Astronomy 100 Homework 4: Solutions Dr. Warner

1. Equilibrium in any star requires a balance between gravity and pressure. If there is not a balance, the star will either collapse or expand depending upon whether gravity or pressure dominates. In white dwarfs and neutron stars the pressure comes, respectively, from electron degeneracy pressure and neutron degeneracy pressure. There is a limit on this form of pressure generation, and so if the mass of the star gets too high, gravity will win and the star will collapse.

The Chandrasekhar limit for a white dwarf is about 1.4 solar masses. Beyond this the gravitational forces dominate over the *electron* degeneracy pressure, and the white dwarf must collapse. In the collapse the electrons are crushed out of existence: they combine with the protons to form neutrons, and the end result is a neutron star.

The Chandrasekhar limit for a neutron is about 3.0 solar masses. Beyond this, the gravitational forces dominate over the *neutron* degeneracy pressure, and the neutron star must collapse, and the end result is a black hole.

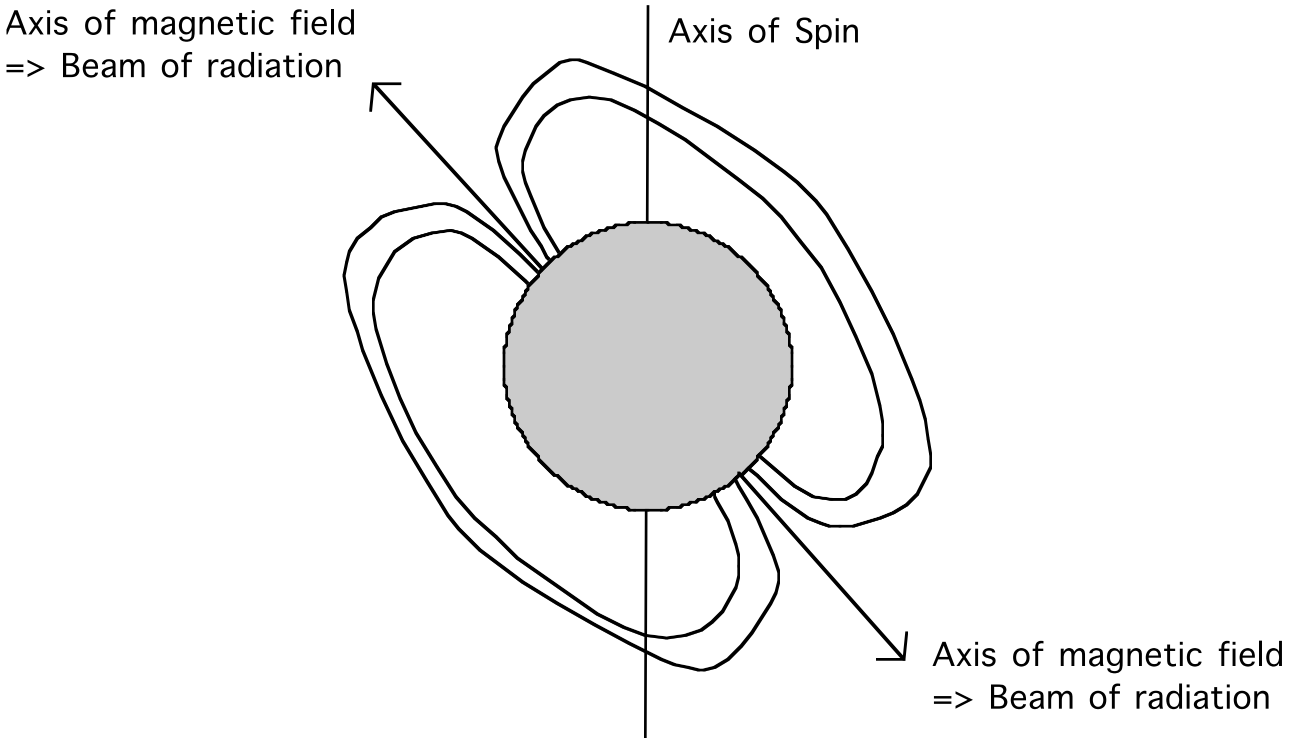
There are thus no white dwarfs of mass greater than 1.4 solar masses, and there are no neutron stars of mass greater than 3 solar masses. Thus a 10 solar mass object can be neither a white dwarf nor a neutron star. But the logic only goes one way: it is perfectly possible to have a neutron star of black hole of a lower mass, like 1 solar mass. Indeed, many observed pulsars come from neutron stars of about 1 solar mass.

The last part of the question is aimed at a elucidating a subtlety:

If you add hydrogen or helium to a white dwarf you can get “Nova” activity: Hydrogen and helium will fuse giving new life to the star. However the primary intent of my question here is to remind you that if you add enough mass of any type to a white dwarf then when you reach (or get near to) the Chandrasekhar limit it ***does not*** collapse to a neutron star. Instead, carbon fusion starts explosively throughout the star in a runaway process: You get a type Ia, or carbon detonation supernova. To make a neutron star you generally need all the outer layers of the star to be in place to prevent this detonation from blowing the star to bits, then, when the core collapse finally occurs, it will make a neutron star or black hole.

