}'^{\,i}_{\mu\nu} = (c\Psi + d)\mathcal{F}^i_{\mu\nu} + ce^{-\phi}(ML)_{ij}\tilde{\mathcal{F}}^j_{\mu\nu}, \tag{15}$$

where $S \equiv \Psi + ie^{-\phi}$, $\tilde{\mathcal{F}}^{i\,\mu\nu} = \frac{1}{2}(\sqrt{-g})^{-1}\varepsilon^{\mu\nu\rho\sigma}\mathcal{F}^i_{\rho\sigma}$, and $a,b,c,d \in R$ satisfy $ad - bc = 1$. The space-time instanton effects break $SL(2,R)$ down to $SL(2,Z)$, referred to as S-duality. S-duality, which is non-perturbative in nature, is conjectured [21] to be an exact symmetry of $N=4$ supersymmetric string vacua. It relates strongly coupled vacua to those of the weakly coupled ones.

The allowed discrete magnetic $\vec{P}$ and electric $\vec{Q}$ charges are determined [20] by T- and S-duality constraints of toroidally compactified heterotic string and by the Dirac-Schwinger-Zwanzinger-Witten (DSZW) quantization condition [22,23]; both of the "lattice charge vectors" [20], $\vec{\beta} \equiv L\vec{P}$ and $\vec{\alpha} \equiv \mathrm{e}^{-\phi_\infty} M_\infty^{-1}\vec{Q} - \Psi_\infty \vec{\beta}$, belong to an even, self-dual Lorentzian lattice Λ with the signature $(6,22)$. Here ϕ_∞, Ψ_∞ and M_∞ correspond to the asymptotic values of the dilaton, axion and moduli fields, respectively. Note, that the lattice charge vectors $\vec{\alpha}$ and $\vec{\beta}$ tranform covariantly [20] under T- and S-duality transformations.

Within this effective theory, we shall present explicit results for a class of spherically symmetric BPS saturated configurations [13] as well as a class of their non-extreme counterparts. The Killing spinor equations for the BPS saturated states are obtained by setting to zero the ten-dimensional supersymmetry transformations for the gravitino, dilatino and 16 gaugini, now expressed in terms of the four-dimensional fields. The spectrum of BPS saturated, static, spherically symmetric configurations is both $O(6,22,Z)$ and $SL(2,Z)$ invariant. Namely, the Killing spinors ε are invariant under T-duality and transform covariantly under S-duality. Therefore one can generate new BPS saturated solutions as well as non-extreme solutions by imposing S- and T-duality transformations on a particular "generating" solution.

For the purpose of obtaining a general set of solutions with an arbitrary choice of M_∞ and $S_\infty = \Psi_\infty + i\mathrm{e}^{-\phi_\infty}$, one can use the following procedure:

- First one performs [20,5] $O(6,22,R)$ transformations of the form $M_\infty \to \hat{M}_\infty = \hat{\Omega} M_\infty \hat{\Omega}^T$ and $\Lambda \to \hat{\Lambda} = L\hat{\Omega}L\Lambda$ ($\hat{\Omega} \in O(6,22,R)$). This procedure allows one to bring an arbitrary asymptotic value of the moduli fields M to the identity matrix, *i.e.*, $\hat{M}_\infty = I_{28}$, while the electric and magnetic lattice charge vectors live in the new lattice $\hat{\Lambda}$. Note, that the transformation Ω that gives $\hat{\Lambda}$ is determined only up to the $O(6,22,Z)$ automorphisms of the lattice Λ. A subset of $O(6,22,Z)$ transformations that preserves the asymptotic value $\hat{M}_\infty = I_{28}$ is $SO(6,Z) \times SO(22,Z)$. The latter subset of T-duality transformations generates $O(6,22,Z)$ orbits of solutions with the same $\hat{M}_\infty = I_{28}$.

- Secondly one can use $SL(2,R)$ transformations to bring $S_\infty \to \breve{S}_\infty = i$ and correspondingly transform the lattice charge vectors living in $\hat{\Lambda}$ into those living in a new lattice $\breve{\Lambda}$. A subset of S-duality transformations that preserves the asymptotic value $\breve{S}_\infty = i$ are $SO(2,Z)$ transformations which generate $SL(2,Z)$ orbits of solutions with the same $\breve{S}_\infty = i$.

- Finally, in order to obtain a set of solutions with arbitrary asymptotic values of M and S, one has to undo the above $O(6,22,R)$ and $SL(2,R)$ transformations.

