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PHASES OF $N = 1$ SUPERSYMMETRIC GAUGE THEORIES AND ELECTRIC-MAGNETIC TRIALITY*

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ABSTRACT

We discuss the phases of four dimensional gauge theories and demonstrate them in solvable examples. Some of our simple examples exhibit confinement and oblique confinement. The theory has dual magnetic and dual dyonic descriptions in which these phenomena happen at weak coupling. Combined with the underlying electric theory, which gives a weak coupling description of the Higgs phase, we have electric-magnetic-dyonic triality. In an appendix we clarify some points regarding the use of 1PI superpotentials in these theories.

1. Introduction - the phases of four dimensional gauge theories

Recently, it has become clear that certain aspects of four dimensional supersymmetric field theories can be analyzed exactly, thus providing a laboratory for the analysis of the dynamics of gauge theories (for a recent elementary presentation and a list of references see [1]). For example, the phases of gauge theories and the mechanisms for phase transitions can be explored in this context. The dynamical mechanisms explored are standard to gauge theories and thus the insights obtained are expected to also be applicable for non-supersymmetric theories. Here we will focus on some of these insights. We summarize the ideas of [2-5] and demonstrate them in simple examples.

A gauge invariant order parameter which characterizes the phases of gauge theories is the Wilson loop:

$$W_w = \mathrm{Tr}_r \; P e^{i\int A}. \tag{1}$$

When the loop is a rectangle of length T and width R, it has the following physical interpretation. Two electrically charged sources in the representations r and $\bar{r}$ of the gauge group are created a distance R apart. They then propagate for time T when they are annihilated. We can use the expectation value

$$\lim_{T\to\infty} \langle W_w \rangle = e^{-TV(R)}, \tag{2}$$

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to find the potential, $V(R)$, between the sources. The various phases are characterized by the large R behavior of the potential which, up to a non-universal additive constant, is

$$\begin{aligned}
\text{Coulomb} \qquad & V(R) \sim \frac{1}{R} \\
\text{free electric} \qquad & V(R) \sim \frac{1}{R\log(R\Lambda)} \\
\text{free magnetic} \qquad & V(R) \sim \frac{\log(R\Lambda)}{R} \\
\text{Higgs} \qquad & V(R) \sim constant \\
\text{confining} \qquad & V(R) \sim \sigma R,
\end{aligned} \tag{3}$$

where σ is the string tension. The free electric phase happens when the theory has massless photons and electrons. Then, the electric charge is renormalized to zero at long distances and leads to the factor $\log R\Lambda$ in the potential. Similar behavior occurs when the long distance theory is a non-Abelian theory which is not asymptotically free. The free magnetic phase occurs when there are massless magnetic monopoles which renormalize the electric coupling constant to infinity (the $\log R\Lambda$ in the numerator) at large distance.

For a general loop shape, in the Higgs phase the Wilson loop has exponential falloff in the the perimeter of the loop and in the confining phase there is exponential falloff in the area of the loop.

In addition to the familiar Abelian Coulomb phase, there are theories which have a non-Abelian Coulomb phase with massless interacting quarks and gluons exhibiting the above Coulomb potential. This phase occurs when there is a non-trivial, infrared fixed point of the renormalization group. These are thus non-trivial, interacting four dimensional conformal field theories.

Another order parameter is the 'tHooft loop W_t constructed by cutting a loop out of the space and considering non-trivial (twisted) boundary conditions around it. In a fashion similar to the Wilson loop, it can be interpreted as creating and annihilating a monopole anti-monopole pair. The potential between the monopoles, obtained from the 'tHooft loop via $\lim_{T\to\infty}\langle W_t\rangle = e^{-TV(R)}$, satisfies for large R

$$\begin{aligned}
\text{Coulomb} \qquad & V(R) \sim \frac{1}{R} \\
\text{free electric} \qquad & V(R) \sim \frac{\log(R\Lambda)}{R} \\
\text{free magnetic} \qquad & V(R) \sim \frac{1}{R\log(R\Lambda)} \\
\text{Higgs} \qquad & V(R) \sim \rho R \\
\text{confining} \qquad & V(R) \sim constant
\end{aligned} \tag{4}$$

up to an additive non-universal constant. The linear potential in the Higgs phase reflects the string tension in the Meissner effect.

