hep-th/9507121, IASSNS-HEP-95-63

# Some Comments On String Dynamics

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Three subjects are considered here: a self-dual non-critical string that appears in Type IIB superstring theory at points in K3 moduli space where the Type IIA theory has extended gauge symmetry; a conformal field theory singularity at such points which may signal quantum effects that persist even at weak coupling; and the rich dynamics of the real world under compactification, which may be relevant to some attempts to explain the vanishing of the cosmological constant.

July, 1995

[1] To appear in the proceedings of *Strings '95*, USC, March 1995

My lecture at *Strings '95* focussed on determining the strong coupling behavior of various string theories in various dimensions. Among the main points were the following: $U$-duality of Type II superstrings requires that the strong coupling limit of the Type IIA superstring in ten dimensions is eleven-dimensional supergravity (on $\mathbf{R}^{10} \times \mathbf{S}^1$); one can make sense of heterotic string dynamics in five, six, and seven dimensions and deduce $S$-duality in four dimensions if one assumes that the heterotic string on $\mathbf{R}^6 \times \mathbf{T}^4$ is equivalent to the Type IIA theory on $\mathbf{R}^6 \times \mathrm{K3}$. The detailed arguments have appeared elsewhere [1] and will not be repeated here. Instead I will try to clarify a few related issues, in some cases involving questions that were asked at the meeting.

The issues I will discuss in sections one and two involve mainly the extended gauge symmetry of the Type IIA superstring on $\mathbf{R}^6 \times \mathrm{K3}$ at certain points in moduli space. In section one, I analyze how the Type IIB theory behaves when Type IIA has extended gauge symmetry, and in section two, I discuss the nature of the singularity that occurs in conformal field theory at these points. In section three, I consider instead some issues involving the behavior of the real world under dimensional reduction; these issues may be relevant to the vanishing of the cosmological constant.

## 1. The Type IIB Theory On $\mathbf{R}^6 \times \mathrm{K3}$

The best-established string-string duality is the equivalence between the heterotic string on $\mathbf{R}^6 \times \mathbf{T}^4$ and the Type IIA string on $\mathbf{R}^6 \times \mathrm{K3}$. According to this equivalence, the Type IIA model on $\mathbf{R}^6 \times \mathrm{K3}$ gets extended non-abelian gauge symmetry at certain points in K3 moduli space. Our first question is to determine how the Type IIB theory – likewise compactified on $\mathbf{R}^6 \times \mathrm{K3}$ – behaves at the points in moduli space at which the Type IIA theory develops enhanced gauge symmetry.

It is certainly not the case that the Type IIB theory develops enhanced gauge symmetry at those points. In fact, the Type IIB theory on $\mathbf{R}^6 \times \mathrm{K3}$ has a chiral supersymmetry that simply does not admit gauge multiplets of any kind, abelian or non-abelian. It is very hard for the Type IIB theory to get extra massless particles at special points in K3 moduli space, and these are not needed to account for singularities in the Zamolodchidov metric. The moduli space of vacua of the Type IIB theory on $\mathbf{R}^6 \times \mathrm{K3}$ is apparently [1,2] the locally homogeneous space $SO(21,5;\mathbf{Z})\backslash SO(21,5;\mathbf{R})/(SO(21)\times SO(5))$. The singularities of this space are orbifold singularities, and instead of looking for a description of these in terms of extra massless particles, we can simply interpret them as a sign of restoration of a discrete local gauge symmetry.

However, such discrete symmetry restoration cannot be the whole story of what happens to the Type IIB theory at special points in K3 moduli space. This becomes clear if one makes a further compactification to $\mathbf{R}^5 \times \mathbf{S}^1 \times \mathrm{K3}$. Once this is done, the Type IIB theory becomes equivalent to the Type IIA theory, which does get extra massless particles at certain points in K3 moduli space. The Type IIB theory has to do something peculiar such that one does not get extra massless particles on $\mathbf{R}^6 \times \mathrm{K3}$, but one does get extra massless particles on $\mathbf{R}^5 \times \mathbf{S}^1 \times \mathrm{K3}$, for any radius of the circle.

Let us write down the precise comparison of the Type IIA and Type IIB theories in this situation. We will be a little more general than the six-dimensional case. First, if a $d$-dimensional string theory is compactified to $d-1$ dimensions on a circle of circumference $R$, then the relation between the $d$ and $d-1$-dimensional string coupling constants is

$$\frac{1}{\lambda_{d-1}^2} = \frac{R}{\lambda_d^2}. \tag{1.1}$$

For Type IIA and Type IIB theories compactified from $d$ to $d-1$ dimensions on a circle to be equivalent, they must have the same $\lambda_{d-1}$, so the relation among couplings in $d$ dimensions is

$$\frac{R_A}{\lambda_{d,A}^2} = \frac{R_B}{\lambda_{d,B}^2}. \tag{1.2}$$

Here $R_A$ and $R_B$ are the circumference of the circle as measured in the Type IIA and Type IIB theories, and similarly $\lambda_{d,A}$ and $\lambda_{d,B}$ are the respective string couplings. Bearing in mind also the $T$-duality relation $R_A = 1/R_B$, we can write (1.2) as

$$\frac{1}{\lambda_{d,A}} = \frac{R_B}{\lambda_{d,B}}, \tag{1.3}$$

a relation that of course also holds if $A$ and $B$ are exchanged. We will henceforth write the $d$-dimensional couplings as simply $\lambda_A$ and $\lambda_B$.