Using the procedures described above, we shall now present a class of dyonic BPS saturated states (and their non-extreme counterparts), which correspond to $O(6,22,Z)$ and $SL(2,Z)$ orbits of the most general dyonic solution with zero axion. When the axion field is turned off, the Killing spinor equations ensure that the BPS saturated states with 28 electric $\vec{Q}$ and 28 magnetic $\vec{P}$ charges are subject to two (orthogonality) constraints [13]:

$$\vec{P}^T \mathcal{M}_\pm \vec{Q} = 0 \quad (\mathcal{M}_\pm \equiv (LM_\infty L \pm L)). \tag{16}$$

The S-duality transformations provide one more parameter for the charge degrees of freedom along with the non-zero axion field. In this case 28 electric and 28 magnetic charges are subject to the following one constraint [13]:

$$\vec{P}^T \mathcal{M}_- \vec{Q}\, [\vec{Q}^T \mathcal{M}_+ \vec{Q} - \vec{P}^T \mathcal{M}_+ \vec{P}] - \vec{P}^T \mathcal{M}_+ \vec{Q} [\vec{Q}^T \mathcal{M}_- \vec{Q} - \vec{P}^T \mathcal{M}_- \vec{P}] = 0. \tag{17}$$

The general class of solutions subject to the charge constraint (17) is obtained following the procedure described above:
(i) First, on a solution with chosen asymptotic values M_∞ and S_∞ one performs the above mentioned $SL(2,R)$ and $O(6,22,R)$ transformations, rendering $S_\infty \to \breve{S}_\infty = i$ and $M_\infty \to \hat{M}_\infty = I_{28}$, respectively, along with the corresponding transformations of the charge lattices.
(ii) The generating solution for a general class of solutions with the charge constraint (16) (and with $\hat{M}_\infty = I_{28}$ and $\breve{S}_\infty = i$) turns out to correspond to the $\breve{U}(1)^{(1)}_{m,M} \times \breve{U}(1)^{(1)}_{n,E} \times \breve{U}(1)^{(2)}_{m,M} \times \breve{U}(1)^{(2)}_{n,E}$ configuration ($1 \le m \ne n \le 6$) [13]. Namely this configuration is parameterized by two magnetic and two electric charges (with the corresponding lattice charge vectors living in lattice $\breve{\Lambda}$), which arise from different $U(1)$ groups; the two magnetic [electric] charges arise from the Kaluza-Klein sector gauge field $\breve{A}^{(1)\,m}_\phi$ [$\breve{A}^{(1)\,n}_t$] and the corresponding two-form $U(1)$ field $\breve{A}^{(2)}_{\phi\, m}$ [$\breve{A}^{(2)}_{t\, n}$]. Without loss of generality we choose the non-zero charges to be $\breve{P}^{(1)}_1, \breve{P}^{(2)}_1, \breve{Q}^{(1)}_2, \breve{Q}^{(2)}_2$.
(iii) $[SO(6,Z) \times SO(22,Z)]/[SO(4,Z) \times SO(20,Z)]$ transformations, *i.e.*, a subset of $O(6,22,Z)$ transformations preserving the asymptotic value $\hat{M}_\infty = I_{28}$, on the generating solutions provide 50 additional parameters specifying the $O(6,22,Z)$ orbits with the general charges subject to two orthogonality constraints (16).
(iv) In addition $SO(2,Z)$ transformations, *i.e.*, a subset of $SL(2,Z)$ transformations preserving $\breve{S}_\infty$, provide one more parameter specifying $SL(2,Z)$ orbits consistent with the constraint (17).
(v) Finally one has to undo the $SL(2,R)$ and $O(6,22,R)$ transformations to obtain configurations with chosen asymptotic values M_∞ and S_∞.

Note that the set of transformations used in the above procedure *does not affect the four-dimensional space-time* and thus all the solutions in the class have the same four-dimensional space-time structure.

The explicit form for the static, spherically symmetric generating solution is [13]:

$$
\begin{aligned}
\lambda &= r^2/[(r-\eta_P\breve{P}_1^{(1)})(r-\eta_P\breve{P}_1^{(2)})(r-\eta_Q\breve{Q}_2^{(1)})(r-\eta_Q\breve{Q}_2^{(2)})]^{\frac{1}{2}},\\
R &= [(r-\eta_P\breve{P}_1^{(1)})(r-\eta_P\breve{P}_1^{(2)})(r-\eta_Q\breve{Q}_2^{(1)})(r-\eta_Q\breve{Q}_2^{(2)})]^{\frac{1}{2}},\\
e^{\phi} &= \left[\frac{(r-\eta_P\breve{P}_1^{(1)})(r-\eta_P\breve{P}_1^{(2)})}{(r-\eta_Q\breve{Q}_2^{(1)})(r-\eta_Q\breve{Q}_2^{(2)})}\right]^{\frac{1}{2}}, \quad \Psi = 0,\\
g_{11} &= \frac{r-\eta_P\breve{P}_1^{(2)}}{r-\eta_P\breve{P}_1^{(1)}},\ g_{22} = \frac{r-\eta_Q\breve{Q}_2^{(1)}}{r-\eta_Q\breve{Q}_2^{(2)}},\ g_{mm} = 1 \quad (m \neq 1,2),\\
g_{mn} &= B_{mn} = 0 \quad (m \neq n), \quad a^I_m = 0.
\end{aligned}
\tag{18}
$$

Here the radial coordinate is chosen so that the horizon is at $r=0$. $\eta_{P,Q} = \pm$ correspond to parameters appearing in the Killing spinor constraints. Namely the upper ε_u and lower ε_ℓ two-component Killing spinors are subject to the constraints [13]: $\Gamma^1\varepsilon_{u,\ell} = i\eta_P\varepsilon_{\ell,u}$ if $\breve{P}_1^{(1)} \neq 0$ and/or $\breve{P}_1^{(2)} \neq 0$, and $\Gamma^2\varepsilon_{u,\ell} = \mp\eta_Q\varepsilon_{\ell,u}$ if $\breve{Q}_2^{(1)} \neq 0$ and/or $\breve{Q}_2^{(2)} \neq 0$. Thus non-zero magnetic and electric charges each break $\frac{1}{2}$ of the remaining supersymmetries; purely electric [or magnetic] and dyonic configurations preserve $\frac{1}{2}$ and $\frac{1}{4}$ of $N=4$ supersymmetry, respectively. The first and the second sets of configurations fall into vector and highest spin $\frac{3}{2}$ supermultiplets [24], respectively.

The requirement that the ADM mass of the above configurations saturates the Bogomol'nyi bound restricts the choice of parameters $\eta_{P,Q}$ to be such that $\eta_P\,\mathrm{sign}(\breve{P}_1^{(1)} + \breve{P}_1^{(2)}) = -1$ and $\eta_Q\,\mathrm{sign}(\breve{Q}_2^{(1)} + \breve{Q}_2^{(2)}) = -1$, thus yielding the positive semidefinite ADM mass of the following form [3]:

$$
M_{\mathrm{BPS}} = |\breve{P}_1^{(1)} + \breve{P}_1^{(2)}| + |\breve{Q}_2^{(1)} + \breve{Q}_2^{(2)}|.
\tag{19}
$$

One can also obtain the non-extreme solutions, parameterized by the above four non-zero charges and the non-extremality parameter β, by solving the Einstein field equations and Euler-Lagrange equations, which yield the following result:

$$
\begin{aligned}
\lambda &= r(r+2\beta)/[(r+\breve{P}_1^{(1)\prime})(r+\breve{P}_1^{(2)\prime})(r+\breve{Q}_2^{(1)\prime})(r+\breve{Q}_2^{(2)\prime})]^{\frac{1}{2}},\\
R(r) &= [(r+\breve{P}_1^{(1)\prime})(r+\breve{P}_1^{(2)\prime})(r+\breve{Q}_2^{(1)\prime})(r+\breve{Q}_2^{(2)\prime})]^{\frac{1}{2}},\\
e^{\phi} &= \left[\frac{(r+\breve{P}_1^{(1)\prime})(r+\breve{P}_1^{(2)\prime})}{(r+\breve{Q}_2^{(1)\prime})(r+\breve{Q}_2^{(2)\prime})}\right]^{\frac{1}{2}}, \ \Psi = 0,
\end{aligned}
$$

[3] General BPS saturated states have the following $O(6,22,Z)$ and $SL(2,Z)$ invariant form of the ADM mass [13]: $M^2_{\mathrm{BPS}} = e^{-\phi_\infty}\{\vec{P}^T\mathcal{M}_+\vec{P} + \vec{Q}^T\mathcal{M}_+\vec{Q} + 2[(\vec{P}^T\mathcal{M}_+\vec{P})(\vec{Q}^T\mathcal{M}_+\vec{Q}) - (\vec{P}^T\mathcal{M}_+\vec{Q})^2]^{\frac{1}{2}}\}$, which is a generalization of the one for BPS states preserving $\frac{1}{2}$ of the original supersymmetry.