If you add matter to a neutron star then there may be some fusion of the matter added, but there are no fusion processes that can occur within the neutron star itself, and so it will not explode, and will indeed collapse to a black hole when you exceed the Chandrasekhar limit.

2. Pulsars are rapidly rotating neutron stars with magnetic fields that do not line up with the axes of spin:

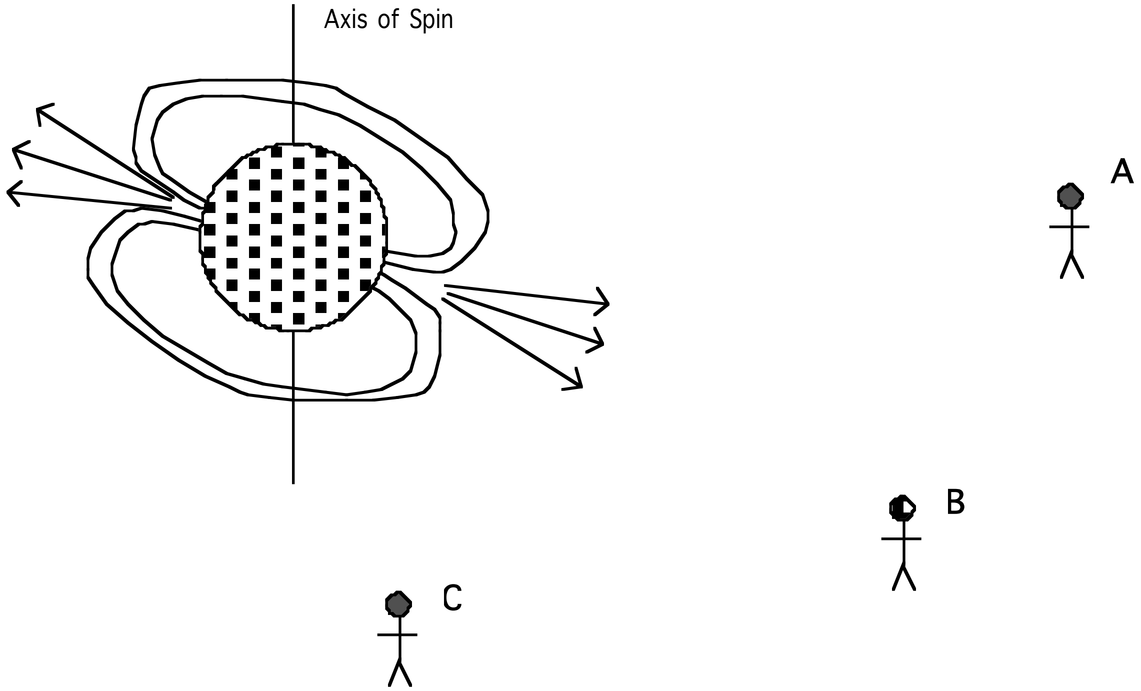


They were first detected by Jocelyn Bell through the reception of incredibly regular pulsed radio signals originating from deep space.

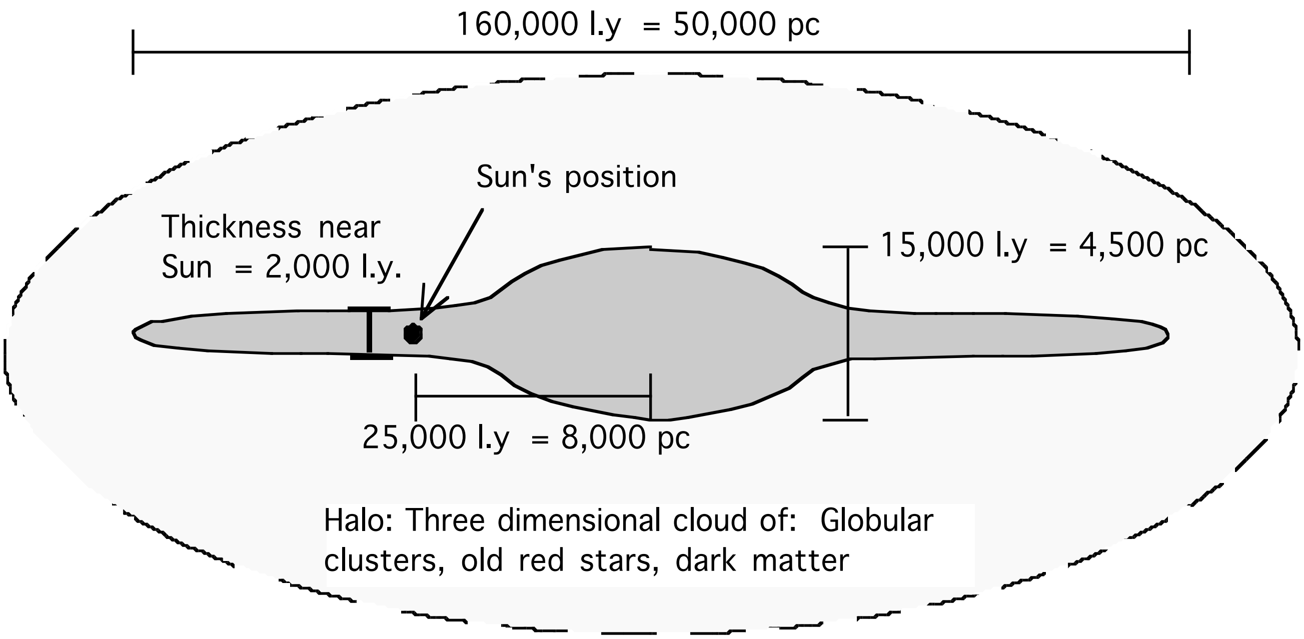
Particles and matter are heated and focused by the spinning magnetic field and are beamed out along the magnetic field axis. The radio signals come from the radiation generated by these hot beams of matter. The radiation is thus emitted from the north and south magnetic poles in a pair of beams rather like those of a light house.

