Shell Effects in Atomic Nuclei

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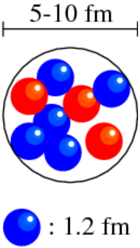
Shell Effects in Finite Quantum Systems
Erice-Sicily July 25-31, 2010

energie atomique • energies alternatives

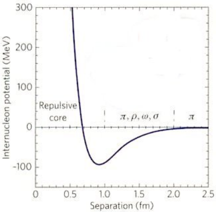
The atomic nucleus

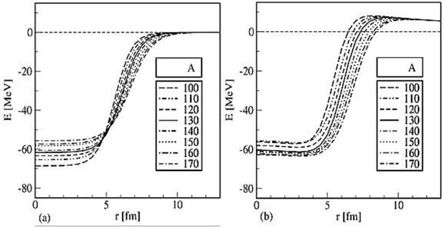


General properties

- Z Protons: $J^{\pi} = 1/2^{+}$;
  N Neutrons: $J^{\pi} = 1/2^{+}$;
  $A = N + Z$ fermions.
- Strong interaction range: $\simeq$ 2 fm
- Nuclear radius: $R \simeq r_0 A^{1/3}$ fm,
  $r_0 \simeq 1.2$ fm.
- Nucleon mean free path: $> R$.

From nucleon-nucleon to nuclear interaction





Nuclei description

- Strong short-range repulsion;
- A (N+Z) interacting fermions;
- Ab initio approach

Nuclear mean field

- Created by the (A-1) nucleons;
- Replaces NN-interaction.
- Shell Model or Mean Field approaches.

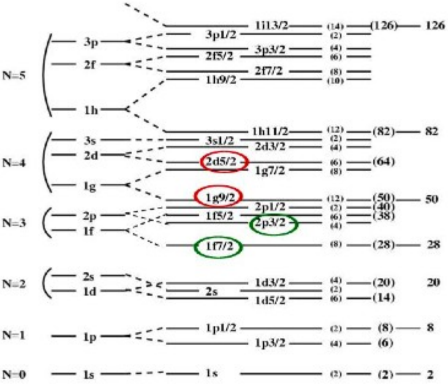
Magic numbers: 2, 8, 20, 28, 50, 82, 126

Goeppert Mayer & Jensen

From M. Goeppert Mayer *Nobel Lecture* (1963)

"What makes a number magic is that a configuration of a magic number of neutrons, or of protons, is unusually stable whatever the associated number of the other nucleons.[...]
We found that there were a few nuclei which had greater isotopic as well as cosmic abundance than our theory or any other reasonable theory could explain. Then I found those nuclei had something in common: they either had 82 neutrons, [...] or 50 neutrons."

Spin-Orbit interaction



Harmonic oscillator potential

$U(r) = \frac{1}{2} M\omega^2 r^2$

- Magic numbers: 2, 8, 20, 40, 70

Angular momentum and spin-orbit

$U'(r) = U(r) + \ell^2 + \ell s$

- Magic numbers: 2, 8, 20, 28, 50, 82

Success and failure of the nuclear shell model

Good features

1. Accounts for known magic numbers.
2. Reproduces J^π, E^*, Q, μ...

Bad features

1. Built from knowledge on stable nuclei.
2. (Dis)appearance of magic numbers in unstable nuclei.

Outline

Today

1. Few body systems.
    - Haloes.
    - Clusters.
2. Heavier systems.
    - Shell evolution: general view.
    - Studies at $N = 28$.

Tomorrow

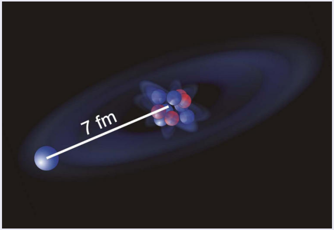
1. Shapes and coexistence.
2. Super heavy elements.

Few-body systems

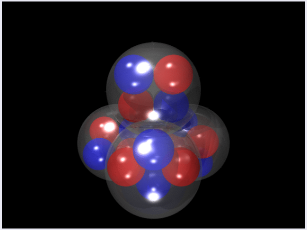
Why?

1. Nuclear interaction $\propto A^{-1/3}$
2. Strong shell effects expected
3. Exotic phenomena

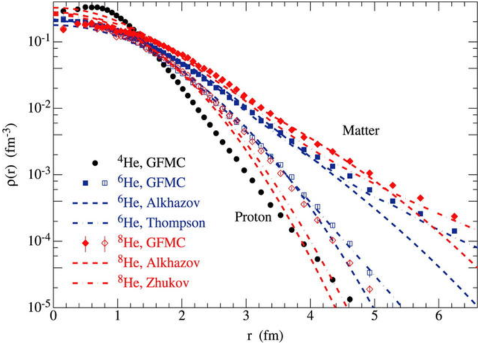
Haloes



Clusters



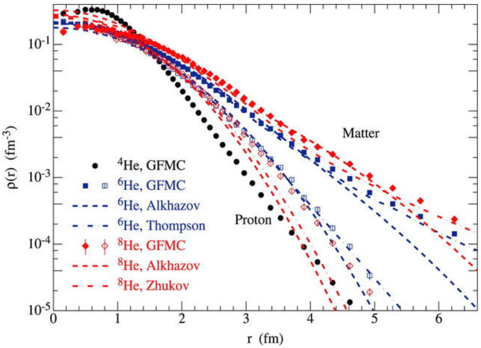
Density distributions in He isotopes



S.C. Pieper & R.B. Wiringa
Annu. Rev. Nucl. Part. Sci.51, 53(2001).

- Add **2** neutrons to ^{4}He
  $\Rightarrow \rho(r > 2) \nearrow$ factor of **10**.
- $^6\text{He} \simeq {}^8\text{He}$

Density distributions in He isotopes



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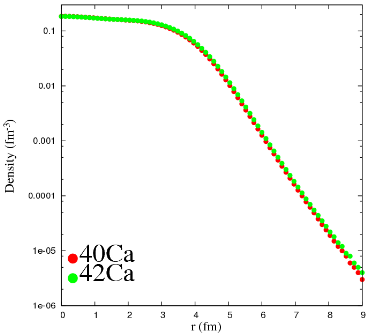

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- From ^{40}Ca to ^{42}Ca
  $\Rightarrow$ No significant change.