Now, suppose that in the K3 moduli space, one is a distance $\epsilon$ from a point at which the Type IIA theory gets an enhanced gauge symmetry. Then the Type IIA theory on $\mathbf{R}^6 \times \mathrm{K3}$ has $W$ bosons with mass a constant times $\epsilon/\lambda_A$; this mass is unchanged in compactification to $\mathbf{R}^5 \times \mathbf{S}^1 \times \mathrm{K3}$. (The $W$ mass is exactly independent of $R_A$, not just approximately so for large $R_A$, because the $W$ boson is in a BPS-saturated "small" supermultiplet.) According to (1.3), the mass of the $W$ meson in the Type IIB theory on $\mathbf{R}^5 \times \mathbf{S}^1 \times \mathrm{K3}$ is then

$$M_W = \frac{\epsilon R_B}{\lambda_B}. \tag{1.4}$$

What are we to make of (1.4)? What sort of state has a mass proportional to $R_B$? The answer to this question, clearly, is that this is the mass of a string wrapping around the circle of circumference $R_B$. So we can interpret (1.4) to mean that the Type IIB theory on $\mathbf{R}^6 \times \mathrm{K3}$ has some kind of cosmic string with a string tension

$$T = \frac{\epsilon}{\lambda_B}. \tag{1.5}$$

After compactification on a circle, the $W$ boson then arises as a particular winding state of this string.

The string whose tension is given in (1.5) is certainly not the fundamental Type IIB superstring. Rather, we must apparently begin with the self-dual super-three-brane solution of the Type IIB theory in ten dimensions [3], whose tension is of order $1/\lambda_B$. As $\epsilon$ goes to zero, a two-sphere $\mathbf{S}$ in the K3 collapses, having an area proportional to $\epsilon$ [1]. $\mathbf{S}$ is self-dual, in the sense that its Poincaré dual cohomology class is self-dual. As in Strominger's discussion of the conifold singularity in four dimensions [4], when one compactifies below ten dimensions, one can get a $p$-brane for $p < 3$ by wrapping the ten-dimensional super-three-brane around a cycle of dimension $3 - p$. In particular, upon K3 compactification, one can wrap the three-brane around $\mathbf{S}$ to get a string in six dimensions. The tension of this string will be $1/\lambda_B$ (the tension of the three-brane) times $\epsilon$ (the area of $\mathbf{S}$), in agreement with (1.5). Since $\mathbf{S}$ is self-dual, the string we get in six dimensions is likewise self-dual (that is, the three-form $H = dB$ that the string produces is self-dual). It is thus similar to the six-dimensional self-dual string described in [5].

This self-dual string is a non-critical string in six dimensions; its tension (1.5) can be vastly below the string and Planck scales. For very small $\epsilon$, one should interpret this as a string that is far too light to influence gravity and which simply propagates in six-dimensional Minkowski space. There is such a string theory for each possible type of isolated singularity ($A$, $D$, or $E$) of the K3. (The formulation in the last paragraph with a single collapsing two-sphere was strictly appropriate only for $A_1$.) Obviously, these non-critical six-dimensional strings are quite different from anything we really understand presently. The fact that these objects have not been discovered in traditional constructions of string theories actually follows from the fact that they are self-dual, so that (as in Dirac quantization of electric and magnetic charge) the string coupling is necessarily of order one.

A weakly coupled string theory with string tension $T$ has long-lived excitations with masses proportional to $\sqrt{T}$. If this formula can be used in the present case - which is not entirely clear - then the Type IIB theory near the special K3 points has long-lived non-perturbative "string" states with masses in string units proportional to $\sqrt{\epsilon/\lambda_B}$. In Einstein units, these states have masses of order $\sqrt{\epsilon}$. In heterotic string units, the mass is of order $\sqrt{\epsilon/\lambda_h}$ (times the masses of elementary string states), with $\lambda_h$ the heterotic string coupling constant.

### *1.1. Reduction To Four Dimensions*

Now, let us recall that, although the Type IIB theory on $\mathbf{R}^6 \times$ K3 does not have any gauge fields, it does have a plethora of two-forms (twenty-one with self-dual field strength and five with anti-self-dual field strength). One of them - say $B$ - arises by writing the four-form $C$ of the ten-dimensional Type IIB theory as $C = B \wedge G$, where $G$ is a self-dual harmonic two-form on K3 supported (for small $\epsilon$) very near $\mathbf{S}$, and $B$ is a two-form on $\mathbf{R}^6$. $B$ is self-dual (that is, it has a self-dual field strength) because $C$ and $G$ are self-dual.

If we compactify from $\mathbf{R}^6$ to $\mathbf{R}^5 \times \mathbf{S}^1$, then the components $B_{i6}$ $(i = 1 \ldots 5)$ of $B$ become a gauge field $A_i$ in five dimensions. One might think that one would also get a five-dimensional two-form from $B_{ij}$, but in five-dimensions a two-form is dual to a one-form, and self-duality of $B$ in six dimensions becomes in five dimensions the statement that $B_{ij}$ is dual to $A_i$. Thus the independent degrees of freedom are all in $A_i$. The string winding states discussed above carry the electric charge that is coupled to $A_i$.

The further compactification to four dimensions, replacing $\mathbf{R}^6$ by $\mathbf{R}^4 \times \mathbf{T}^2$, has been discussed at the field theory level in [6]. The self-dual two-form $B$ in six dimensions again gives rise in four dimensions to only *one* $U(1)$ gauge field - as $A_i = B_{i6}$ and $\widetilde{A}_i = B_{i5}$ are dual.

Going back to string theory, it follows that the two types of winding states of the non-critical string - strings wrapping around the first or second circle in $\mathbf{T}^2 = \mathbf{S}^1 \times \mathbf{S}^1$ - carry electric and magnetic charge for this one $U(1)$ gauge field. The coupling parameter $\tau$ of the four-dimensional $U(1)$ theory is simply the $\tau$ of the $\mathbf{T}^2$. The four-dimensional theory has manifest $S$-duality coming from the diffeomorphisms of the $\mathbf{T}^2$. (If we bear in mind that $SL(2,\mathbf{Z})_U$ of Type IIB is $SL(2,\mathbf{Z})_T$ of Type IIA, this is equivalent to the fact

[1] that string-string duality transforms $S$-duality of the heterotic string into $T$-duality of the Type IIA string.)