$$
\begin{aligned}
g_{11} &= \frac{r+\breve{P}_1^{(2)\prime}}{r+\breve{P}_1^{(1)\prime}}, \quad g_{22} = \frac{r+\breve{Q}_2^{(1)\prime}}{r+\breve{Q}_2^{(2)\prime}}, \quad g_{mm} = 1 \quad (m \neq 1,2), \\
g_{mn} &= B_{mn} = 0 \quad (m \neq n), \quad a^I_m = 0,
\end{aligned} \tag{20}
$$

with the ADM mass:

$$
M_{\text{non-ext}} = \breve{P}_1^{(1)\prime} + \breve{P}_1^{(2)\prime} + \breve{Q}_2^{(1)\prime} + \breve{Q}_2^{(2)\prime} - 4\beta. \tag{21}
$$

Here $\breve{P}_1^{(1)\prime} \equiv \beta \pm \sqrt{(\breve{P}_1^{(1)})^2 + \beta^2}$, *etc.* and β parameterizes the deviation from the corresponding supersymmetric solutions.

The signs ($\pm$) in the expressions for $\breve{P}_1^{(1)\prime}$, *etc.* should be chosen so that in the limit $\beta \to 0$, $M_{\text{non-ext}} \to M_{\text{BPS}}$. Note that $O(6,22,Z)$ and $SL(2,Z)$ orbits of the non-extreme solutions are subject to the charge constraint (17) and thus constitute only a subset of a general class of non-extreme configurations. The full set of non-extreme solutions should be obtained by performing a subset of $O(8,24,Z)$ transformations, *i.e.*, symmetry transformations of the effective three-dimensional action [25], on (20).

In the following two subsections we shall address the four-dimensional space-time structure of these solutions.

A. Regular Dyonic Solutions

For a black hole solution, *i.e.*, a spherically symmetric configuration with the regular horizon, one has to choose the relative signs of the two electric and two magnetic charges of the BPS saturated generating solutions (18) to be the same [13], so that the space-time singularity (the point at which $R(r)$ vanishes) lies inside or on the horizon (the point at which $\lambda(r)$ vanishes). In this case the non-extreme solutions are given by (20) with the *positive* signs in the expressions for $\breve{P}_1^{(1)\prime}$, *etc.* and have the ADM mass:

$$
M_{\text{non-ext}} = \sqrt{(\breve{P}_1^{(1)})^2 + \beta^2} + \sqrt{(\breve{P}_1^{(2)})^2 + \beta^2} + \sqrt{(\breve{Q}_2^{(1)})^2 + \beta^2} + \sqrt{(\breve{Q}_2^{(2)})^2 + \beta^2}, \tag{22}
$$

which is always compatible with the Bogomol'nyi bound:

$$
M_{\text{BPS}} = |\breve{P}_1^{(1)}| + |\breve{P}_1^{(2)}| + |\breve{Q}_2^{(1)}| + |\breve{Q}_2^{(2)}|. \tag{23}
$$

These solutions always have nonzero mass [4].

---

[4]Interestingly one can draw parallels between the relation of regular dyonic BPS saturated states to their non-extreme counterparts and that of Type-II supergravity walls [26,27] to their non-extreme counterparts [28,29]. Type-II supergravity walls [27] are planar configurations in $N=1$ supergravity theory, interpolating between two isolated supersymmetric anti-deSitter vacua, whose ADM mass density (in the thin wall approximation) is determined to be $\kappa\sigma_{\text{ext}} = 2(\alpha_1 + \alpha_2)$, where $\Lambda_i \equiv -3\alpha_i^2$ is the cosmological constant on each side of the wall. The corresponding set of non-extreme domain wall solutions [28,29] corresponds to spherically symmetric bubbles with two insides whose ADM mass density $\kappa\sigma_{\text{non-ext}} = 2(\sqrt{\alpha_1^2+\beta^2} + \sqrt{\alpha_2^2+\beta^2})$ is larger than that of the Type-II supergravity wall. Here β parameterizes, analogously as in the case of the non-extreme black holes, a deviation from the extreme limit.

The singularity structure of and thremal properties of regular solutions can be summarized in the following way [13]:

- The case with *all the four charges non-zero* corresponds to black holes with two horizons at $r = 0, -2\beta$ and a time-like singularity hidden behind the inner horizon, *i.e.*, the global space-time is that of the Reissner-Nordström black holes. The Hawking temperature is $T_H = \beta/(\pi\sqrt{\breve{P}_1^{(1)}{}'\breve{P}_1^{(2)}{}'\breve{Q}_2^{(1)}{}'\breve{Q}_2^{(2)}{}'})$ and the entropy is finite $S = \pi\breve{P}_1^{(1)}{}'\breve{P}_1^{(2)}{}'\breve{Q}_2^{(1)}{}'\breve{Q}_2^{(2)}{}'$. As $\beta \to 0$ the space-time is that of extreme Reissner-Nordström black holes.

