Shell Effects in Atomic Nuclei

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Shell Effects in Finite Quantum Systems
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The atomic nucleus

**General properties**

- **Z** Protons: $J^\pi = 1/2^+$; 
  - **N** Neutrons: $J^\pi = 1/2^+$; 
  - $A = N + Z$ fermions.
- Strong interaction range: $\simeq 2$ fm
- Nuclear radius: $R \simeq r_0 A^{1/3}$ fm, 
  - $r_0 \simeq 1.2$ fm.
- Nucleon mean free path: $> R$. 

![Diagram](https://via.placeholder.com/150)
From nucleon-nucleon to nuclear interaction

Nuclei description
- Strong short-range repulsion;
- A $(N+Z)$ interacting fermions;
- Ab initio approach

Nuclear mean field
- Created by the $(A-1)$ nucleons;
- Replaces NN-interaction.
- Shell Model or Mean Field approaches.
Magic numbers: 2, 8, 20, 28, 50, 82, 126

"What makes a number magic is that a configuration of a magic number of neutrons, or of protons, is unusually stable whatever the associated number of the other nucleons.[...]

We found that there were a few nuclei which had greater isotopic as well as cosmic abundance than our theory or any other reasonable theory could explain. Then I found those nuclei had something in common: they either had 82 neutrons, [...] or 50 neutrons."
Spin-Orbit interaction

**Harmonic oscillator potential**

\[ U(r) = \frac{1}{2} M \omega^2 r^2 \]

- Magic numbers: 2, 8, 20, 40, 70

**Angular momentum and spin-orbit**

\[ U'(r) = U(r) + \ell^2 + \ell s \]

- Magic numbers: 2, 8, 20, 28, 50, 82
Success and failure of the nuclear shell model

**Good features**

1. Accounts for known **magic numbers**.
2. Reproduces $J^\pi$, $E^*$, $Q$, $\mu$…

**Bad features**

1. Built from knowledge on stable nuclei.
2. (Dis)appearance of magic numbers in unstable nuclei.
Outline

Today

1. Few body systems.
   - Haloes.
   - Clusters.

2. Heavier systems.
   - Shell evolution: general view.
   - Studies at $N = 28$.

Tomorrow

1. Shapes and coexistence.

2. Super heavy elements.
Few-body systems

Why?

1. Nuclear interaction $\propto A^{-1/3}$
2. Strong shell effects expected
3. Exotic phenomena

Haloes

Clusters
Density distributions in He isotopes

Add 2 neutrons to $^4\text{He}$

$\Rightarrow \rho(r > 2) \uparrow$ factor of 10.

$^6\text{He} \simeq ^8\text{He}$
Density distributions in He isotopes

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  $\Rightarrow \rho(r > 2) \uparrow$ factor of 10.

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- From $^{40}$Ca to $^{42}$Ca
  $\Rightarrow$ No significant change.
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- idem for $^{44}$Ca...
- 20 neutrons latter $\rho(r > 2) \uparrow$. 

S.C. Pieper & R.B. Wiringa

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Halo nuclei

An exotic phenomenon

- Weakly bound nuclei.
- Extension of neutron wave function out of the interaction range!
- Linked to shell structure (s or p waves).
Halo nuclei: Experimental evidence

\[ R \text{ from reaction cross section:} \]
\[ \sigma = \pi (R_{\text{Target}} + R_{\text{Proj}})^2. \]


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Halo nuclei: Experimental evidence

- $R$ from reaction cross section: 
  \[ \sigma = \pi (R_{\text{Target}} + R_{\text{Proj}})^2. \]
- Does not follow $A^{1/3}$ law for: $(^6,^8\text{He}), ^{11}\text{Li}, ^{11,14}\text{Be}$ and $^{17}\text{B}$. 
- (Near) Drip line nuclei.


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Halo nuclei: Shell effect

\[ \Psi(r) \propto e^{-Sn_{sn}}. \]

Loosely bound systems

Centrifugal force.

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Halo nuclei: Shell effect

Drip-line

- Loosely bound systems
- $\Psi(r) \propto e^{-Snr/r}$
- Low $\ell$
- Centrifugal force.
Halo nuclei: Shell effect

Drip-line

- Loosely bound systems
  - $\Psi(r) \propto e^{-\frac{S_{nr}}{r}}$
- Low $\ell$
- Centrifugal force.
Halo nuclei: Shell effect

From $N = 9$ to $N = 14$: $\nu d_{5/2}$ filling.

Strong shell effect $\Rightarrow$ Shell rearrangement.
Halo nuclei: Shell effect

**Structural effect**

- From $^{15}\text{C}_9$ to $^{22}\text{C}_{16}$ $\Rightarrow \nu s_{1/2}$ orbit as GS.
- Not yet quantitatively understood.
# Halo nuclei

## Summary

1. Extension of nucleon wave function out of interaction range.
2. Appear in light loosely bound nuclei.
3. Shell effects $\Rightarrow$ orbital reordering.
Density distribution in $^8\text{Be}$.