What makes this interesting is that it gives a manifestly $S$-dual formulation of $N = 4$ supersymmetric Yang-Mills theory. In fact, for very small $\epsilon$, the $W$ bosons and monopoles (which come from string winding states and have masses of order $\epsilon$) are much lighter than other string excitations (which as we noted above generically have masses of order $\epsilon^{1/2}$). Thus, in this limit, the manifestly $S$-dual theory of the self-dual string on $\mathbf{R}^4 \times \mathbf{T}^2$ should go over to $N = 4$ supersymmetric Yang-Mills theory on $\mathbf{R}^4$.

This may well be the proper setting for understanding $S$-duality of the $N = 4$ gauge theory. Thus, if one asks, "How can the $S$-duality of $N = 4$ Yang-Mills theory be made obvious?" one answer is that this can be done by embedding $N = 4$ supersymmetric Yang-Mills theory in the heterotic string and then mapping to a Type IIA theory by using string-string duality. The weakness of this answer is that it embeds the gauge theory in a problem with many other features - such as gravity - that may not be material. One would like to "flow to the infrared," eliminating as many degrees of freedom as possible, and obtaining the minimal theory in which the $S$-duality is still manifest. The self-dual string in six dimensions may be the answer to this question.

The self-dual string in six dimensions does not look easier than the Type IIB model that we started with; certainly we understand it less. Nevertheless, it might be the right structure for understanding the four-dimensional field theory. The situation would be somewhat similar to the study of critical phenomena. In that subject, one can start with an elementary, manifestly well-defined system such as a lattice Ising model. In seeking to describe the critical behavior, the right object to introduce turns out to be a continuum quantum field theory even though this is superficially far less elementary (existence is far less obvious, for instance) and superficially there are far more degrees of freedom. The field theory is the right object for critical phenomena because it contains all the universal information (about the critical point) and nothing else. The more elementary-looking Ising model has the field theory as a difficult-to-extract limit; the additional information it contains is extraneous. The self-dual string may similarly be the minimal manifestly $S$-dual extension of the $N = 4$ super Yang-Mills theory.

### *1.2. Non-Local Critical Points In Four Dimensions*

Likewise, natural answers to other questions about gauge theory dynamics may involve non-critical strings of one kind or another. For instance, there appears to be [7] an $N = 2$ superconformal critical point in four dimensions with massless electrons and monopoles alike. A natural understanding of this critical point may be difficult to achieve in field theory – where it is hard to put electrons and monopoles on the same footing. Perhaps one should seek a natural description by some sort of non-critical string theory.

Certainly critical string theory gives a natural framework for describing generalizations of the critical point considered in [7]. That critical point, first of all, can be embedded in string theory by simply considering a Calabi-Yau manifold with a singularity that looks like

$$t^2 + w^2 + y^2 + x^3 = \epsilon. \tag{1.6}$$

This manifold contains [8] two $\mathbf{S}^3$'s that collapse as $\epsilon \to 0$. These two $\mathbf{S}^3$'s have a non-zero intersection number, with the result that the charged black holes that arise as $\epsilon \to 0$ are respectively electrically and magnetically charged with respect to the *same* $U(1)$. In fact, the description of the critical point in [7] involves essentially the family of complex curves

$$y^2 + x^3 = \epsilon. \tag{1.7}$$

In this case, a pair of $\mathbf{S}^1$'s with a non-zero intersection number collapse as $\epsilon \to 0$. Obviously, (1.6) is obtained from (1.7) by adding new variables that appear quadratically, a standard operation that preserves many aspects of the singularity.

An $SU(N)$ generalization of this critical point that was explained briefly in [7] involves the family of curves $y^2 + x^N = \epsilon$ and could be imitated by a Calabi-Yau singularity $w^2 + z^2 + y^2 + x^N = \epsilon$. More generally, the $N = 2$ $SU(N)$ gauge theory with a massive adjoint hypermultiplet can realize an arbitrary singularity of the form $F(x, y) = \epsilon$ [9], corresponding to a Calabi-Yau singularity

$$t^2 + w^2 + F(x, y) = \epsilon. \tag{1.8}$$

From the Calabi-Yau point of view, we can write many more objects, such as a general hypersurface singularity $F(t, w, x, y) = 0$. To restrict oneself to singularities that are at a finite distance in the Zamolodchikov metric, one should consider what are called the canonical singularities (reviewed in [10]) of which (1.8) is an example. As there are many other canonical singularities, it may turn out that the natural classification and description of such non-local fixed points involves the canonical singularities and the string theory dynamics they produce.

## 2. The Singularity Of The Conformal Field Theory

At the time of *Strings '95*, two points about the extended gauge symmetry of the Type IIA superstring on $\mathbf{R}^6 \times$ K3 were particularly puzzling:

(1) Although it was clear that the extended gauge symmetry occurred only when one or more two-spheres collapse to zero area, it was not clear why such collapse would lead to the appearance of extra massless gauge bosons.

(2) More generally, it seemed that the collapse of a two-sphere could lead to an interesting novelty in string theory only if there is some sort of breakdown of the conformal field theory. The example of orbifolds, which certainly contain collapsed two-spheres (which are restored to non-zero area if one blows up the orbifold singularities by adding suitable twist fields to the world-sheet Lagrangian) seemed to show that there was absolutely no singularity in the conformal field theory when a two-sphere collapses.

The first point was soon settled by Strominger [4]: a two-brane wrapped around a two-sphere goes to zero mass when the two-sphere collapses to zero area. (Strominger discussed mainly compactification on Calabi-Yau threefolds, but the application to K3 is evident.)

The second point was settled more recently by Aspinwall [11] who showed that extended gauge symmetry arises only when there is a collapsed two-sphere *and in addition a certain world-sheet theta angle vanishes.* Orbifolds, that is K3's that are of the form $\mathbf{T}^4/\Gamma$ with $\Gamma$ a finite group, contain collapsed two-spheres, but the relevant theta angles are non-zero.

In fact, associated with the $\mathbf{S}^2$ are four real parameters: the area, the theta angle, and two parameters associated with the complex structure. Aspinwall's claim is that all four parameters must vanish to get extended gauge symmetry.