- The case with *three nonzero charges*, say, $\breve{P}_1^{(1)} = 0$, corresponds to solutions with a space-like singularity located at the inner horizon ($r = -2\beta$), $T_H = \beta^{\frac{1}{2}}/(\pi\sqrt{2\breve{P}_1^{(2)}{}'\breve{Q}_2^{(1)}{}'\breve{Q}_2^{(2)}{}'})$ and $S = 2\pi\beta\breve{P}_1^{(2)}{}'\breve{Q}_2^{(1)}{}'\breve{Q}_2^{(2)}{}'$. As $\beta \to 0$ the singularity coincides with the horizon at $r = 0$.

- The case with *two charges nonzero*, say, $\breve{P}_1^{(1)} \neq 0 \neq \breve{P}_1^{(2)}$, corresponds to solutions with a space-like singularity at $r = -2\beta$, $T_H = 1/(2\pi\sqrt{\breve{P}_1^{(1)}{}'\breve{P}_1^{(2)}{}'})$ and $S = 4\pi\beta^2\breve{P}_1^{(1)}{}'\breve{P}_1^{(2)}{}'$. As $\beta \to 0$ the singularity coincides with the horizon at $r = 0$.

- The case with *one nonzero charge*, say, $\breve{P}_1^{(1)} \neq 0$, corresponds to black holes with a space-like singularity at $r = -2\beta$, $T_H = 1/(2\pi\sqrt{2\beta\breve{P}_1^{(1)}{}'})$ and $S = 8\pi\beta^3\breve{P}_1^{(1)}{}'$. As $\beta \to 0$ the singularity becomes naked.

B. Singular BPS Saturated Solutions

When the relative signs for the two magnetic and/or two electric charges are opposite [14] the BPS saturated generating solutions (18) are always singular, *i.e.*, the singularity takes place at $r_{\text{sing}} > 0$ [5]. In this case the non-extreme solutions are given by (20) with the negative sign in either $\breve{P}_1^{(1)}{}'$ [and/or $\breve{Q}_2^{(1)}{}'$] or $\breve{P}_1^{(2)}{}'$ [and/or $\breve{Q}_2^{(2)}{}'$], in such a way that their ADM mass (21) reduces to that of the BPS saturated solutions (19) as $\beta \to 0$. In particular, when the relative signs of the two magnetic and two electric charges are opposite, the ADM mass for non-extreme solutions:

$$M_{\text{ultra-ext}} = \left|\sqrt{(\breve{P}_1^{(1)})^2 + \beta^2} - \sqrt{(\breve{P}_1^{(2)})^2 + \beta^2}\right| + \left|\sqrt{(\breve{Q}_2^{(1)})^2 + \beta^2} - \sqrt{(\breve{Q}_2^{(2)})^2 + \beta^2}\right| \tag{24}$$

[5] Such purely electrically charged configurations are related to massless black holes, recently found by Behrndt [30], which were obtained by dimensionally reducing supersymmetric gravitational waves of the effective ten-dimensional heterotic string theory. Generalizations to the corresponding multi-black hole solutions and the corresponding exact (in α' expansion) magnetic solutions were given by Kallosh [31], and by Kallosh and Linde [32], respectively. In the latter work the physical properties of such configurations were further addressed; they repel massive particles.

is always *smaller* than the ADM mass for the corresponding BPS saturated states [6]:

$$M_{\rm BPS} = \left||\breve{P}_1^{(1)}| - |\breve{P}_1^{(2)}|\right| + \left||\breve{Q}_2^{(1)}| - |\breve{Q}_2^{(2)}|\right|. \tag{25}$$

Thus these non-extreme solutions are not in the spectrum; singular BPS solutions with the relative signs for both the two electric and two magnetic charges opposite *do not have non-extreme counterparts* compatible with the corresponding Bogomol'nyi bound.

These BPS saturated solutions are singular with a naked singularity at $r = r_{\rm sing} \equiv \max\{\min[|\breve{P}_1^{(1)}|, |\breve{P}_1^{(2)}|], \min[|\breve{Q}_2^{(1)}|, |\breve{Q}_2^{(2)}|]\} > 0$. They repel massive particles, just like in the special case of purely electric [or purely magnetic] solutions [32], however, massless particles with zero angular momentum reach a naked singularity in a finite proper time. In addition these configurations become massless [14] when the magnitudes of the two magnetic and the two electric charges are equal, *i.e.*, when $|\breve{P}_1^{(1)}| = |\breve{P}_1^{(2)}|$ and $|\breve{Q}_2^{(1)}| = |\breve{Q}_2^{(2)}|$.