Note that in going from the Wilson loop to the 'tHooft loop the behavior in the free electric and the free magnetic phases are exchanged. This is a reflection of the fact that under electric-magnetic duality, which exchanges electrically charged fields with magnetically charged fields, the Wilson loop and the 'tHooft loop are exchanged. Mandelstam and 'tHooft suggested that, similarly, the Higgs and confining phases are exchanged by duality. Confinement can thus be understood as the dual Meissner effect associated with a condensate of monopoles.

Dualizing a theory in the Coulomb phase, we remain in the same phase (the behavior of the potential is unchanged). For an Abelian Coulomb phase with free massless photons this follows from a standard duality transformation. What is not obvious is that this is also the case in a non-Abelian Coulomb phase. This was first suggested by Montonen and Olive [6]. The simplest version of their proposal is true only in $N = 4$ supersymmetric field theories [7] and in finite $N = 2$ supersymmetric theories [8]. The extension of these ideas to asymptotically free $N = 1$ theories appeared in [4].

Another order parameter is the product $W_d = W_w W_t$, which corresponds to a dyon loop. Making a table with the dependence of the order parameters in the phases suggests an "oblique" confinement phase [9,10]

	W_w	W_t	W_d
Higgs	perimeter	area	area
Confinement	area	perimeter	area
Oblique Conf.	area	area	perimeter

Whereas the Higgs phase is associated with an electrically charged condensate, the confining phase can be associated with a condensate of monopoles and the oblique confinement phase can be associated with a condensate of dyons.

In $SU(N_c)$ theories with matter in the fundamental representation, the elementary quarks can screen the charges involved in the above loops and thus all loops have perimeter behavior. Indeed, there is no distinction between Higgs and confinement in these theories [11]. This suggests consideration of gauge theories with matter not in the fundamental representation of the gauge group. More precisely, we need matter fields in a non-faithful representation of the center of the gauge group.

In the next two sections we will discuss an $SU(2)$ gauge theories with matter Q in the adjoint representation. It is then possible to study confinement by considering Wilson loops in the fundamental representation of $SU(2)$. The quarks in the adjoint representation are unable to screen the Z_2 center of the gauge group.

In the confining phase there is often a mass gap with no massless particles (or the massless particles are free). In that case the 1PI effective Lagrangian for operators does not suffer from infrared divergences. The superpotential of this effective action can be obtained following techniques discussed in [12-16]. In an appendix we will present our understanding of this effective action and its proper use. In particular, we will show when it leads to incorrect conclusions when interpreted as a Wilsonian effective action.

2. $SU(2)$ with one adjoint, Q; an Abelian Coulomb phase

This is the $N = 2$ theory discussed in [2]. The theory has a quantum moduli space of vacua labeled by the expectation value of the massless meson field $M = Q^2$. The $SU(2)$ gauge symmetry is broken to $U(1)$ on this moduli space, so the theory has a Coulomb phase with a massless photon.

As discussed in [2], there is a massless magnetic monopole field $q_{(+)}$, at $M = 4\Lambda^2$ and a massless dyon $q_{(-)}$ at[1] $M = -4\Lambda^2$. Therefore, these two points are in a free magnetic and a free dyonic phase, respectively. Here $q_{(+)}$ is a doublet charged under the magnetic $U(1)_M$, which is related to the electric $U(1)_E$ by the electric-magnetic transformation S: $F \to \tilde{F}$ (modulo $\Gamma(2) \subset SL(2,Z)$). Similarly, $q_{(-)}$ is a doublet charged under a dyonic $U(1)_D$, related to $U(1)_E$ by the $SL(2,Z)$ transformation ST (again, modulo $\Gamma(2) \subset SL(2,Z)$), where T is a rotation of the theta angle by 2π. Near where these fields are massless, they couple through the effective superpotentials