Since orbifolds no longer serve as a counterexample, the likelihood now arises that the K3 conformal field theory is singular at the points at which extended gauge symmetry appears. That is the question that I wish to address in the present section. I will analyze the question by a mean field theory approach, and suggest an answer that seems natural. First we will look at a problem – which proves to be analogous – of an instanton shrinking to zero size; then we will consider the K3 case; and finally we will discuss conifold singularities of threefolds in a similar spirit.

*2.1. Instanton Shrinking To Zero Size*

In [12], I described a mean field approach to sigma models that are related to Yang-Mills instantons. This was achieved by constructing two-dimensional linear sigma models with $(0,4)$ world-sheet supersymmetry, which appear to flow in the infrared to conformal field theories related to Yang-Mills instantons on $\mathbf{R}^4$.

I will not here recall the full details of the construction. Suffice it to note that the bosons are four massless fields $X^{BY}$, and additional fields $\phi^{B'Y'}$ that are generically massive (each of the four types of index $B, B', Y, Y'$ is acted on by a different symmetry group); inclusion of the massive fields makes it possible to write a simple polynomial Lagrangian that leads (after integrating them out) to very complicated Yang-Mills instantons. In the one-instanton sector, the description is particularly simple; there are four $\phi$'s, and the potential energy is

$$V = \frac{1}{8}\left(X^2 + \rho^2\right)\phi^2 \tag{2.1}$$

with $\rho$ the instanton size. For $\rho$ large (compared to the string scale), $\phi$ is everywhere very heavy, and after integrating it out one gets something very much like an ordinary Yang-Mills instanton, embedded in string theory. For $\rho$ of order the string scale, the stringy corrections to the instanton may be large. The point on which we wish to focus here is the behavior of the conformal field theory when $\rho$ goes to zero. If we take (2.1) literally, we appear to learn that the "target space," obtained by setting $V = 0$, acquires a second branch precisely at $\rho = 0$. Apart from the usual space-time $M$ with $X$ unrestricted and $\phi = 0$, we get a second world $M'$ with $\phi$ unrestricted and $X = 0$. The linear sigma model at $\rho = 0$ in fact has a symmetry that exchanges $X$ and $\phi$.

Before accepting the strange idea that when an instanton reaches zero size, a second branch in space-time appears, let us compare to another approach to the problem, in which one simply solves the space-time equation for the instanton including terms of order $\alpha'$ [13,14]. In this approximation, the metric on a space-time that contains an instanton of scale parameter $\rho$ centered on the origin turns out to be

$$ds^2 = (dX)^2 \cdot \left(e^{2\phi_0} + 8\alpha' \frac{X^2 + 2\rho^2}{(X^2+\rho^2)^2}\right) \tag{2.2}$$

(with $\phi_0$ the value of the dilaton at infinity). The picture is quite different from what one seems to get from mean field theory. As $\rho$ goes to zero, instead of a second branch appearing, the space-time develops a long tube, with the result that at $\rho = 0$, $X = 0$ is infinitely far away.

It is true that (2.2) is based only on solving the low energy equations to lowest order in $\alpha'$. However, one can show [14] that at $\rho = 0$, the long tube that arises near $X = 0$ corresponds to an exact soluble conformal field theory (a WZW model times a free field with a linear dilaton), and this gives credence to the idea that the structure seen in (2.2) is essentially correct.

On the other hand, there is the following problem in the "two-branch" scenario that mean field theory seems to suggest. The global $(0,4)$ supersymmetry algebra admits an $SU(2) \times SU(2)$ group of $R$ symmetries. To extend the global $(0,4)$ algebra to a superconformal algebra, one of the two $SU(2)$'s is included in the algebra and so is generated, in particular, by purely right-moving currents. (The second $SU(2)$ is not part of the superconformal algebra, but might be realized as a symmetry group acting by outer automorphisms on the algebra; if so the conserved current generating this symmetry can have both left and right-moving pieces.)

Now, the linear sigma model of the instanton has at $\rho = 0$ the full $SU(2) \times SU(2)$ symmetry, called $F \times F'$ in [12]. If this model flows in the infrared to a $(0,4)$ superconformal theory, is it $F$ or $F'$ that appears in the superconformal algebra? The basic fact here is that $F$ acts by rotations of $X$ but acts trivially on $\phi$, and $F'$ rotates $\phi$ but acts trivially on $X$. The currents generating the $F$ action on $X$ are $X^{AY}\partial_\alpha X^B{}_Y$ and have both left and right-moving parts which are not separately conserved even if $X$ is treated as a free field (which is valid for $X$ large enough); the currents generating the $F'$ action on $\phi$ are similar.

Therefore, in any superconformal description that contains $X$, $F$ cannot appear in the superconformal algebra, and in any description that contains $\phi$, $F'$ cannot appear in the superconformal algebra. If this theory flows to a superconformal field theory in the infrared, then (short of more exotic possibilities in which the pertinent symmetries cannot be seen in the linear sigma model at all) the symmetry between $X$ and $\phi$ must be "spontaneously broken": there must be two different superconformal limits, one living on the $X$ branch with $F'$ in the algebra, and one living on the $\phi$ branch with $F$ in the algebra.

Given that in the linear sigma model the distance between the $X$ and $\phi$ branches appears to be finite (they even meet at $X = \phi = 0$) how is this possible? It must be that as one flows to the infrared, the distance grows from any given point on either branch to the point $X = \phi = 0$ where they meet; in the limit of the conformal field theory, this distance must become infinite and the two branches separate. What makes this plausible is that near $X = \phi = 0$, the classical linear sigma model does not give a good approximation to the metric on the target space; loop diagrams are proportional to negative powers of the mass, that is, to powers of $1/X^2$ or $1/\phi^2$.