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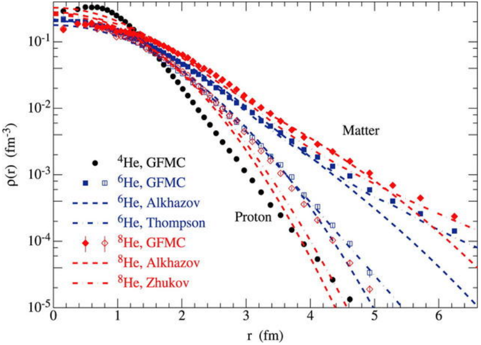

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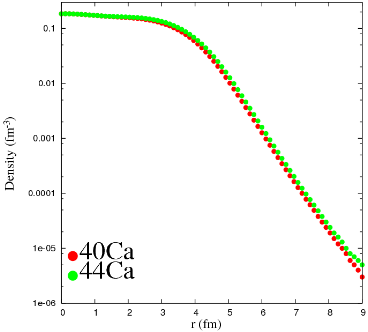

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- idem for ^{44}Ca. . .

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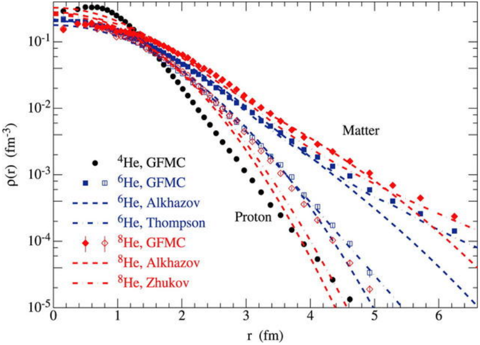

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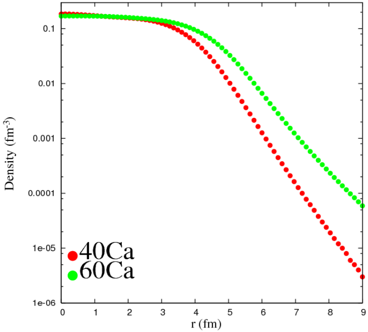

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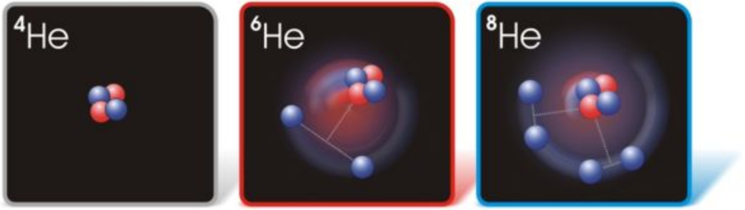
- $^{6}\text{He} \simeq {}^{8}\text{He}$

- From ^{40}Ca to ^{42}Ca
  $\Rightarrow$ No significant change.
- idem for ^{44}Ca...
- 20 neutrons latter $\rho(r > 2) \nearrow$.

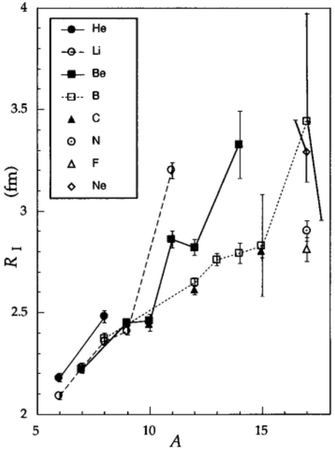
Halo nuclei

An exotic phenomenon

- Weakly bound nuclei.
- Extension of neutron wave function out of the interaction range!
- Linked to shell structure (s or p waves).



Halo nuclei: Experimental evidence



I. Tanihata, J. Phys. G: Nucl. Part. Phys.22, 157 (1996).

- R from reaction cross section: $\sigma = \pi(R_{Target} + R_{Proj})^2$.

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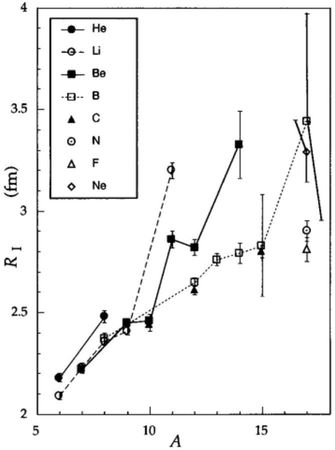

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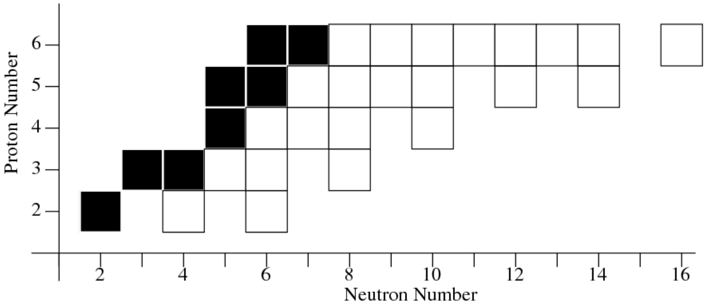
- R from reaction cross section: $\sigma = \pi(R_{Target} + R_{Proj})^2$.
- Does not follow $A^{1/3}$ law for: (^{6,8}He), ^{11}Li, ^{11,14}Be and ^{17}B.
- (Near) Drip line nuclei.

Halo nuclei: Shell effect



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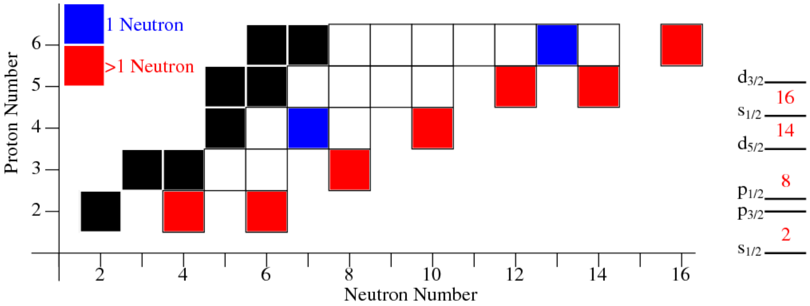
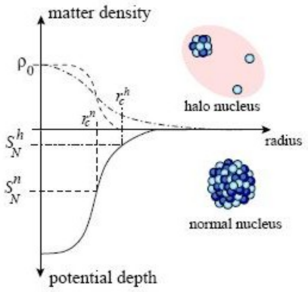


Drip-line

- Loosely bound systems
- $\Psi(r) \propto \frac{e^{-S_n r}}{r}$.
- Low ℓ
- Centrifugal force.