$$W_\pm \sim \left(M \mp 4\Lambda^2\right) q_{(\pm)} \cdot q_{(\pm)}. \tag{5}$$

Referring to the underlying $SU(2)$ theory as "electric," we can say that it has two dual theories. One of them, which we can refer to as the "magnetic dual," describes the physics around $M = 4\Lambda^2$ with the superpotential W_+. The other dual, which can be called the "dyonic dual," is valid around $M = -4\Lambda^2$ and is described by W_-. Consider giving Q a mass by adding a term $W_{tree} = \frac{1}{2}mM$ in the electric theory. Adding W_{tree} to (5), the equations of motion give $\langle q_{(\pm)} \cdot q_{(\pm)}\rangle \sim m$ and lock $\langle M\rangle = \pm 4\Lambda^2$. The condensate of monopoles/dyons Higgses the dual theory and thus gives confinement/oblique confinement of the electric theory by the dual Meissner effect [2].

[1] We use the conventions of [15,17] where the normalization of Λ^2 (in the $\overline{DR}$ scheme) differs by a factor of 2 from that of [2]; our order parameter M is related to u of [2] as $u = \frac{1}{2}M$.

3. $SU(2)$ with two adjoints; A non-Abelian Coulomb phase

3.1 The "electric" theory

This theory has $N = 1$ (not $N = 2$) supersymmetry. Writing the matter fields as Q^i with $i = 1, 2$ a flavor index, there is a 3 complex dimensional moduli space of classical vacua parametrized by the expectation values of the gauge singlet fields $M^{ij} = Q^i \cdot Q^i$. In the generic vacuum $\langle Q^1 \rangle$ breaks $SU(2)$ to a $U(1)$ which is then broken by $\langle Q^2 \rangle$. For $\det\langle M^{ij} \rangle \neq 0$, the gauge group is completely broken and the theory is in the Higgs phase. On the non-compact two complex dimensional subspace of vacua with $\det M = 0$, there is an unbroken $U(1)$ gauge symmetry and thus a light photon along with a pair of massless electrically charged fields. At the point $\langle M \rangle = 0$ the $SU(2)$ gauge group is unbroken.

We now turn to the quantum theory. The theory has the global symmetry group $SU(2) \times U(1)_R$, with Q transforming as $\mathbf{2}_{\frac{1}{2}}$, which determines that any dynamically generated superpotential must be of the form

$$W = \frac{c}{\Lambda} \det M, \tag{6}$$

with c a dimensionless constant. Its behavior at $M \to \infty$ is incompatible with asymptotic freedom, as signaled by the presence of the scale Λ in the denominator. Therefore, no superpotential can be generated and the classical vacuum degeneracy outlined above is not lifted quantum mechanically.

The generic ground state with generic M is in the Higgs phase. Consider now the subspace of the moduli space with $\det M = 0$. The low energy degrees of freedom there are a single photon, a pair of massless electrically charged fields and some neutral fields. This theory cannot become strong in the infrared. In fact, the loops of the massless charged fields renormalize the electric charge to zero. Therefore, this subspace of the moduli space is in a free electric phase.

Now consider adding a tree level superpotential $W_{tree} = \frac{1}{2}\mathrm{Tr}\ mM$. Taking $m = \begin{pmatrix} 0 & 0 \\ 0 & m_2 \end{pmatrix}$, Q^2 gets a mass and can be integrated out. The low energy theory is $SU(2)$ with a single massless adjoint matter field, which is the example of the previous section. Its scale Λ_L can be expressed in terms of the scale Λ of the high energy theory and the mass as $\Lambda_L^4 = m_2^2 \Lambda^2$. Therefore, the massless monopole and dyon are at $\langle M^{11} \rangle = \pm 4 m_2 \Lambda$. Note that as $m_2 \to 0$ the point $\langle M \rangle = 0$ has both massless monopoles and dyons. These are mutually non-local[2] and signal another phase at this point in the theory with $m_2 = 0$. We interpret this as a non-Abelian Coulomb phase [3].