Thus, we have recovered, or at least rationalized, the qualitative structure of (2.2) from the linear sigma model. To avoid a contradiction with the properties of $F$ and $F'$, the two branches must be decoupled at $\rho = 0$, and this most reasonably happens by $X = 0$ being infinitely far away (from finite points on the $X$ space) when $\rho = 0$, as we see in (2.2).

### *2.2. Singularities Of* K3*'s*

Now I wish to describe a similar mean field theory by which we can study orbifold singularities of K3. For simplicity, we will discuss only the $\mathbf{Z}_2$ orbifold singularity, so we will analyze simply the $(4,4)$ superconformal field theory with target space $\mathbf{R}^4/\mathbf{Z}_2$. [2] In the twisted sector of this orbifold, there are four moduli (three of them involving the classical blow-up and deformation of the singularity and one the world-sheet theta angle). Since it is difficult to add twist fields to the Lagrangian with finite coefficients (as one must do, according to [11], to reach the point relevant to extended gauge symmetry), we will study instead a $(4,4)$ linear sigma model in which all four parameters can be exhibited. The goal is to recover the claim that a singularity only arises when all four parameters have special values and to learn something about the singularity.

Most of the construction and analysis of the linear sigma model are quite similar to the discussion of the $(2,2)$ case in [17], so we will be brief. The model we will discuss is a two-dimensional $(4,4)$ globally supersymmetric theory consisting of a $U(1)$ gauge theory coupled to two hypermultiplets $H_i$, $i = 1,2$, of the same charge. From an $N = 2$ point of view (in what follows we count supersymmetries in two dimensions, so what we call $N = 2$ and $N = 4$ are related to $N = 1$ and $N = 2$ in four dimensions), each $H_i$ consists of a chiral multiplet $M^i$ of charge 1 and another chiral multiplet $\widetilde{M}_i$ of charge $-1$. The $(4,4)$ $U(1)$ gauge multiplet contains four scalars $\phi_i$ (as one can see by dimensional reduction from more familiar facts in four or six dimensions). The potential energy of the theory is

$$V = \frac{1}{2e^2}\left(\vec{D}(H) - \vec{r}\right)^2 + \frac{1}{2}|H|^2|\phi|^2. \tag{2.3}$$

[2] The general $A-D-E$ case can be discussed using a linear sigma model constructed via Kronheimer's description of the $A-D-E$ singularities as hyper-Kahler quotients [15]. Kronheimer's construction specializes for $A_1$ to the description we will give below, which also entered in [16].

This formula is analogous to equation (3.2) in [17] for the $(2,2)$ case. The notation is as follows. For $N=4$ in two dimensions, there are three $D$ functions $\vec{D}(H)$, quadratic and homogeneous in the components of $H$, generalizing the more familiar one $D$ function for two-dimensional $N=2$ theories. (They transform as a vector under an $SU(2)$ $R$ symmetry of the model that will be described later.) The three constants $\vec{r}$ generalize the familiar Fayet-Iliopoulos interaction of $N=2$ theories. The four relevant operators associated with the $A_1$ singularity are in fact the three components of $\vec{r}$ and the $\theta$ angle of the $U(1)$ gauge theory.

The space of zero energy classical states with $H \neq 0$ (and therefore $\phi = 0$) is obtained by setting $\vec{D} - \vec{r} = 0$ and diving by the gauge group $U(1)$. (The combined operation is the hyper-Kahler quotient [18], which was discovered in precisely the present context.) Let us carry this out explicitly for the case that $\vec{r} = 0$. From an $N=2$ point of view, the three $D$ terms are the real and imaginary part of a holomorphic function of chiral superfields

$$D_+ = M^1 \widetilde{M}_1 + M^2 \widetilde{M}_2 \tag{2.4}$$

and the usual $N=2$ $D$ term $D_0 = \sum_i (|M^i|^2 - |\widetilde{M}_i|^2)$. Dividing by $U(1)$ and setting $D_0 = 0$ is equivalent (according to geometric invariant theory [19]) to working with the $U(1)$ invariant holomorphic functions ${S^i}_j = M^i \widetilde{M}_j$. Upon setting $D_+ = 0$, there are three such independent functions, $A = M^1 \widetilde{M}_1 = -M^2 \widetilde{M}_2$, $B = M^1 \widetilde{M}_2$, $C = M^2 \widetilde{M}_1$. These obey the identity

$$A^2 + BC = 0. \tag{2.5}$$

This complex equation in $\mathbf{C}^3$ is a standard description of the $A_1$ singularity, so we have established the fact that the classical moduli space of $\phi = 0$ vacua, at $\vec{r} = 0$, is $\mathbf{R}^4/\mathbf{Z}_2$. If one repeats the computation at $\vec{r} \neq 0$, one gets a non-singular space, exhibiting the $\vec{r}$ as the three parameters associated with deforming and resolving the singularity. How the theta angle enters the story will be seen momentarily.

So far we have discussed only the zero energy states of $\phi = 0$. What about zero energy states of $\phi \neq 0$? Inspection of (2.3) shows that such states exist only if $H = 0$, and therefore that one needs also $\vec{r} = 0$. Classically, these are sufficient requirements, but quantum mechanically, as explained in [17], one requires also $\theta = 0$. The reason for this is that on the branch of $\phi \neq 0$ but $H = 0$, the low energy theory is a free $U(1)$ gauge theory;

turning on a non-zero theta angle gives a term in the energy $|\theta/2\pi|^2$. So the Coulomb branch of zero energy states with $\phi \neq 0$, $H = 0$[3] exists only for $\vec{r} = \theta = 0$.