Halo nuclei: Shell effect



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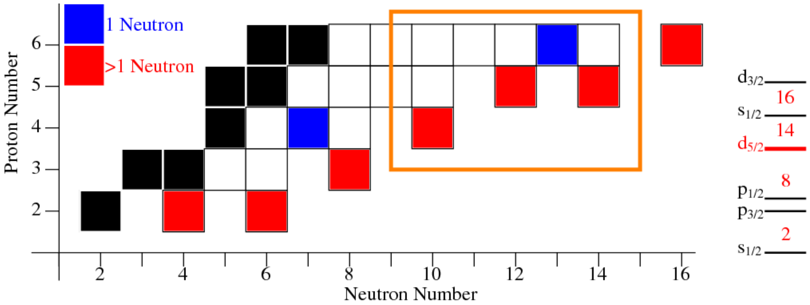
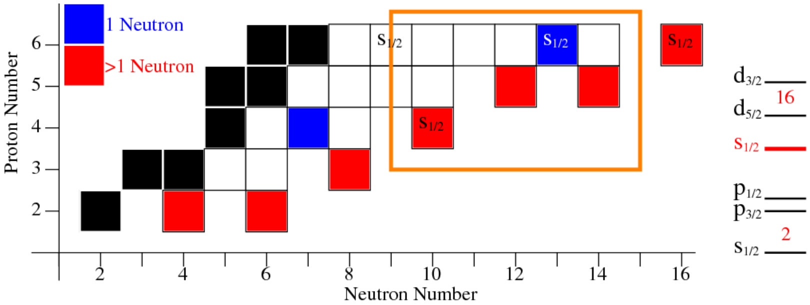


Structural effect

- From $N = 9$ to $N = 14$: $\nu d_{5/2}$ filling.
- Strong shell effect ⇒Shell rearrangement.

Halo nuclei: Shell effect



Structural effect

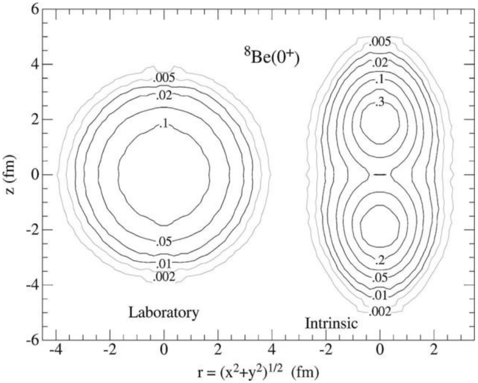
- From $^{15}C_9$ to $^{22}C_{16}$ $\Rightarrow$ $\nu s_{1/2}$ orbit as GS.
- Not yet quantitatively understood.

Halo nuclei

Summary

1. Extension of nucleon wave function out of interaction range.
2. Appear in light loosely bound nuclei.
3. Shell effects $\Rightarrow$ orbital reordering.

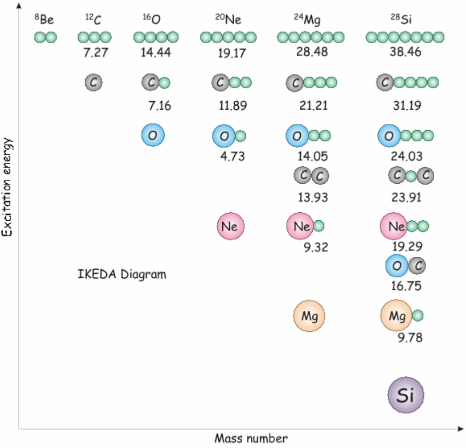
Density distribution in ^{8}Be.



R. B. Wiringa *et al.*, Phys. Rev. C 62, 014001 (2000).

- Unbound GS: $T_{1/2} \simeq 10^{-16}s$
  $\Rightarrow^8 Be \rightarrow ^4$He + ^{4}He.
- 0^+: two structures
  $\Rightarrow^4$He cluster.
- α: $N = Z = 2$.
- Clusters might appear in light $N = Z$ nuclei.

Clusters in nuclei

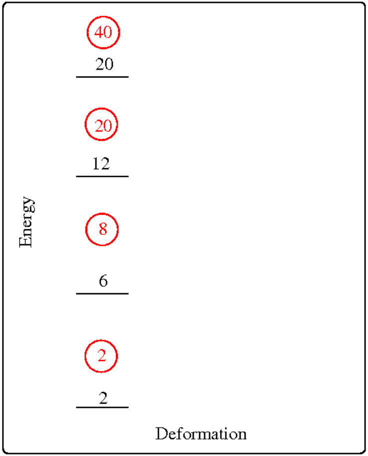


K. Ikeda *et.al*, Prog. Theor. Phys. (Suppl.), 464. (1968)

Energy threshold for clustering

- Must be energetically allowed.
- $^8Be \longrightarrow 2\alpha$
- $^{4n}X \longrightarrow n\alpha$
- Cluster phase expected around $E^* =$ decay threshold.