There are also hybrid singular solutions with the opposite relative signs for one type of charges, say, magnetic ones with $|\breve{P}_1^{(1)}| > |\breve{P}_1^{(2)}|$, and the same relative signs for the other type of charges, *i.e.*, electric ones. These solutions are singular with a singularity, say, at $r_{\rm sing} = \sqrt{(\breve{P}_1^{(2)})^2 + \beta^2} - \beta > 0$, however, the non-extreme solutions with $M_{\text{non-ext}} \geq M_{\rm BPS}$ are in the spectrum, provided $\sqrt{(\breve{P}_1^{(2)})^2 + \beta^2}\left(1/\sqrt{(\breve{P}_1^{(1)})^2 + \beta^2} + 1/\sqrt{(\breve{Q}_2^{(1)})^2 + \beta^2} + 1/\sqrt{(\breve{Q}_2^{(2)})^2 + \beta^2}\right) \geq 0$. Note that this set of solutions always has non-zero ADM mass.

C. Implications of Massless BPS Saturated States for $N = 4$ String Vacua

By exploring the ADM mass formula for the BPS saturated states, preserving $\frac{1}{2}$ of $N = 4$ supersymmetry, Hull and Townsend [3,5] show that massless BPS saturated states can occur at certain points of moduli space of four-dimensional $N = 4$ effective superstring vacua, parameterized in terms of fields and allowed charges of toroidally compactified heterotic string. Such massless BPS states contribute to a phenomenon [3,5] which is a generalization of the Halpern-Frenkel-Kač (HFK) mechanism; namely at special points of moduli space along with the perturbative electrically charged massless string states, which enhance the gauge symmetry of the perturbative string states to the non-Abelian one, there are massless BPS saturated magnetic monopoles and a tower of $SL(2, Z)$ related BPS saturated dyons, which may contribute to a new phase of the enhanced non-Abelian gauge symmetry.

---

[6] One can also draw parallels between the relation of the singular dyonic BPS saturated states to their non-extreme counterparts and that of Type-III supergravity walls [26,27] to their ultra-extreme counterparts [28,29]. Type-III supergravity walls in $N = 1$ supergravity correspond [26,27] to planar configurations, interpolating between two specific isolated supersymmetric anti-deSitter vacua with cosmological constants $\Lambda_i \equiv -3\alpha_i^2$, whose ADM mass density is given by $\kappa\sigma_{\rm ext} = 2(\alpha_1 - \alpha_2)$ $(\alpha_1 > \alpha_2)$. The set of non-extreme solutions corresponds to bubbles of the false vacuum decay whose ADM mass density $\kappa\sigma_{\text{ultra-ext}} = 2(\sqrt{\alpha_1^2 + \beta^2} - \sqrt{\alpha_2^2 + \beta^2})$ is always *smaller* than that of Type-III supergravity walls.

Singular massless BPS saturated states, whose generating solutions are purely magnetic [or purely electric] configurations with magnetic [or electric] charges $\breve{P}_1^{(1),(2)}$ [or $\breve{Q}_2^{(1),(2)}$] opposite in relative signs and equal in magnitude, provide an explicit realization of a class of massless BPS saturated states contributing the generalized HFK mechanism. In addition there are also massless dyonic solutions whose generating solutions have both the two magnetic $\breve{P}_1^{(1),(2)}$ *and* the two electric $\breve{Q}_2^{(1),(2)}$ charges opposite in signs and equal in magnitudes. Since such states preserve $\frac{1}{4}$ of $N = 4$ supersymmetry they belong to massless highest spin $\frac{3}{2}$ supermultiplets and may allow for an appearance of additional massless gravitini, thus providing hints of a new phase with enhanced local supersymmetry [14].