Now the Higgs branch – that is, the branch of low energy states with $H \neq 0$ – presumably flows for any values of $\vec{r}, \theta$ to a $(4,4)$ conformal field theory in the infrared. Our question is: for what values of $\vec{r}$, $\theta$ is this conformal field theory singular? As explained in [17], a singularity can only arise when the vacuum state on the Higgs branch can spread onto the Coulomb branch. The situation is most easily described if (as in [17]) we work on a compact K3 manifold that is developing an $A_1$ singularity rather than, as above, working simply on $\mathbf{R}^4/\mathbf{Z}_2$. (Unfortunately, working on a compact K3 would have made it difficult to explicitly exhibit the four parameters associated with the singularity.) Then we would simply say that unless $\vec{r} = \theta = 0$, the theory has a normalizable vacuum state, which ceases to be normalizable at $\vec{r} = \theta = 0$ when the vacuum can spread onto the Coulomb branch. On $\mathbf{R}^4/\mathbf{Z}_2$, the vacuum is not normalizable to begin with, but the new non-compactness from the appearance of the Coulomb branch still gives a singularity.

So we have learned that a singularity appears in the conformal field theory precisely upon setting all four parameters to zero – and thus the conformal field theory is singular precisely where, according to [11], the extended gauge symmetry appears. We would like to learn more about the nature of the singularity.

To do so, as in the $(0,4)$ problem that was discussed above, we want to look at the possible global symmetries that can appear in the $(4,4)$ superconformal algebra in the infrared. These symmetries are very conveniently seen by starting in *six* dimensions with a $U(1)$ gauge multiplet coupled to the two hypermultiplets $H^i$. There is a global $SU(2)$ symmetry $G$: the group of linear transformations of the eight real components of the $H^i$ that preserves the hyper-Kahler structure and commutes with the gauge group. The fact that $G$ preserves the hyper-Kahler structure means that it commutes with all the supersymmetries and so will not be seen as an $R$ symmetry under any conditions. There is also, already in six dimensions, an $SU(2)$ $R$ symmetry $K$; it acts trivially on the gauge fields (and non-trivially, therefore, on their fermionic partners), while the bosonic part of $H^i$ transforms with $K = 1/2$. Dimensional reduction from six to two dimensions produces

[3] I will use the terms "Coulomb branch" and "Higgs branch" that were applied in [16] to related theories in four dimensions, but the meaning is rather different in two dimensions: because of two-dimensional infrared divergences, these branches do not parametrize a family of quantum vacua; rather, they are target spaces of low energy supersymmetric sigma models.

an extra $SO(4)$ symmetry, which we write as $L \times L'$, with $L$ and $L'$ being copies of $SU(2)$. $L$ and $L'$ act trivially on the bosons in $H^i$ (because they are scalars in six dimensions), but the scalars in the two-dimensional vector multiplet (because of the way they arise from a vector in six dimensions) transform in the $(1/2, 1/2)$ representation of $L \times L'$.

Now we can analyze the possible $R$ symmetries. A $(4,4)$ superconformal field theory will have left and right-moving $SU(2)$ $R$ symmetries. For a conformal field theory arising from the Higgs branch, these $R$-symmetries must act trivially on the scalar components of $H^i$. From the above description, the symmetries with the right properties are $L$ and $L'$, and it is easy to see in perturbation theory (valid for large $H$) that $L$ and $L'$ do emerge as the $R$ symmetries on the conformal field theory of the Higgs branch.

Setting $\vec{r} = \theta = 0$, we can also analyze the singularities of the Coulomb branch. On the Coulomb branch, the $R$ symmetries must act trivially on the scalars in the vector multiplet, so $L$ and $L'$ are forbidden; the only possibility from what we have seen above is $K$. Perhaps $K$ decomposes in the infrared into separately conserved left and right-moving pieces.

Just as in our discussion of the $(0,4)$ case, the fact that different $R$ symmetries enter in the superconformal algebra on the different branches must mean that by the time one flows to a conformal field theory, the branches no longer meet as they do classically. The most natural way for this to happen is once again that in the conformal field theory limit, the point $H = \phi = 0$ should be infinitely far away from the rest of the Higgs branch.

So we are led to look for a $(4,4)$ superconformal field theory, with $\hat{c} = 4$, and the following characteristics. The model should be a sigma model with a four-dimensional target space that is asymptotic to $\mathbf{R}^4/\mathbf{Z}_2$ at spatial infinity, while also one point has been deleted from $\mathbf{R}^4/\mathbf{Z}_2$ and in some way projected to infinity. Happily, such a conformal field theory is known. It is essentially the so-called symmetric five-brane [14], which can be described as a four-dimensional sigma model with a target space metric that coincides with the $\rho = 0$ limit of equation (2.2). This (or more exactly its quotient by $\mathbf{Z}_2$ to get the right asymptotic behavior) has just the properties that we want. (There is a puzzle, however, about the presence in the symmetric five-brane of a $B$ field with non-zero field strength - absent for conventional K3's at least in sigma model perturbation theory. Perhaps it is a novel non-perturbative effect.)

Our proposal, then, is that at a point of extended gauge symmetry, the singular behavior of the conformal field theory is that it develops an infinite tube like that of the symmetric five-brane. Hopefully, this understanding of the singularity may lead in future to a better understanding of how extended gauge symmetry comes about. One simple remark that can be made right away is that, no matter how small the string coupling constant may be on most of the K3, it blows up (because of the linear dilaton) as one goes down the infinite tube of the five-brane. Thus, once the K3 in conformal field theory develops such a tube, there is no further surprise in the fact that – no matter how small the string coupling constant is – there are quantum effects that do not get turned off. The place where these effects occur just moves "down the tube" as the string coupling constant is made smaller. This seems to shed some light on some of the puzzles of the last few months.

*Zero Area?*

Finally, I want to resolve a small paradox that this discussion may present. Classically, as one takes $\vec{r} \to 0$, a two-sphere collapses to zero size – and in the "two-brane" picture, this is why massless charged gauge bosons appear. At first sight we have lost this explanation upon replacing $\mathbf{R}^4/\mathbf{Z}_2$ with the symmetric five-brane. But what is written in (2.2) is (at $\rho = 0$) the *sigma-model metric* of the symmetric five-brane. This metric is conformally flat, as is evident in the way it has been written, and in fact the *Einstein metric* of the symmetric five-brane is simply the flat metric on $\mathbf{R}^4/\mathbf{Z}_2$. So the "collapsing two-sphere" mechanism survives the better understanding of the singularity – but must be implemented in the Einstein metric.