Clusters & Shell effects



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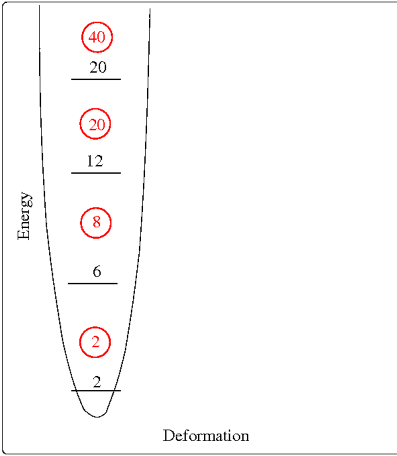

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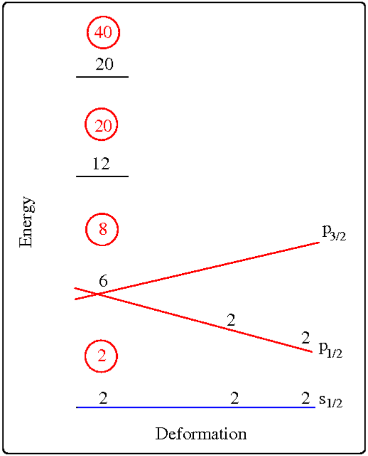

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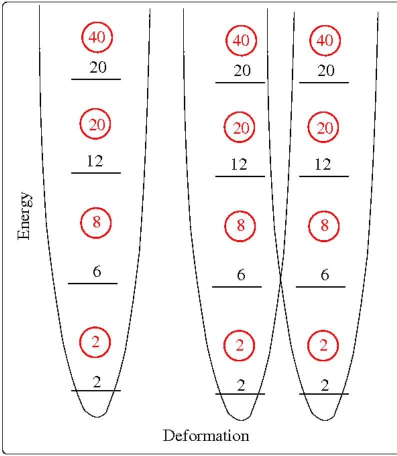

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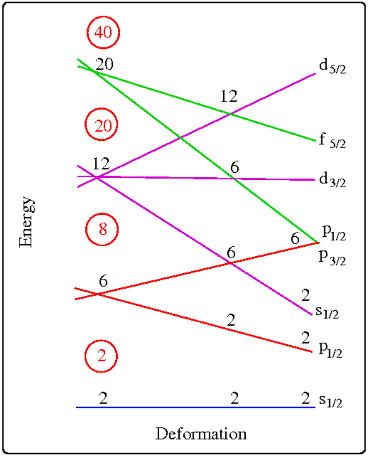

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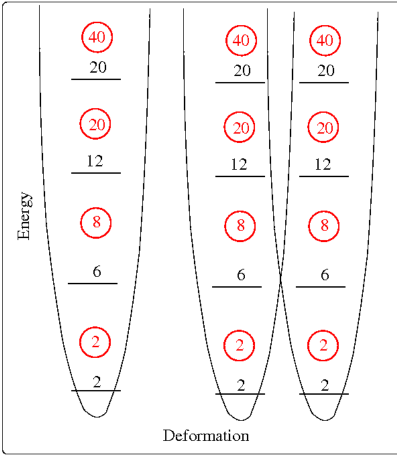

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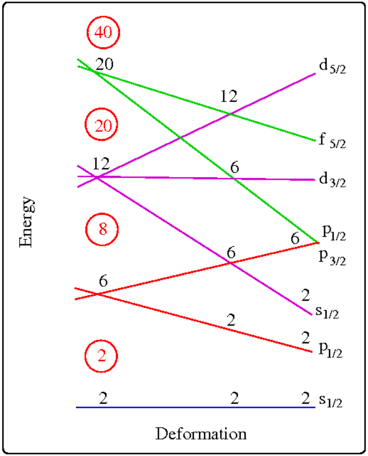

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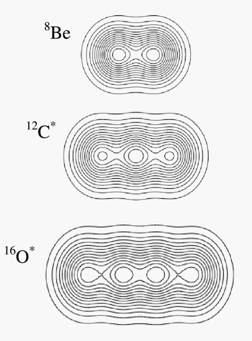
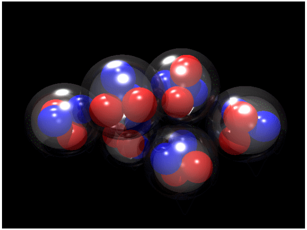


Clusters in nuclei.

Summary

- α clusters appear in N=Z light nuclei.
- Close to decay threshold.
- Strong deformation leading to shell rearrangement.
- Experimental evidence: Eg. look for deformed structure.

Digression: ^{12}C, life and clusters.



Synthesis of ^{12}C

- Insufficient production for ^{12}C;
- F. Hoyle (1954) predicted a $\simeq$ 7.27 MeV state

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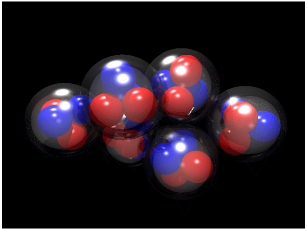

Synthesis of ^{12}C

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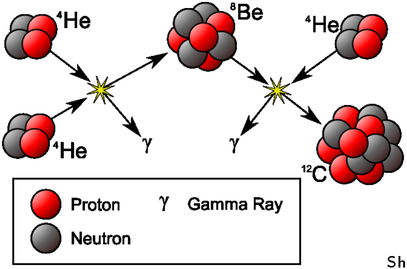
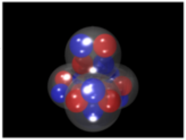
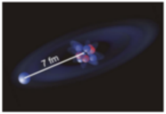
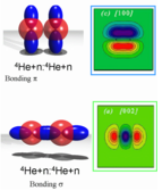
- Triple α process: Fowler (Nobel Prize 1983).

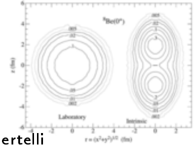


Few Body Systems









Summary

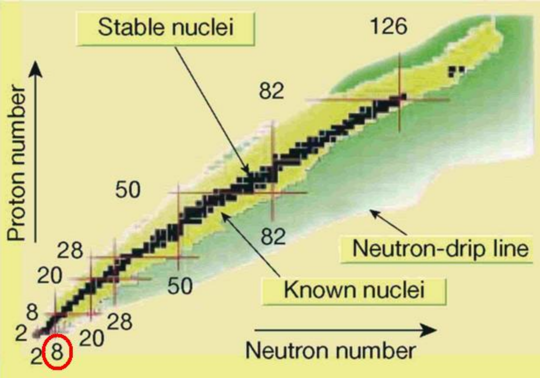
1. Benchmarks for models.
2. Strong shell effects.
3. exotic phenomena:
4. haloes, clusters, molecules, . . .