[2] A similar situation was found in $N = 2$ $SU(3)$ Yang Mills theory [18].

Starting from the theory with $m_2 \neq 0$, turning on $m_1 \neq 0$ drives the monopole or dyon to condense and the vacuum is locked at $\langle M^{11} \rangle = \pm 4 m_2 \Lambda$. The $+$ sign is a vacuum with monopole condensation and thus confinement. The $-$ sign is a vacuum with dyon condensation and thus oblique confinement. More generally, these vacua are at $\langle M^{ij} \rangle = \pm 4\Lambda \det m (m^{-1})^{ij}$. These expectation values can be obtained from

$$W_e = \frac{e}{8\Lambda} \det M + \frac{1}{2} \text{Tr } mM, \tag{7}$$

with $e = \mp 1$ for confinement and oblique confinement, respectively.

The theory has various phase branches. For mass $m = 0$ there is a Higgs phase which, in terms of W_e, corresponds to $e = 0$. There is a subspace $\det M = 0$ in the free electric phase and the point $M = 0$ in a non-Abelian Coulomb phase. For $m \neq 0$ but with $\det m = 0$ the theory is in the Coulomb phase with a free magnetic point and a free dyonic point. For $\det m \neq 0$ the theory is either confining and described by the superpotential with $e = -1$ or it is oblique confining and described by the superpotential with $e = 1$.

Note that the theory has three branches. Every one of them has its own superpotential ($e = 0, 1, -1$ in (7)). We will return to the meaning of this superpotential and how it could be different in the various phases in the appendix.

3.2 Dual non-Abelian theories

The analysis of [5] reveals that this theory has two dual theories, labeled by $\epsilon = \pm 1$. The two theories are based on an $SU(2)$ gauge group with two fields q_i in its adjoint representation and three gauge singlet fields M^{ij}. The difference between the two theories is in the superpotential

$$W_\epsilon = \frac{1}{12\sqrt{\Lambda\tilde{\Lambda}}} M^{ij} q_i \cdot q_j + \epsilon \left(\frac{1}{24\Lambda} \det M + \frac{1}{24\tilde{\Lambda}} \det q_i \cdot q_j \right), \tag{8}$$

where $\tilde{\Lambda}$ is the scale of the dual $SU(2)$. Here the elementary field M^{ij} was rescaled to have dimension 2 just as its counterpart $M^{ij} = Q^i \cdot Q^j$ in the electric theory. The theory with $\epsilon = 1$ is a "magnetic" dual and that with $\epsilon = -1$ a "dyonic" dual.

We now analyze the dynamics of these dual theories. Since they are similar to the theory studied in the previous subsection, we proceed as we did there. These theories have three phases: Higgs, confining and oblique confining. We study them using the gauge invariant order parameters $N_{ij} \equiv q_i \cdot q_j$. Its effective superpotential is obtained by writing the tree level superpotential (8) in terms of N and adding to it $\frac{\tilde{e}}{8\tilde{\Lambda}} \det N$ where, in the Higgs, confining and oblique confinement branches, $\tilde{e} = 0, -1, 1$, respectively

$$W_{\epsilon,\tilde{e}} = \frac{1}{12\sqrt{\Lambda\tilde{\Lambda}}} \text{Tr } MN + \epsilon \left(\frac{1}{24\Lambda} \det M + \frac{1}{24\tilde{\Lambda}} \det N \right) + \frac{\tilde{e}}{8\tilde{\Lambda}} \det N. \tag{9}$$

Now we can integrate out the massive field N to find

$$W_{\rm eff} = \frac{1}{8\Lambda}\frac{\tilde e - \epsilon}{1+3\tilde e\epsilon}\det M. \tag{10}$$

This is the same as the effective superpotential (7) of the electric theory with

$$e = \frac{\tilde e - \epsilon}{1+3\tilde e\epsilon}. \tag{11}$$

We see that the various phases are permuted in the different descriptions as:

Theory		**Phases**	
electric	Higgs ($e=0$)	conf. ($e=-1$)	obl. conf. ($e=1$)
magnetic ($\epsilon=1$)	obl. conf. ($\tilde e=1$)	Higgs ($\tilde e=0$)	conf. ($\tilde e=-1$)
dyonic ($\epsilon=-1$)	conf. ($\tilde e=-1$)	obl. conf. ($\tilde e=1$)	Higgs ($\tilde e=0$)

It is a simple exercise to check that by dualizing the magnetic and dyonic theories as we above dualized the electric theory (two duals of each), we find permutations of the same three theories. The S_3 triality permuting the phases and branches is associated with a quotient of the $SL(2,Z)$ electric-magnetic duality symmetry group: the theories are preserved under $\Gamma(2)\subset SL(2,Z)$, leaving the quotient $S_3 = SL(2,Z)/\Gamma(2)$ with a non-trivial action.

This discussion leads to a new interpretation of the first term in (7). In the electric theory this term appears as a consequence of complicated strong coupling dynamics in the confining and the oblique confinement branches of the theory. In the dual descriptions it is already present at tree level.

Consider the theory with a mass m_2 for Q^2. As discussed above, the low energy electric theory has a Coulomb phase with massless monopoles or dyons at the strong coupling singularities $\langle M^{11}\rangle = \pm 4m_2\Lambda$. We now derive this result in the dual theories. Adding $W_{tree}=\frac{1}{2}m_2M^{22}$ to the superpotential (8) of the dual theory, the equations of motion give

$$\begin{aligned} \frac{1}{12\sqrt{\Lambda\tilde\Lambda}}q_2\cdot q_2 + \frac{8\epsilon}{24\Lambda}M^{11} + \frac{1}{2}m_2 = 0 \qquad & M^{22} = -\frac{1}{2}\epsilon\sqrt{\frac{\Lambda}{\tilde\Lambda}}q_1\cdot q_1 \\ q_1\cdot q_2 = 0 \qquad & M^{12} = 0. \end{aligned} \tag{12}$$

For $q_2^2\neq 0$, $\langle q_2\rangle$ breaks the gauge group to $U(1)$ and the remaining charged fields $q_1^\pm$ couple through the low energy superpotential

$$\frac{1}{16\sqrt{\Lambda\tilde\Lambda}}(M^{11} - 4\epsilon m_2\Lambda)q_1^+q_1^-. \tag{13}$$

(This superpotential is corrected by contributions from instantons in the broken magnetic $SU(2)$ theory. However, these are negligible near $M^{11} = 4\epsilon m_2 \Lambda$.) We see that the theory has a charged doublet of massless fields $q_1^{\pm}$ at $M^{11} = 4\epsilon m_2 \Lambda$, exactly as expected from the analysis of the electric theory. There these states appeared as a result of strong coupling effects. Here we see them as weakly coupled states in the dual theories. This is in accord with the interpretation of the $\epsilon = 1$ ($\epsilon = -1$) theory as magnetic (dyonic).

The other monopole point on the moduli space of the theory with $m_1 = 0$ but $m_2 \neq 0$ is at $M^{11} = -4\epsilon m_2 \Lambda$. It arises from strong coupling dynamics in the dual theories. To see that, note that the above analysis is not valid when the expectation value of q_2 is on the order of or smaller than the mass of q_1. In that case, q_1 should be integrated out first. The equations of motion in the low energy theory yield a single massless monopole point at $M^{11} = -4\epsilon m_2 \Lambda$ [5].

An analysis similar to the one above leads to a strongly coupled state in the dual theories along the flat directions with $\det M = 0$ in the $m = 0$ case. This state can be interpreted as the massless quark of the electric theory in that free electric phase.