### *2.3. Conifolds In Calabi-Yau Threefolds*

Now we will – more briefly – discuss conifolds in Calabi-Yau threefolds in a similar fashion. The general argument really applies to any isolated singularities that will arise by varying the complex structure of a Calabi-Yau manifold. Via mirror symmetry (or more explicitly via linear sigma models [17,20]), a similar story can be told for singularities that arise upon varying Kahler parameters.

We consider a $(2,2)$ model in two dimensions with chiral superfields $P, X, Y, Z$, and $T$ and superpotential

$$W = P(XY + ZT - \epsilon). \tag{2.6}$$

For $\epsilon = 0$, the classical states of zero energy – which are precisely the critical points of $W$ – are described by

$$XY + ZT = \epsilon, \qquad P = 0. \tag{2.7}$$

For $\epsilon \neq 0$, the equation $XY + ZT = \epsilon$ describes a smooth hypersurface $V_\epsilon$ in $\mathbf{C}^4$. For $\epsilon = 0$, $V_\epsilon$ develops a conifold singularity. The singularity does not in itself show that the low energy conformal field theory is singular; we are familiar with classical singularities (such as orbifold singularities at $\theta \neq 0$) that do not correspond to singularities in conformal field theory. What really shows that a singularity appears in the field theory is that precisely at $\epsilon = 0$, a second branch of critical points appears, with $P \neq 0$, $X = Y = Z = T = 0$. We will call this the $P$ branch. The vacuum constructed on the original "$V$ branch" spreads onto the $P$ branch and (even when such a conifold singularity is embedded in a compact Calabi-Yau manifold) its normalizability is lost. In [20], a pole in Yukawa couplings at $\epsilon = 0$ was deduced directly from the appearance of the $P$ branch.

Now let us use the $R$ symmetries to learn a little more about the superconformal field theories to which these theories presumably flow in the infrared. To get the necessary $R$ symmetry, [4] we need a holomorphic $U(1)$ action on $P, X, Y, Z, T$ under which $W$ has charge two. The only appropriate symmetry, for $\epsilon \neq 0$, is the one that assigns charge two to $P$ and charge zero to $X, Y, Z$, and $T$. This must therefore be the $R$ symmetry for non-zero $\epsilon$, and by continuity it will therefore be the $R$ symmetry on the $V$ branch also at $\epsilon = 0$. (That $W = 0$ for the bosonic fields in $X, Y, Z$, and $T$ makes it possible for $W$ to be the $R$-symmetry on this branch.) Since this symmetry acts non-trivially on the bosonic part of $P$, it must be that at $\epsilon = 0$, by the time one flows to a conformal field theory, the $P$ branch is disconnected from the $V$ branch. Precisely at $\epsilon = 0$, the theory has a new $R$ symmetry – the one under which $P$ is neutral and the other fields all have charge one – which has the right properties to appear in the superconformal algebra on the $P$ branch.

So we learn again – as in the earlier discussion of $(0,4)$ and $(4,4)$ models – that when one flows to conformal field theory, the various branches are disconnected. As before, the most plausible interpretation of this is that the sigma model metric of the conformal field theory at $\epsilon = 0$ is an incomplete metric, with $X = Y = Z = T = 0$ being infinitely far away.

*Relation To Quantum Description*

[4] We need separate left and right-moving $R$ symmetries for a $(2,2)$ model in two dimensions. One combination of these (which if one constructs the model by dimensional reduction from four dimensions arises as the rotation of the two extra dimensions) is present in all models of this kind, so there is precisely one model-dependent symmetry to be described.

In each case that we have examined, the target space apparently becomes effectively non-compact at the point where the conformal field theory is singular. For instance, in compactification on $\mathbf{R}^4 \times V$, with $V$ a Calabi-Yau three-fold, space-time is four-dimensional macroscopically as long as $V$ is smooth and compact. But when $V$ acquires a conifold (or other) singularity, we have argued that the sigma model metric on $V$ becomes incomplete, strongly indicating that the space-time becomes at least five-dimensional macroscopically, in the sigma model description. It may be quite different in the Einstein metric. For the $(0,4)$ and $(4,4)$ cases, where the sigma model metric at the point analogous to $\epsilon = 0$ is known, precisely one new macroscopic dimension appears in the sigma model metric, but not in the Einstein metric. For threefolds, we have much less information.

The singularities we have found for K3's and Calabi-Yau threefolds in this worldsheet treatment are much more drastic than what has been argued quantum mechanically. Let $\lambda$ be the string coupling constant and $\epsilon$ a parameter measuring the distance in coupling constant space from the singularity. For $\epsilon = 0$ with $\lambda \neq 0$, it has been argued that what happens at the singularity is that finitely many massless particles appear (charged gauge bosons or charged hypermultiplets for Calabi-Yau twofolds or threefolds). For $\epsilon = \lambda = 0$, we are instead finding a noncompactness which means that infinitely many particles are going to zero mass in the low energy description. Turning on quantum mechanics makes the behavior much gentler; in particular, the effective dimension of space-time is not changed. Perhaps a better understanding of the singular behavior of the conformal field theory would enable one to understand in a more *a priori* fashion what happens quantum mechanically.

There are other reasons, apart from what I have given here, for suspecting that infinitely many particles become massless at $\lambda = \epsilon = 0$. First of all, *some* particles must become massless in the conformal field theory when one sets $\epsilon = 0$, because for instance the one loop conformal field theory calculation in [21] develops a singularity at $\epsilon = 0$. This singularity somehow comes from massless elementary string states running around the loop (and not from charged Ramond-Ramond black holes, which are not present in the conformal field theory!). The charged black holes are probably the only natural way to get this sort of singularity from the contributions of finitely many particles, so it is not too surprising that the conformal field theory method of generating this singularity would turn out to involve infinitely many light states.