Outline

Today

1. Few body systems.
2. Heavier systems.
    - Shell evolution: quick tour.
    - Studies at $N = 28$.

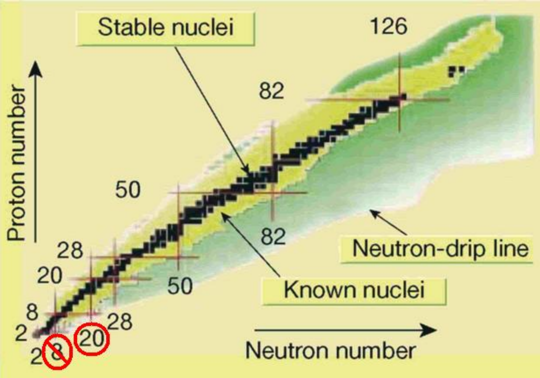
Shell evolution: overview



$N = 8$

- ${}^{16}_{8}O_8$: E(2^+)$\simeq$ 7 MeV
- ${}^{12}_{4}Be_8$: E(2^+)$\simeq$ 2 MeV
- $2s_{1/2}$ intruding and breaking the gap.

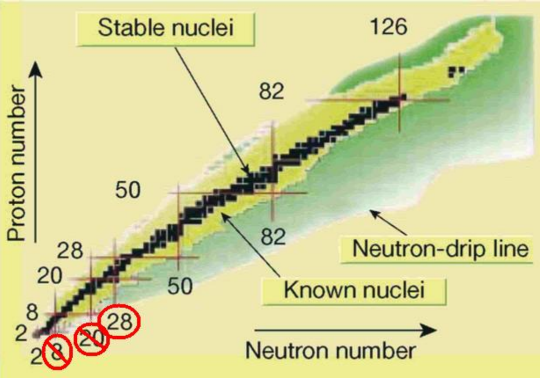
Shell evolution: overview



$N = 20$

- $^{40}_{20}Ca_{20}$: $E(2^+) \simeq 7$ MeV
- $^{32}_{12}Mg_{20}$: $E(2^+) \simeq 0.9$ MeV
- Island of deformation near $^{32}_{12}Mg_{20}$.

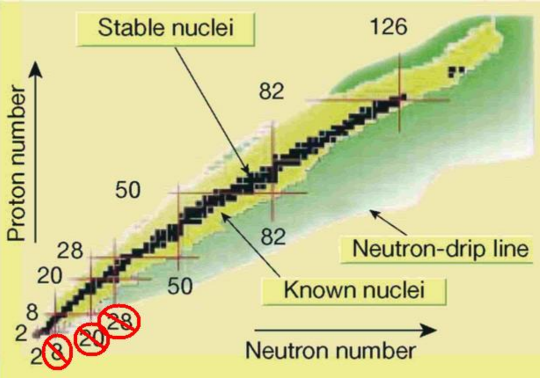
Shell evolution: overview



$N = 28$

- ${}^{48}_{20}Ca_{28}$: $E(2^+) \simeq 4$ MeV
- ${}^{42}_{14}Si_{28}$: $E(2^+) \simeq 0.8$ MeV
- Island of deformation near ${}^{42}_{14}Si_{28}$.

Shell evolution: overview



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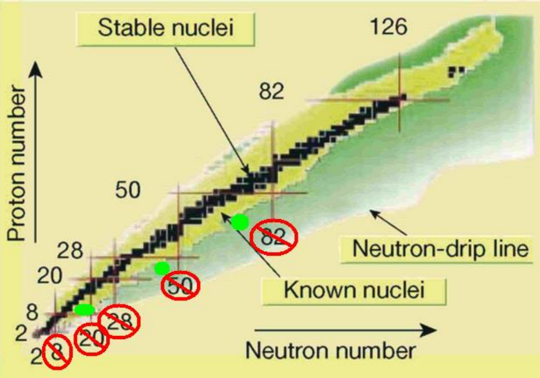


50, 82

Predicted to disappear in exotic (enough) nuclei.

14, 16, 32, 40

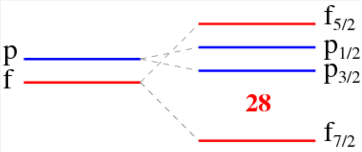
Observed magic properties in neutron-rich nuclei.

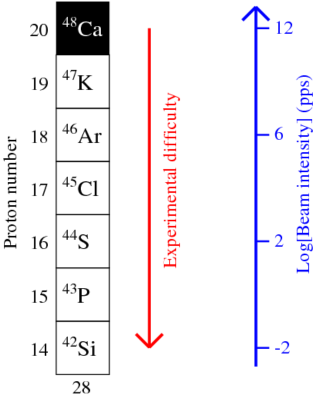
70

Predicted as magic number in exotic nuclei.

The $N = 28$ magic number

1^{st} Spin-Orbit magic number





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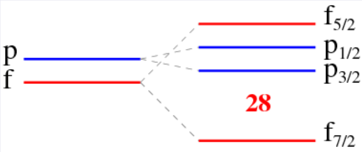

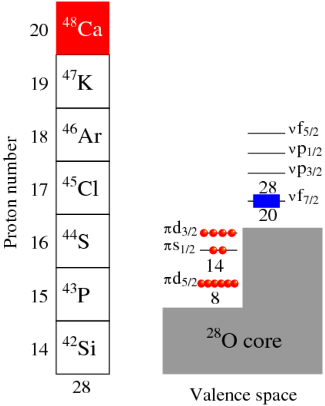

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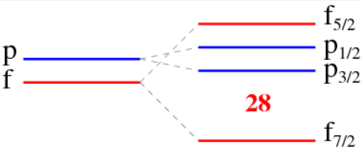