- **Unbound GS**: $T_{1/2} \simeq 10^{-16}\text{s}$
  \[ \Rightarrow ^8\text{Be} \rightarrow ^4\text{He} + ^4\text{He}. \]
- **0$^+$**: two structures
  \[ \Rightarrow ^4\text{He} \text{ cluster}. \]
- **$\alpha$**: $N = Z = 2$.
- **Clusters** might appear in light $N = Z$ nuclei.

Clusters in nuclei

Energy threshold for clustering

- Must be energetically allowed.
- $^8\text{Be} \rightarrow 2\alpha$
- $^{4n}\text{X} \rightarrow n\alpha$
- Cluster phase expected around $E^* = \text{decay threshold}$.


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Clusters & Shell effects

Adapted from:

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Clusters in nuclei.

Summary

- $\alpha$ clusters appear in $N=Z$ light nuclei.
- Close to decay threshold.
- Strong deformation leading to shell rearrangement.
- Experimental evidence: Eg. look for deformed structure.
Digression: $^{12}\text{C}$, life and clusters.

**Synthesis of $^{12}\text{C}$**

- Insufficient production for $^{12}\text{C}$;
- F. Hoyle (1954) predicted a $\approx 7.27$ MeV state
Digression: $^{12}$C, life and clusters.

**Synthesis of $^{12}$C**

- Insufficient production for $^{12}$C;
- F. Hoyle (1954) predicted a $\simeq 7.27$ MeV state;
- Triple $\alpha$ process: Fowler (Nobel Prize 1983).
Few Body Systems

Summary

1. Benchmarks for models.
2. Strong shell effects.
3. Exotic phenomena: haloes, clusters, molecules, ...
Outline

Today

1. Few body systems.
2. Heavier systems.
   - Shell evolution: quick tour.
   - Studies at $N = 28$. 
Shell evolution: overview

\[ N = 8 \]

- \(^{16}\text{O}_8\): \( E(2^+) \approx 7 \text{ MeV} \)
- \(^{12}\text{Be}_8\): \( E(2^+) \approx 2 \text{ MeV} \)
- \(2s_{1/2}\) intruding and breaking the gap.
Shell evolution: overview

\[ N = 20 \]

- \(^{40}\text{Ca}_{20}\): \(E(2^+) \approx 7\) MeV
- \(^{32}\text{Mg}_{20}\): \(E(2^+) \approx 0.9\) MeV
- Island of deformation near \(^{32}\text{Mg}_{20}\).
Shell evolution: overview

- $N = 28$
  - $^{48}\text{Ca}_{28}$: $E(2^+) \simeq 4$ MeV
  - $^{42}\text{Si}_{28}$: $E(2^+) \simeq 0.8$ MeV
  - Island of deformation near $^{42}\text{Si}_{28}$.
Shell evolution: overview

- $^{48}_{20}\text{Ca}_{28}$: $E(2^+) \simeq 4$ MeV
- $^{42}_{14}\text{Si}_{28}$: $E(2^+) \simeq 0.8$ MeV
- Island of deformation near $^{42}_{14}\text{Si}_{28}$. 
Shell evolution: overview

50, 82
Predicted to disappear in exotic (enough) nuclei.

14, 16, 32, 40
Observed magic properties in neutron-rich nuclei.

70
Predicted as magic number in exotic nuclei.
The $N = 28$ magic number

1st Spin-Orbit magic number

$p_f$ \quad $f_{5/2}$ \quad $f_{7/2}$

Heavier systems

Summary

Few-body systems

The $N = 28$ magic number

1st Spin-Orbit magic number

$p_f$ \quad $f_{5/2}$ \quad $f_{7/2}$

Heavier systems

Summary

Few-body systems

Study of exotic nuclei

Away to access part of NN interaction not at play in stable nuclei.

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Shell Effects in Atomic Nuclei
The $N = 28$ magic number

First Spin-Orbit magic number:

- $f$ states:
  - $f_{7/2}$
  - $f_{5/2}$

- $p$ states:
  - $p_{1/2}$
  - $p_{3/2}$

Proton numbers and nuclei:

- $^{48}\text{Ca}$
- $^{47}\text{K}$
- $^{46}\text{Ar}$
- $^{45}\text{Cl}$
- $^{44}\text{S}$
- $^{43}\text{P}$
- $^{42}\text{Si}$

Valence space and $^{28}\text{O}$ core

Study of exotic nuclei: Away to access part of NN interaction not at play in stable nuclei.
The $N = 28$ magic number

**1st Spin-Orbit magic number**

- $f$ states
  - $f_{5/2}$
  - $f_{7/2}$
  - $28$
- $p$ states
  - $p_{1/2}$
  - $p_{3/2}$

**Proton number**

- $20$
- $19$
- $18$
- $17$
- $16$
- $15$
- $14$

**Valence space**

- $^{28}\text{O core}$
- $^{42}\text{Si}$
- $^{43}\text{P}$
- $^{44}\text{S}$
- $^{45}\text{Cl}$
- $^{46}\text{Ar}$
- $^{47}\text{K}$
- $^{48}\text{Ca}$