To conclude, this theory has three branches which are in three different phases: Higgs, confining and oblique confinement (various submanifolds of these branches are in Coulomb, free electric, free magnetic and free dyonic phases). They touch each other at a point in a non-Abelian Coulomb phase. Corresponding to the three branches there are three different Lagrangian descriptions of the theory: electric, magnetic and dyonic. Each of them describes the physics of one of the branches, where it is Higgsed, in weak coupling and the other two in strong coupling.

In both the example of the previous section and this one, the theory has a discrete symmetry which relates the confining and the oblique confinement phases[3]. Therefore, in these cases the effects of confinement are indistinguishable from the effects of oblique confinement. Correspondingly, the magnetic and the dyonic descriptions are similar – they differ only in the sign of ϵ. In other examples, discussed in [5], these two phases are not related by a symmetry and the two dual descriptions look totally different.

Appendix: The superpotential in the confining phase, 1PI effective action, Legendre transform and "integrating in"

There are two different objects which are usually called "the effective action:" the 1PI effective action and the Wilsonian one. When there are no interacting

[3] This symmetry is manifest only in the electric description. In the dual descriptions it is realized as a quantum symmetry [5].

massless particles, these two effective actions are identical. This is often the case in the confining phase. However, when interacting massless particles are present, the 1PI effective action suffers from IR ambiguities and might suffer from holomorphic anomalies [19]. These are absent in the Wilsonian effective action.

Consider the theory with a tree level superpotential with sources for the gauge invariant polynomials X^r in the matter fields, $W_{tree} = \sum_r g_r X^r$, with the g_r regarded as background chiral superfield sources [20]. The functional integral with the added source terms gives the standard generating function for the correlation functions, $\Gamma(g)$. If supersymmetry is not broken, $\Gamma(g)$ is supersymmetric (otherwise we should include the Goldstino field and supersymmetry will be realized non-linearly) and[4] $\Gamma(g) = \ldots + \int d^2\theta W_L(g)$. Using $W_L(g)$ we can compute the expectation values

$$\frac{\partial W_L(g)}{\partial g_r} = \langle X^r \rangle. \tag{14}$$

It is standard to perform a Legendre transform to find the 1PI effective action for the operators X_r:

$$W_{dyn}(X^r) = \left(W_L(g_r) - \sum_r g_r X^r \right)_{\langle g_r \rangle}, \tag{15}$$

where the $\langle g_r \rangle$ are the solutions of (14). The transformation from $W_L(g_r)$ to $W_{dyn}(X_r)$ can be inverted by the inverse Legendre transform as

$$W_L(g) = \left(W_{dyn}(X^r) + \sum_r g_r X^r \right)_{\langle X^r \rangle}, \tag{16}$$

where the X^r are evaluated at their expectation values $\langle X^r \rangle$, which solve

$$\frac{\partial W_{dyn}}{\partial X_r} + g_r = 0. \tag{17}$$

The 1PI effective superpotential

$$W_{eff}(X, g) = W_{dyn}(X^r) + \sum_r g_r X^r \tag{18}$$

[4] In writing this expression we should think of the coupling constants g_r as background superfields. Otherwise, $W_L(g)$ is a constant superpotential, which has no effect in global supersymmetry. Indeed, the following equation can be interpreted as differentiating the action with respect to the F component of g_r.

has the property that the equations of motion for the fields X^r derived from it (17) determine their expectation values. In some cases the superpotential W_{eff} obtained by the above Legendre transform is the same as the Wilsonian superpotential for the light fields. In applying this procedure we should be careful of the following pitfalls:

1. The theory with the sources should have a gap. Otherwise, the 1PI action is ill defined.
2. The theory with the sources might break supersymmetry. In that case W_L is ill defined.
3. As the sources are turned off, some particles become massless. Their interpolating fields should be among the composite fields X^r. If some massless particles cannot be represented by a gauge invariant operator X^r, the effective superpotential derived this way will not include them. This often leads to singularities.
4. The theory might also have other branches (as in the examples above) which are present only when some sources vanish. In this case there are new massless particles at that point and this W_{eff} might miss some of the branches. In other words, then the Legendre transform does not exist.
5. If some composites do not represent massless particles, they should be integrated out. Although we can use the effective superpotential to find their expectation values, we cannot think of them as fields corresponding to massive particles except near a point where they become massless.