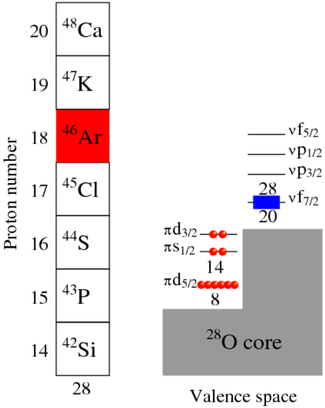

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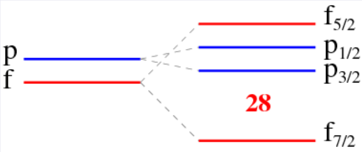
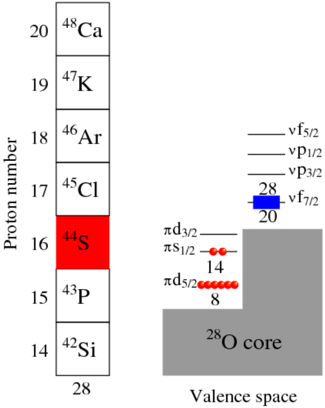
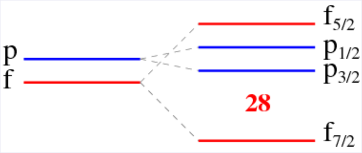
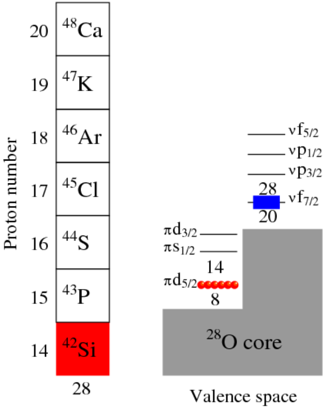




The $N = 28$ magic number

1^{st} Spin-Orbit magic number



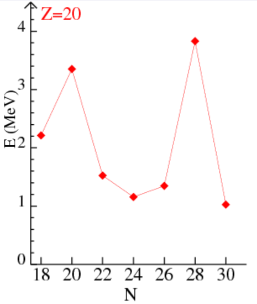


Study of exotic nuclei

A way to access part of NN interaction not at play in stable nuclei.

2^+ excitation energy

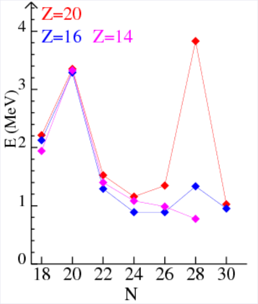
Onset of correlations - Indirect evidence



1. $N = 20, 28$: magic at $Z = 20$.

2^+ excitation energy

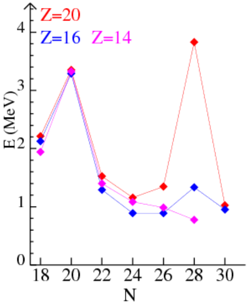
Onset of correlations - Indirect evidence



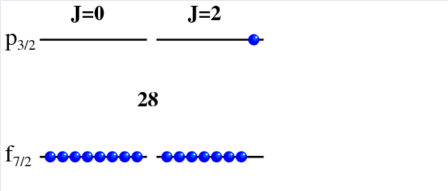
1. $N = 20, 28$: magic at $Z = 20$.
2. Deacrease at $Z = 16$ ...
3. ... and at $Z = 14$ as well.

$N = 20$ remains rigid up to $Z = 14$, while $N = 28$ vanishes.

Basic interpretation



2^+ configurations



- Neutron excitations across $N = 28$.
- $(\nu f_{7/2} \otimes \nu p_{3/2})^{J^\pi=2^+}$.

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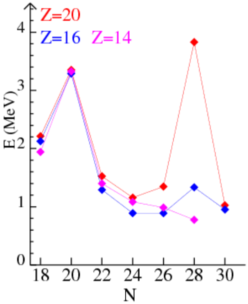
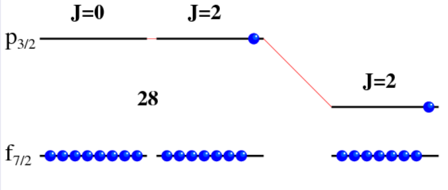


2^+ configurations



- Neutron excitations across $N = 28$.
- $(\nu f_{7/2} \otimes \nu p_{3/2})^{J^\pi=2^+}$.
- Shell gap reduced $\Rightarrow$ E(2^+) reduced.
- Neglects correlations.

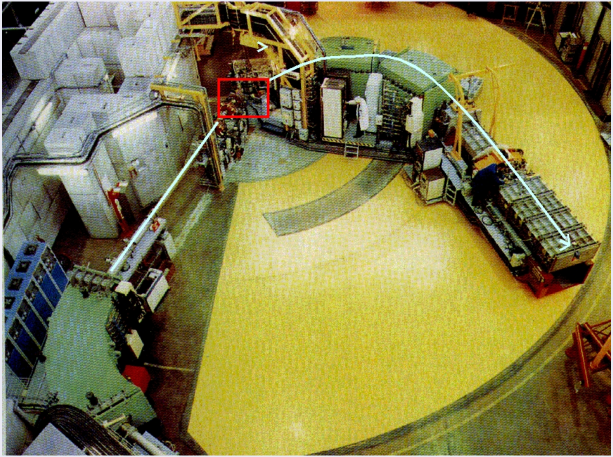
Transfer reaction

Interest

- Direct way to probe shell structure
- Possible for relatively high intensity beam ($> 10^4$pps)
- Performed on the radioactive $^{46}_{18}Ar_{28}$ nucleus.

Transfer reaction: $^{46}Ar(d,p)^{47}Ar$

Experimental Setup: SPEG at GANIL



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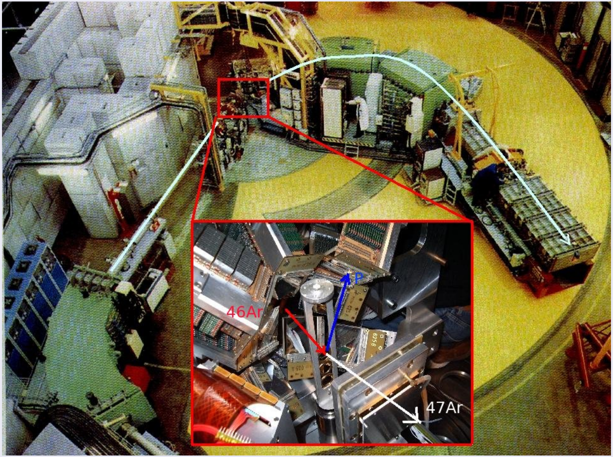