**Shell Effects in Atomic Nuclei**

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The $N = 28$ magic number

1\textsuperscript{st} Spin-Orbit magic number

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Valence space

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Shell Effects in Atomic Nuclei 27/37
The $N = 28$ magic number

**1st Spin-Orbit magic number**

- $p$
- $f$
- $f_{5/2}$
- $f_{7/2}$
- $28$

Study of **exotic nuclei**

A way to access part of NN interaction not at play in stable nuclei.
Onset of correlations - Indirect evidence

$N = 20, 28$: magic at $Z = 20$. 
$2^+$ excitation energy

Onset of correlations - Indirect evidence

2. Decrease at $Z = 16$ . . .
3. . . . and at $Z = 14$ as well.

$N = 20$ remains rigid up to $Z = 14$, while $N = 28$ vanishes.
Neutron excitations across $N = 28$.

$(\nu f_{7/2} \otimes \nu p_{3/2})^{J\pi=2^+}$. 
Neutron excitations across $N = 28$.

$$(\nu f_{7/2} \otimes \nu p_{3/2})^{J\pi=2^+}.$$  

Shell gap reduced $\Rightarrow E(2^+) \text{ reduced}$.

Neglects correlations.
Transfer reaction

**Interest**
- Direct way to probe shell structure
- Possible for relatively high intensity beam ($> 10^4$ pps)
- Performed on the radioactive $^{46}_{18}$Ar$_{28}$ nucleus.
Transfer reaction: $^{46}Ar(d, p)^{47}Ar$

Experimental Setup: SPEG at GANIL
Transfer reaction: $^{46}\text{Ar}(d, p)^{47}\text{Ar}$

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**Experimental Setup: SPEG at GANIL**
$^{46}\text{Ar}(d, p)^{47}\text{Ar}$: Results

### Level scheme

- $^{49}\text{Ca}$
- $^{47}\text{Ar}$

### State configurations

- $3/2$:
  - $p_{1/2}$
  - $p_{3/2}$
  - $f_{7/2}$ with $N = 28$

- $1/2$:
  - $p_{1/2}$
  - $p_{3/2}$
  - $f_{7/2}$ with $N = 28$
\( ^{46}\text{Ar}(d, p)^{47}\text{Ar} \): Results

### Level scheme

- \( ^{49}\text{Ca} \)
- \( ^{47}\text{Ar} \)

#### State configurations

- \( ^{3/2} \)
- \( ^{1/2} \)

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<th>State</th>
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46 Ar(d, p)47 Ar: Results

Level scheme

State configurations

Conclusions

1. Still single particle states in 47 Ar.
2. 7/2− intruder state.
Shell evolution from $^{20}\text{Ca}$ to $^{14}\text{Si}$
Shell evolution from $^{20}\text{Ca}$ to $^{14}\text{Si}$

1. $\pi s_{1/2}$ and $d_{3/2}$ orbits degenerate.
2. Attractive $\pi d_{3/2}-\nu f_{7/2}$ interaction.
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1. $\pi s_{1/2}$ and $d_{3/2}$ orbits degenerate.
2. Attractive $\pi d_{3/2} - \nu f_{7/2}$ interaction.
3. Not strong enough effect.

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Shell evolution: what else?

**Correlations**

\[ \mathcal{H} = \mathcal{H}_{Mono} + \mathcal{H}_{Multi} \]

**\( \mathcal{H}_{Mono} \)** main component:

\[ V_{Mono} = \frac{\sum J (2J + 1) V_{ij}^J}{\sum J (2J + 1)} \]

**\( \mathcal{H}_{Multi} \)**: correlations (pairing, quadrupole, ...).
Onset of correlation at $N = 28$

- $^{48}\text{Ca}$: Less than gap size
Onset of correlation at $N = 28$

- $^{48}\text{Ca}$: Less than gap size
- $^{46}\text{Ar}$: Promote 2 neutrons

Onset of correlation at $N = 28$

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- $^{46}\text{Ar}$: Promote 2 neutrons

$L. \text{Gaudefroy et al.}, \text{Phys. Rev. Lett.}\text{97, 092501(2006)}.$

- $^{44}\text{S}$: Spher./Def. shape coex.

$S. \text{Grévy et al.}, \text{Submit. to Phys. Rev. Lett.}$

- $^{42}\text{Si}$: Deformed nucleus.

Shell evolution at $N = 28$: Summary

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- $^{47}\text{K}$
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Proton number
Shell evolution at $N = 28$: Summary
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![Graph showing the energy levels of atomic nuclei with a downward arrow indicating a decrease in energy.](image)
Shell evolution at $N = 28$: Summary
Concluding remarks

1. Atomic nuclei: A interacting fermions.
2. Shell structure and magic numbers.
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5. Original models from stable nuclei.
7. Larger systems: from magic to strongly correlated.
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8. Correlations \(\iff\) deformation - Alexandre’s lecture.