## 3. Dimensional Reduction Below Four Dimensions

The final subject that I will discuss here concerns an attempt to apply some of the new string theory ideas directly to nature. Recently, I suggested [22] that the vanishing of the cosmological constant in nature results from the existence of an interpretation of the four-dimensional world as a strong coupling limit of a supersymmetric world in three dimensions. The idea is that a mode which a three-dimensional observer interprets as the dilaton is interpreted by a four-dimensional observer as the radius of the fourth dimension. Thus in the strong coupling limit of the three-dimensional theory, the world becomes four-dimensional and the dilaton is reinterpreted as part of the four-dimensional metric tensor, so that there is no dilaton in the four-dimensional sense. In three dimensions, for generic coupling, the cosmological constant vanishes but [23] the bosons and fermions are not degenerate; the limiting four-dimensional world hopefully inherits these properties.

A crucial question about this scenario is what the dynamics looks like from the three-dimensional point of view, as one approaches the limit of four dimensions. In particular, one want to retain the vanishing of the cosmological constant but very few other implications of three-dimensional supersymmetry. It is not clear precisely how this can work. I will here discuss instead a more straightforward question, which is what things look like from the *four*-dimensional point of view when one is near the four-dimensional limit. That is, we will consider the dimensional reduction of the real world on $\mathbf{R}^3 \times \mathbf{S}^1$, and ask what one sees when the radius $R$ of the $\mathbf{S}^1$ is extremely large.

In doing so, we will assume that on $\mathbf{R}^4$, the only exactly massless bosons are the photon and the graviton; one could extend the discussion if one knew what additional massless bosons to consider. It would similarly be somewhat natural to assume that the massless fermions are a subset of the known neutrinos, though this is of course far from certain.

Upon reduction on $\mathbf{R}^3 \times \mathbf{S}^1$, the photon becomes a scalar $\phi$ and a vector $a$. The graviton similarly decomposes as a scalar $r$ (the fluctuating radius of the $\mathbf{S}^1$), a vector $b$, and a three-dimensional graviton which does not have have any propagating modes. The intention here is to discuss in turn the dynamics of the modes $\phi$, $a$, $b$, and $r$ – taking them roughly in increasing order of subtlety.

(1) $\phi$ is really an angular variable, with $0 \leq \phi \leq 2\pi$, since it is best interpreted as the holonomy of the photon around $\mathbf{S}^1$. At the classical level, the energy of the vacuum is independent of $\phi$. Quantum mechanically, as electrons are the lightest charged particles, the main influence of $\phi$ is on the vacuum energy of the filled Dirac sea of electrons.

This energy is minimized at $\phi = \pi$ [24] so $\phi$ will acquire that vacuum expectation value. Expanding around the minimum, the mass of $\phi$ is roughly of order $e^{-\pi m R}$ with $m$ the electron mass.

(2) The three-dimensional photon $a$ is massless in perturbation theory. If, however, as is generally believed, magnetic monopoles exist in nature, then by thinking of the $\mathbf{S}^1$ direction as "time," a time-independent magnetic monopole on $\mathbf{R}^3 \times \mathbf{S}^1$ is a localized object that can be interpreted as a kind of instanton. Three-dimensional $U(1)$ gauge theory with such instantons ("compact QED") was first studied by Polyakov [25] and has the remarkable property that the photon acquires a mass – a phenomenon most conveniently described in terms of a scalar $u$ dual to $a$. The mass of $u$ is roughly of order $\exp(-\pi M R)$ with $M$ the mass of the lightest magnetic monopole in nature. If this phenomenon does occur, as one would expect, then electric charges are subject to not just logarithmic but linear confinement.

(3) Now we come to the second photon $b$ of the three-dimensional world. Though the physics involved is not well understood, it is very plausible that also in this case suitable instantons exist and the $b$ field gets a mass. In this case the charge that would be subject to linear confinement is the one that comes from rotations of the $\mathbf{S}^1$; the modes carrying momentum in the fourth dimension would be confined!

(4) Finally, we come to the scalar $r$ that measures fluctuations in the radius of the $\mathbf{S}^1$. If the cosmological constant vanishes in four dimensions, then the potential $V(r)$ for this scalar vanishes for $r \to \infty$. Corrections vanishing as a power of $r$ for $r \to \infty$ can be computed systematically by evaluating Feynman diagrams involving massless particles only. The leading correction for $r \to \infty$, for instance, is the one-loop Casimir effect of the massless bosons and fermions in nature, and is a multiple of

$$-\frac{n_B - n_F}{r^3} \tag{3.1}$$

with $n_B$ and $n_F$ the number of exactly massless bose and fermi helicity states in nature. Feyman diagrams of massless particles with two or more loops give corrections of higher order in $1/r$. To compute to order $1/r^{3+n}$ needs to know the effective Lagrangian of the exactly massless particles in nature including all terms up to dimension $4 + n$. Thus, the more perfectly the low energy effective action of nature is known, the more precisely one could work out the expansion of $V(r)$ in powers of $1/r$.

Without any further theory, one would assume that $V(r)$ is non-zero except in the limit of $r \to \infty$; we know from experimental bounds on the cosmological constant that

$V(\infty)$ is zero or at least incredibly small. The scenario in [22], however, at least in the form presented there, implies that $V(r)$ is identically zero. This is indeed the four-dimensional analog of the three-dimensional statement that because of unbroken supersymmetry the vacuum energy is zero for any value of the dilaton field. Thus, this scenario makes the remarkable prediction that the vanishing of the cosmological constant is only the first of an infinite series of vanishing phenomena that might mystify a low energy observer. The second prediction is that $n_B - n_F = 0$, and subsequent predictions, involving the $r$ dependence of Feynman diagrams with two or more loops, could be worked out given sufficient knowledge of the low energy world. This framework, then, certainly has some predictive power, if not too much.

I would like to thank M. Dine, D. J. Gross, D. Morrison, M. Peskin, M. Reid, N. Seiberg, S. Shenker, E. Silverstein, and A. Strominger for discussions concerning some of these matters.

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