When we can use this procedure to find the Wilsonian action, the linearity of W_{eff} (18) in the sources provides a derivation of the linearity of the Wilsonian effective action in the sources. See [14] for a related discussion.

This approach is particularly useful when we know how to compute $W_L(g_r)$ exactly. Then, W_{dyn} and W_{eff} follow simply from the Legendre transform (15); this is the "integration in" discussed in [15]. One situation where $W_L(g_r)$ can be determined is when the X^r are all quadratic in the elementary fields. In that case, the sources g_r are simply mass terms for the matter fields and $W_L(g)$ is the superpotential for the low energy gauge theory with the massive matter integrated out, expressed in terms of the quantities in the high-energy theory.

As an example, consider supersymmetric $SU(N_c)$ QCD with N_f flavors. For $N_f < N_c$ the gauge invariant operators are $M_{i\tilde{j}} = Q_i\tilde{Q}_{\tilde{j}}$ and their sources are mass terms, $W_{tree} = \mathrm{Tr}\; mM$. When the masses are large the quarks can be integrated out. The low energy $SU(N_c)$ theory then has a scale $\Lambda_L^{3N_c} = \Lambda^{3N_c-N_f} \det m$ (again, we use the conventions of [15,17]). Gluino condensation in this theory leads to the effective superpotential for the sources

$$W_L(m) = N_c(\Lambda^{3N_c-N_f} \det m)^{1/N_c}. \tag{19}$$

Using (15), the effective superpotential for the operators M is

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\det M}\right)^{1/(N_c-N_f)} . \tag{20}$$

In this case W_{dyn} agrees with the Wilsonian effective superpotential [21].

It is also possible to "integrate in" more operators which do not correspond to massless particles. Then, the effective action can be used only to compute their expectation values, rather than for studying them as massive particles. An example is the "glueball" field $S \sim W_\alpha^2$, whose source is $\sim \log \Lambda$. Integrating in S by the Legendre transform of (20) with the source $\log \Lambda^{3N_c-N_f}$ yields

$$W(S, M) = S\left[\log\left(\frac{\Lambda^{3N_c-N_f}}{S^{N_c-N_f}\det M}\right) + (N_c - N_f)\right] , \tag{21}$$

the superpotential obtained in [12]. Working with such an effective potential including massive fields can be convenient when interesting but complicated dynamics is encoded in the integrating out of these massive fields. However, as stressed above, we should not think of S as a field describing a massive particle.

As we said above, the analysis of the effective action with sources might fail to reveal some of the physics. For example, for $SU(2)$ with one adjoint field, discussed in section 2, we can start from the analog of (19) for this theory, $W_L(m) = \pm 2(\Lambda^4 m^2)^{1/2}$. Then equations (14) and (15) give $W = 0$ with the constraint $\langle M \rangle = \pm 4\Lambda^2$. Indeed, adding the source for M drives the theory to the confining or oblique confining phase with $\langle M \rangle = \pm 4\Lambda^2$. The Coulomb phase cannot be explored in the theory with a mass term for Q. Similarly, for $SU(2)$ with two adjoint fields, discussed in section 3, the analog of (19) is $W_L(m) = \pm 2(\Lambda^2 \det m)^{1/2}$. Integrating in gives the confining or oblique confining phase superpotential (7) with $e = \pm 1$, missing the $e = 0$ phase. In both of these situations the theory without the sources has massless particles (the photon and the monopoles or dyons in the theory of section 2 and the quarks and the gluons in the theory of section 3) which cannot be represented by the gauge invariant observables. As we said above, in a situation like that this method must fail to capture some of the physics.