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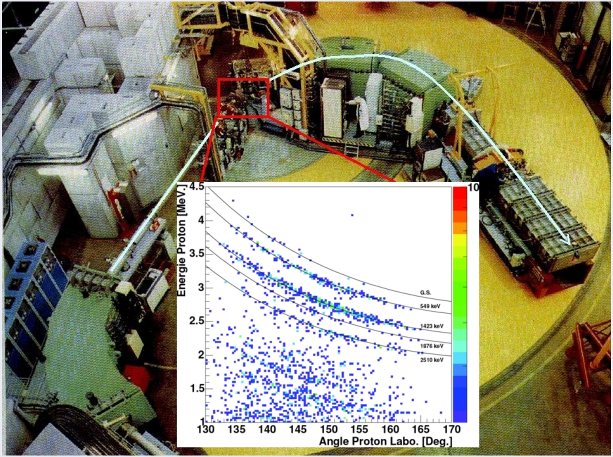

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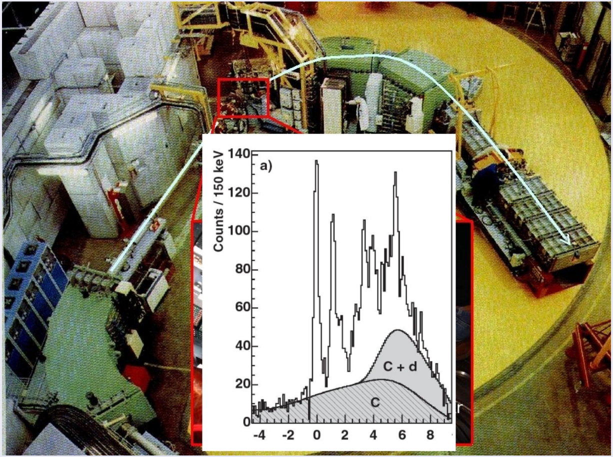
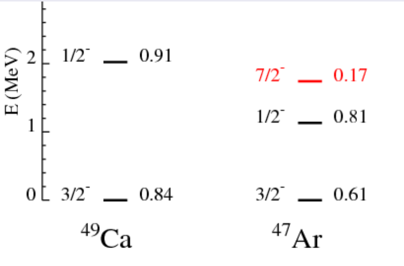
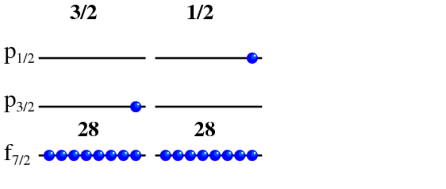


$^{46}Ar(d,p)^{47}Ar$: Results

Level scheme



State configurations



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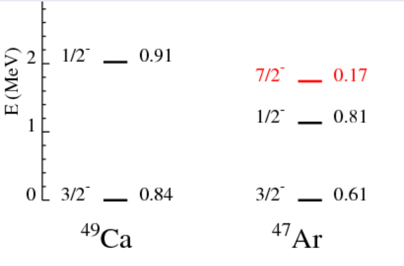

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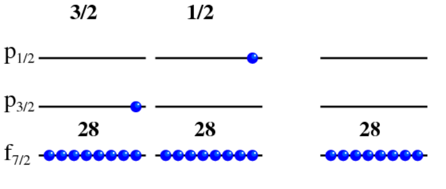

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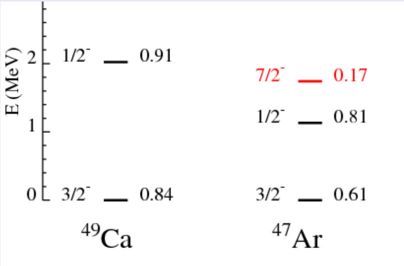

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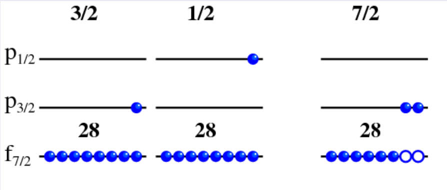
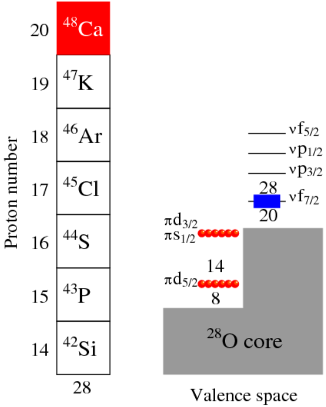


Conclusions

1. Still single particle states in ^{47}Ar.
2. $7/2^-$ intruder state.
3. Slight erosion of $N = 28$ (by 300keV).

Shell evolution from $_{20}$Ca to $_{14}$Si



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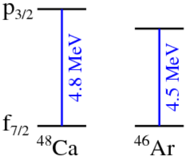

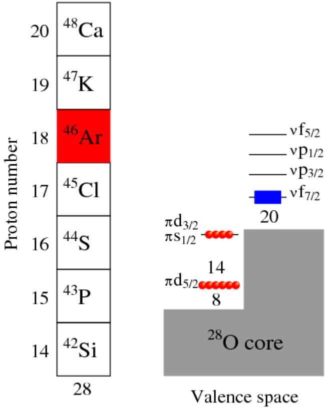

1. $\pi s_{1/2}$ and $d_{3/2}$ orbits degenerate.
2. Attractive $\pi d_{3/2}$-$\nu f_{7/2}$ interaction.

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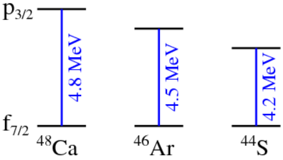

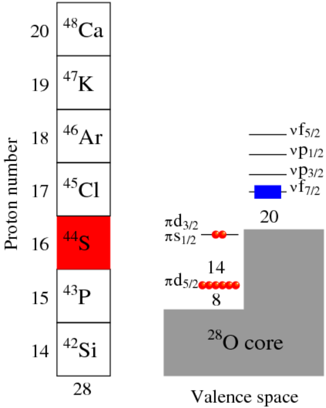

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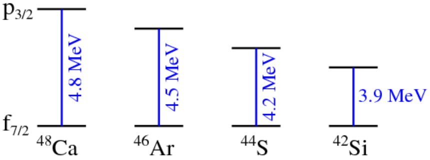
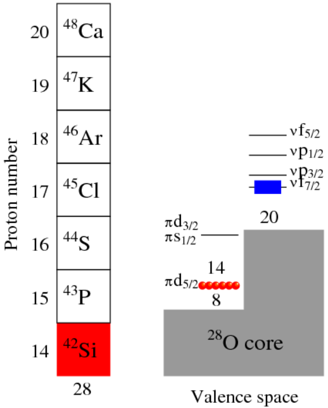




1. $\pi s_{1/2}$ and $d_{3/2}$ orbits degenerate.
2. Attractive $\pi d_{3/2}$-$\nu f_{7/2}$ interaction.
3. Not strong enough effect.

