hep-th/9505190, IASSNS-HEP-94/103, UK-HEP/95-07

CURVES OF MARGINAL STABILITY IN N=2 SUPER–QCD*

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Seiberg and Witten's solution[1] to N=2 $SU(2)$ Yang-Mills with N_f=0 flavors has a one-complex-dimensional Coulomb branch of degenerate vacua labeled by a coordinate u. The effective $U(1)$ theory is described in terms of two functions $a(u)$ and $a_D(u)$. The gauge coupling, $\tau \equiv (\theta/2\pi) + i(4\pi^2/g^2)$ is given by $\tau = da_D/da$; it must satisfy $\mathrm{Im}(\tau) > 0$. The theory is governed by a dynamically generated strong-coupling scale which we set to 1.

The mass of a dyon hypermultiplet with electric and magnetic charges n_e and n_m is given by $M = \sqrt{2}|n_m a_D(u) + n_e a(u)|$. Whenever $\mathrm{Im}(a_D/a) = 0$, any dyon becomes marginally unstable to decay into two or more other dyons (conserving electric and magnetic charges). This note presents a simple argument that determines the shape of the curve of marginal stability $\mathrm{Im}(a_D/a) = 0$.

The effective theory has a duality group that acts on $\binom{a_D}{a}$ as a vector under $SL(2,Z)$, and on τ in the usual way. Note that $f(u) \equiv a_D/a$ transforms like τ.

The $U(1)$ effective theory breaks down at $u = \pm 1$ and ∞, where a dyon hypermultiplet becomes massless. The $SL(2,Z)$ monodromies around these points are ST^2S^{-1}, $(TS)T^2(TS)^{-1}$, and $-T^2$. These matrices generate the group $\Gamma(2) \subset SL(2,Z)$. $u(\tau)$ is a one–to–one map of a single fundamental domain of $\Gamma(2)$ onto the complex plane, which has cusp points at $\tau = 0$, 1, and $i\infty$. These cusp points correspond to the three singularities in the u–plane, and are fixed points of the corresponding $SL(2,Z)$ monodromies—see Fig. 1.

The range of the function $f(u)$ is a subset of the complex plane with similar properties to the fundamental domain of τ. $\Gamma(2)$ acts identically on both the f–plane and the τ–plane, and its generators fix the same 3 points. However, since $\mathrm{Im}\, f$ is not necessarily positive the range of f may extend below the real axis, unlike τ. Indeed, since we know (from expanding the explicit expressions[1] for a and a_D around $u = \pm 1$) that there are whole lines where f is real, it follows that f^{-1} must map an infinite number of $\Gamma(2)$ domains, both above and below the real axis, onto the u-plane.

There is only one possibility for the shape of the range of $f(u)$, due to the fact that the generators ST^2S^{-1} and $(TS)T^2(TS)^{-1}$ are of infinite order, which implies that the opening angles of the corresponding cusps must also be of infinite order, *i.e.*, 0 or 2π. An opening angle of 0 would correspond to a single fundamental domain of $\Gamma(2)$, which we have ruled out. Opening angles of 2π correspond to the domain shown in Fig. 1, a full strip in the f-plane with one $\Gamma(2)$ domain removed. It is easy to see that the monodromies for this region are correct. As a check, it is easily verified using the explicit expressions[1] that $f(0) = -(i \pm 1)/2$.

The curve of marginal stability is the image under f^{-1} of the interval [-1,1], which is a simple closed curve in the u–plane (with $f(-1) = \pm 1$ and $f(+1) = 0$) as conjectured in Ref. 1. Outside of this curve are the images of the infinite number of $\Gamma(2)$ domains between $\mathrm{Re}(f) = +1$ and -1 and with $\mathrm{Im}(f) \geq 0$. Inside the curve are the images of all but one of the $\Gamma(2)$ domains with $\mathrm{Im}(f) < 0$.

The curve $\mathrm{Im}\, f = 0$ has been shown by independent methods to be simple and closed.[2] Also, we have numerically computed it to be the curve shown in Fig. 1.

The methods presented here are easily extended to the massless $N_f = 1$, 2, and 3 cases.[3] For nonzero masses, as well as for $N_c > 2$, the curves of marginal stability become dense in moduli space.

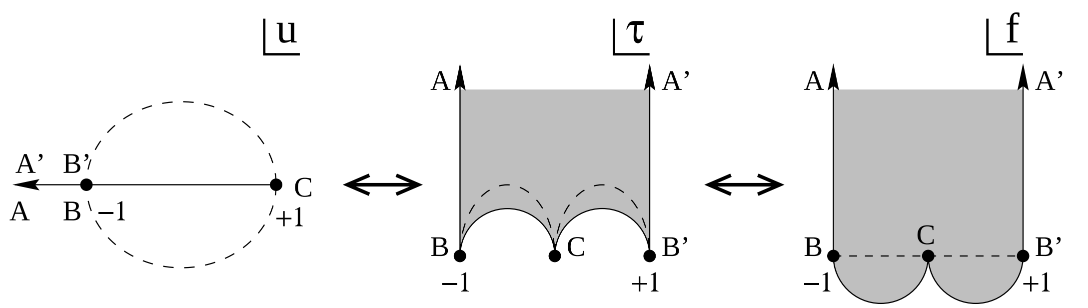

Fig. 1: The shaded regions are the images of the u–plane in the τ and f–planes. The dashed lines are the images of $\mathrm{Im}\, f = 0$.

It is a pleasure to thank M. Douglas, M. Matone, R. Plesser, K. Ranganathan, N. Seiberg, and E. Witten for useful discussions. P.C.A. is supported by NSF grant PHY92-45317 and by the Ambrose Monell Foundation, A.E.F. by DOE grant DE-FG02-90ER40542, and A.D.S. by DOE grant DE-FC02-91ER75661 and by an A.P. Sloan Fellowship.

∗. Talk given by P.C.A. at Strings ’95, U.S.C, March 1995.

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