Another situation in which the Legendre transform analysis is incomplete is when supersymmetry is dynamically broken by the added source terms. A simple example of this is supersymmetric $SU(2)$ with a single field Q in the $\mathbf{4}$ of $SU(2)$ [22]. The theory without added source terms has a one complex dimensional smooth moduli space of vacua labeled by $\langle X \rangle$, where $X = Q^4$ is the basic gauge invariant, with a superpotential $W(X) = 0$. Adding a source $W = gX$ does not lead to a supersymmetric effective superpotential $W(g)$ – rather, it breaks supersymmetry [22]. (As discussed in [22], it is also possible that there is a non-Abelian Coulomb phase at the origin of the moduli space and that supersymmetry is unbroken with the added source term. In that case the 1PI analysis again fails to capture the physics.)

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References

[1] N. Seiberg, hep-th/9506077, RU-95-37, IASSNS-HEP-95/46, to appear in the Proc. of PASCOS 95 and in the Proc. of the Oskar Klein lectures.

[2] N. Seiberg and E. Witten, hep-th/9407087, *Nucl. Phys.* **B426** (1994) 19.

[3] K. Intriligator and N. Seiberg, hep-th/9408155, *Nucl. Phys.* **B431** (1994) 551.

[4] N. Seiberg, hep-th/9411149 , *Nucl. Phys.* **B435** (1995) 129.

[5] K. Intriligator and N. Seiberg, hep-th/9503179, to appear in Nucl. Phys. B.

[6] C. Montonen and D. Olive, *Phys. Lett.* **72B** (1977) 117; P. Goddard, J. Nuyts and D. Olive, *Nucl. Phys.* **B125** (1977) 1.

[7] H. Osborn, *Phys. Lett.* **83B** (1979) 321; A. Sen, hep-th/9402032, *Phys. Lett.* **329B** (1994) 217; C. Vafa and E. Witten, hep-th/9408074, *Nucl. Phys.* **B432** (1994) 3.

[8] N. Seiberg and E. Witten, hep-th/9408099, *Nucl. Phys.* **B431** (1994) 484.

[9] G. 'tHooft, *Nucl. Phys.* **B190** (1981) 455.

[10] J. Cardy and E. Rabinovici, *Nucl. Phys.* **B205** (1982) 1; J. Cardy, *Nucl. Phys.* **B205** (1982) 17.

[11] T. Banks and E. Rabinovici, *Nucl. Phys.* **B160** (1979) 349; E. Fradkin and S. Shenker, *Phys. Rev.* **D19** (1979) 3682.

[12] G. Veneziano and S. Yankielowicz *Phys. Lett.* **113B** (1982) 231; T. Taylor, G. Veneziano and Yankielowicz, *Nucl. Phys.* **B218** (1983) 439.

[13] D. Amati, K. Konishi, Y. Meurice, G.C. Rossi and G. Veneziano, *Phys. Rep.* **162** (1988) 169 and references therein.

[14] V. Kaplunovsky and J. Louis, *Nucl. Phys.* **B422** (1994) 57.

[15] K. Intriligator, R.G. Leigh and N. Seiberg, hep-th/9403198, *Phys. Rev.* **D50** (1994) 1092; K. Intriligator, hep-th/9407106, *Phys. Lett.* **336B** (1994) 409.

[16] C.P. Burgess, J.-P. Derendinger, F. Quevedo, and M. Quiros, hep-th/9595171, CERN-Th/95-111.

[17] D. Finnell and P. Pouliot, RU-95-14, SLAC-PUB-95-6768, hep-th/9503115.

[18] P. Argyres and M. Douglas, hep-th/9505062, IASSNS-HEP-95-31.

[19] M.A. Shifman and A. I. Vainshtein, *Nucl. Phys.* **B277** (1986) 456; *Nucl. Phys.* **B359** (1991) 571.

[20] N. Seiberg, hep-ph/9309335, *Phys. Lett.* **318B** (1993) 469.

[21] I. Affleck, M. Dine and N. Seiberg, *Nucl. Phys.* **B241** (1984) 493; *Nucl. Phys.* **B256** (1985) 557.

[22] K. Intriligator, N. Seiberg, and S. Shenker, *Phys. Lett.* **342B** (1995) 152.