Shell evolution: what else?

Correlations

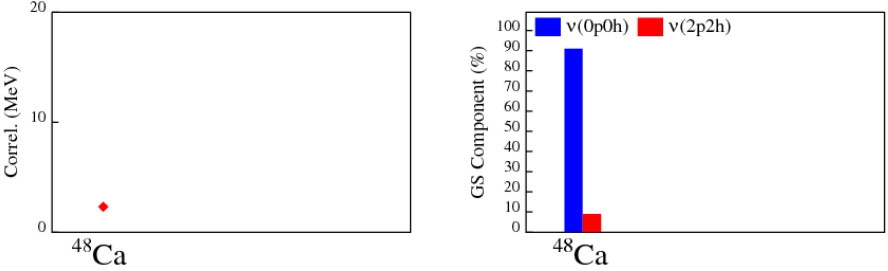
$$\mathscr{H} = \mathscr{H}_{Mono} + \mathscr{H}_{Multi}$$

$\mathscr{H}_{Mono}$ main component:

$$V_{Mono} = \frac{\sum_J (2J+1) V_{ij}^J}{\sum_J (2J+1)}$$

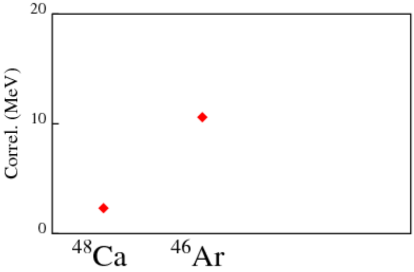
$\mathscr{H}_{Multi}$: correlations (pairing, quadrupole, ...).

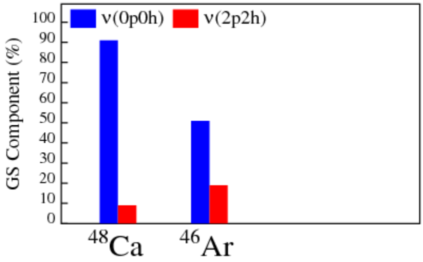
Onset of correlation at $N = 28$



- ^{48}Ca: Less than gap size

Onset of correlation at $N = 28$





- ^{48}Ca: Less than gap size
- ^{46}Ar: Promote 2 neutrons

L. Gaudefroy *et al.*, Phys. Rev. Lett.97, 092501(2006).

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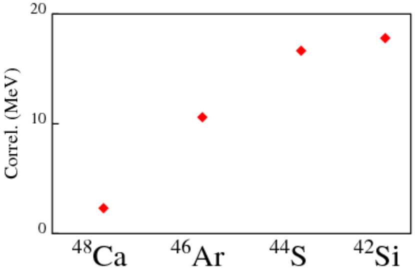
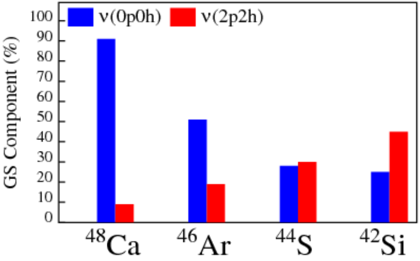




- ^{48}Ca: Less than gap size
- ^{46}Ar: Promote 2 neutrons

L. Gaudefroy *et al.*, Phys. Rev. Lett.97, 092501(2006).

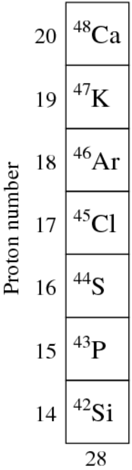
- ^{44}S: Spher./Def. shape coex.

S. Grévy *et al.*, *Submit. to Phys. Rev. Lett.*

- ^{42}Si: Deformed nucleus.

B. Bastin *et al.*, Phys. Rev. Lett.99, 022503(2007).

Shell evolution at $N = 28$: Summary



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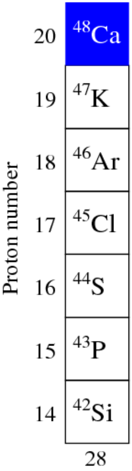

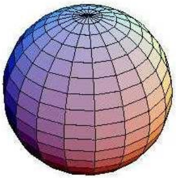

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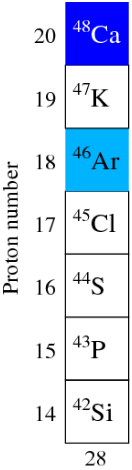

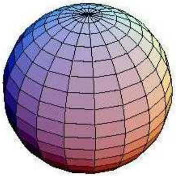

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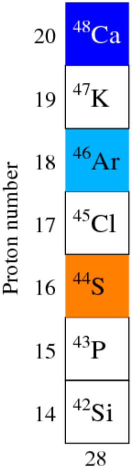

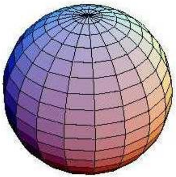

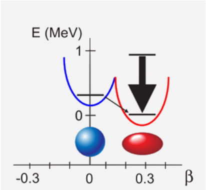

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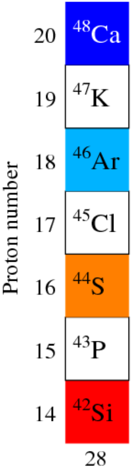

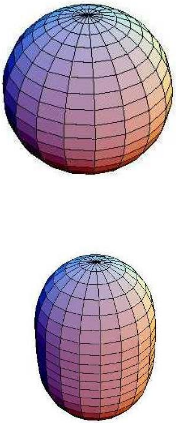

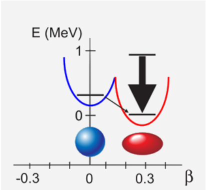

Concluding remarks

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6. Exotic nuclei: a probe for NN-interaction.
7. Larger systems: from magic to strongly correlated.

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8. Correlations $\Longleftrightarrow$ deformation - Alexandre's lecture.