For the purpose of illustrating the enhancement of symmetries at particular points of moduli space [14], we will choose a set of solutions with a particularly simple, however, non-trivial choice for the asymptotic values of the scalar fields (M and S) and the charge configuration [7]; M_∞ is diagonal, *i.e.*, from moduli only the asymptotic values of the diagonal components of the internal metric, g_{mm}, are turned on, and S_∞ is purely imaginary, *i.e.*, the asymptotic value of the axion is set to zero. In this case, the undoing of the corresponding $O(6, 22, R)$ and $SL(2, R)$ transformations on the generating solutions (18) and (20) yields solutions whose ADM mass is now expressed in terms of the asymptotic values M_∞ and S_∞, and charge lattice vectors living in Λ. The ADM mass is then of the form:

$$M_{\rm BPS} = e^{-\frac{\phi_\infty}{2}}|g_{11\,\infty}^{\frac{1}{2}}\beta_1^{(2)} + g_{11\,\infty}^{-\frac{1}{2}}\beta_1^{(1)}| + e^{\frac{\phi_\infty}{2}}|g_{22\,\infty}^{\frac{1}{2}}\alpha_2^{(1)} + g_{22\,\infty}^{-\frac{1}{2}}\alpha_2^{(2)}|. \qquad (26)$$

The allowed magnetic and electric charge lattice vectors, living in a self-dual even Lorentzian lattice Λ, that give rise to massless black holes are, say, $\beta_1^{(1)} = -\beta_1^{(2)} = \pm 1$ and $\alpha_2^{(1)} = -\alpha_2^{(2)} = \pm 1$.

The $SL(2, Z)$ orbits of the generating solution provide a tower of dyonic solutions with the non-zero axion field and the same dependence on the moduli fields M, and thus are massless at the same points of the moduli space as the $O(6, 22, R)$ and $SL(2, R)$ undone generating solution. On the other hand the $O(6, 22, Z)$ orbits of the generating solution provide solutions with a dependence on transformed moduli fields and thus become massless at different points of the moduli points.

We first consider the case of BPS saturated states that preserve $\frac{1}{2}$ of supersymmetries. These are purely, say, electrically charged states with *two* possible charge lattice vectors: $\alpha_2^{(1)} = -\alpha_2^{(2)} = \pm 1$. Such states become massless at the self-dual point of a "one-torus", *i.e.*, when $g_{22\,\infty} = 1$, and form, together with the $U(1)^{(1)+(2)}$ gauge field $(A_{\mu\,2}^{(1)} + A_{\mu\,2}^{(2)})/\sqrt{2}$, the adjoint representation of the non-Abelian $SU(2)_2^{(1)+(2)}$ gauge group, thus enhancing the gauge symmetry from $U(1)_2^{(1)} \times U(1)_2^{(2)}$ to $U(1)_2^{(1)-(2)} \times SU(2)_2^{(1)+(2)}$ at this point of the moduli space [14]. There is also an infinite tower of massless dyonic states, including purely magnetic ones, which correspond to $SL(2, Z)$ orbits of the generating solutions, and thus may contribute to a new phase of enhanced gauge symmetry.

On the other hand for massless BPS saturated states that preserve $\frac{1}{4}$ of supersymmetries, the local supersymmetry is enhanced since the corresponding supermultiplets contain the

[7]Solutions with more general charge configurations and asymptotic values of scalar fields allow for different enhancements of symmetries at different points of moduli space.

gravitino as well as the $U(1)$ gauge field. The possible charge configurations in the charge lattice that could give rise to the massless states are $\beta_1^{(1)} = -\beta_1^{(2)} = \pm 1$ and $\alpha_2^{(1)} = -\alpha_2^{(2)} = \pm 1$. These states become massless at the self-dual point of the corresponding two-torus, *i.e.*, when $g_{11\,\infty} = g_{22\,\infty} = 1$. Since each of these additional massless states belongs to the highest spin $\frac{3}{2}$ supermultiplete, the local supersymmetry is enhanced [14] from $N = 4$ to $N = 8$. As in the previous case, these four massless states combine with the $U(1)_1^{(1)+(2)}$ gauge field $(A_{\mu\,1}^{(1)} + A_{\mu\,1}^{(2)})/\sqrt{2}$ and the $U(1)_2^{(1)+(2)}$ gauge field $(A_{\mu\,2}^{(1)} + A_{\mu\,2}^{(2)})/\sqrt{2}$ to form the adjoint representation of the non-Abelian $SU(2)_1^{(1)+(2)} \times SU(2)_2^{(1)+(2)}$ gauge group, thus enhancing the gauge symmetry from $U(1)_1^{(1)} \times U(1)_1^{(2)} \times U(1)_2^{(1)} \times U(1)_2^{(2)}$ to $U(1)_1^{(1)-(2)} \times U(1)_2^{(1)-(2)} \times SU(2)_1^{(1)+(2)} \times SU(2)_2^{(1)+(2)}$ at this point of moduli space [14]. Additionally there is an infinite tower of dyonic massless states that are related through $SL(2,Z)$ transformations. The occurrence of these new types of states may indicate a transition to a new phase of superstring vacua.

IV. CONCLUSIONS

We have discussed a general set of BPS saturated solutions and their non-extreme counterparts which arise in effective supergravity theories compactified down to four dimensions on manifolds with Abelian isometries. We concentrated on a general class of solutions of the effective $N = 4$ superstring vacua, parameterized in terms of fields of the effective heterotic string theory compactified on a six-torus. Such a program was completed within the Kaluza-Klein sector of the $(4+n)$-dimensional (minimally extended) supergravities compactified on n-tori, *i.e.*, by obtaining explicit results for all the four-dimensional static, spherically symmetric BPS saturated states [9] as well as all of their non-extreme solutions [10]. Within four-dimensional effective $N = 4$ supersymmetric string vacua we presented a class [13,14] of BPS saturated states as well as a class of their non-extreme counterparts which correspond to $O(6,22,Z)$ and $SL(2,Z)$ orbits of dyonic configurations with zero axion; those are configurations whose 28 electric and 28 magnetic charges are subject to one constraint (17).

The generating solution is parameterized by two electric and two magnetic charges and in the non-extreme case additionally by the non-extremality parameter β. The BPS states whose generating solutions are purely electrically charged [or purely magnetically charged] and dyonic preserve $\frac{1}{2}$ and $\frac{1}{4}$ of $N = 4$ supersymmetry, respectively. These solutions fall into different classes depending on the relative signs of the two magnetic and two electric charges of the generating solution. When the relative signs of both two sets of these charges are the same [13] solutions are regular (with a horizon in four dimensions) accompanied by the non-extreme counterparts and always have the ADM mass non-zero. On the other hand when the relative sign of at least one of the two sets of charges of the generating solution is opposite [14] solutions are singular (they have a naked singularity). In the case when both sets of charges have opposite relative sings the singular solutions are unaccompanied by the non-extreme counterparts (whose ADM masses are compatible with the Bogomol'nyi bound) and have zero mass when the magnitudes of the two magnetic and two electric charges are equal [14].

Purely electrically charged BPS saturated states have the same mass spectrum and charge assignments as a subset of perturbative string excitations, and should probably be identified with each other [33]. On the other hand the magnetically charged and dyonic BPS saturated states should be viewed as non-perturbative states of string vacua. Massless BPS states with purely magnetically charged generating solutions (along with a tower of $SL(2,Z)$ orbits) may contribute to the generalized HFK-mechanism [3,5], while massless BPS states with dyonic genetating solutions may contribute to an enhancement of local supersymmetry [14] at such points of moduli space. We would, however, like to caution that at these points of moduli space the effective theory approach breaks down due to the appearance of an infinite tower of new massless modes. In addition since we have studied only classical configurations within an effective theory, quantum corrections may qualitatively alter the features of such massless states. Thus full physical implications of such massless states await further investigation.

The heterotic string compactified on T^4 is conjectured [6,2,1,7] to be equivalent to the type IIA string compactified on $K3$ surface. Since the four-dimensional effective actions of these two theories further compactified on T^2 are related through a field redefinition, one can address the BPS saturated solutions in the type IIA string on $T^2 \times K3$. Whenever the gauge symmetry of the heterotic string is enhanced to a non-Abelian group at particular points of moduli space, $K3$ surface of the type IIA string theory develops quotient singularities (and thus is an orbifold as far as target space geometry is concerned) with certain homology two-cycles of $K3$ collapsing to zero area, giving rise to massless BPS saturated states [2,5]. In the case of the type IIA string on $K3$ surface the point in the moduli space representing the conformal field theory orbifold and the point representing the theory with the enhanced gauge symmetry do not coincide, indicating that the conformal field theory is ill-behaved [34], *i.e.*, perturbative string theory does not describe the full string dynamics.

Since the BPS saturated states presented here are derived within the effective field theory compactified from a higher-dimensional theory, they should presumably have origins as p-brain solutions in higher dimensions [35]; the case of magnetically charged BPS states in the dual type IIA string on $K3 \times T^2$ was studied in [5]. Also, four-dimensional dilatonic black holes can be obtained from dimensionally reduced Brinkmann-type pp-wave solutions in higher dimensions [36,37], which were found for a class of electrically charged black holes which preserve $\frac{1}{2}$ of supersymmetries [37,30]. It is of interest to find out how the dyonic BPS saturated solutions, which preserve $\frac{1}{4}$ of $N = 4$ supersymmetry, are related to the higher-dimensional p-brane solutions and/or pp-wave solutions.

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