Since the beams are strongly directional, the appearance of a pulsar depends upon the relative location of the observer, the pulsar and the beam directions.

2. Continued: If the magnetic axis is highly tilted and the beams spread enough then a suitably placed observer (A below) will see two strong beams. An observer in a different position might see one beam very strongly, but only see a little of the other beam (B below). An observer, like C, may see nothing at all.



3. Rough layout of the galaxy

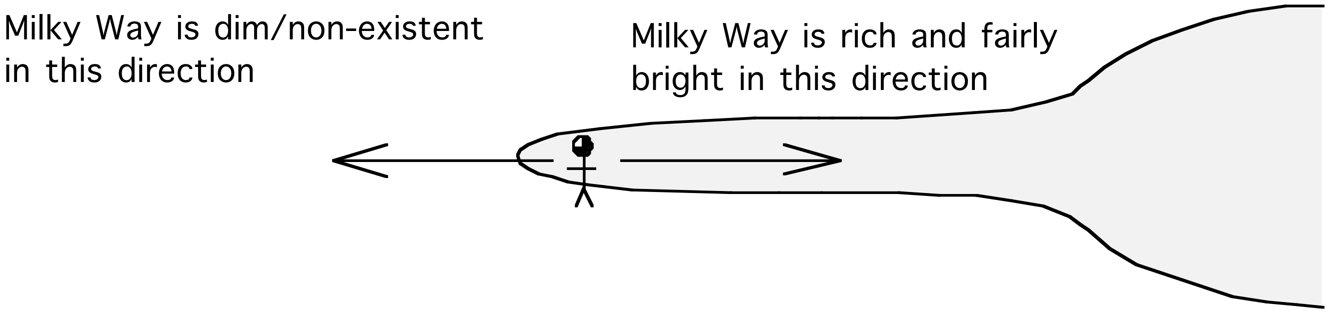


Approximate total mass: 10^{12} solar masses

Approximate Number of Stars: 10^{11}

Approximate total luminosity: 2.5×10^{10} solar luminosities

4. a) If you were located near the edge of our galaxy then the Milky Way would not be a uniform, roughly evenly distributed band around the sky. Instead it would fade away almost completely in one direction (where one would be looking directly away from the galactic core) and it would grow in brightness as one looks around the sky, becoming relatively rather bright as one looks toward the center of the galaxy.



4. b) Interstellar extinction means that starlight is scattered and absorbed on its way to us. As a result, more distant stars within the galactic disk appear to be much dimmer than one would otherwise expect. Beyond a certain limit, more distant stars within the galactic disk would become optically invisible. This meant that in the early studies of the number of visible stars in each direction in the Milky Way, the numbers came out about the same for each direction. This suggested that we are centrally placed. In reality, these studies did not see deeply enough into the galactic disk to discern that there are more stars in one direction than another, and hence we are “off-center” to a significant degree. Because of interstellar extinction this imbalance is only discernable if one looks at very dim stars in the disk, or (as did Harlow Shapley) one looks outside the disk and counts globular clusters.

5. Harlow Shapley measured the size of our galaxy, the Milky Way, and was the first person to determine our position in it. He accomplished this by surveying the cloud of globular clusters in the halo around our galaxy, and using this to infer the size of the galaxy and our position relative to the center. More specifically, he found and observed variable stars (in fact, RR Lyrae variables, not Cepheids) in the globular clusters, and used the period-luminosity relation to figure the distance, and then mapped out the galactic halo. He determined the size of the cloud, and showed that it was centered several kiloparsecs from us in the direction of Sagittarius. He argued that the cloud must be centered upon the core of the galaxy.

Globular clusters are roughly spherical clusters of up to about a million stars in a relatively small volume (about 100 pc across). Globular clusters are generally very old clusters, containing no high-mass main sequence stars. Such clusters are generally found outside the plane of a spiral galaxy, that is, they populate the galactic halo.

The Shapley-Curtis debate was held in 1920, and was intended to resolve the nature of spiral nebulae: Were they separate “island universes,” that is, separate galaxies, outside our own galaxy, or were they another class of nebulae lying within our own galaxy? The issue was implicitly one of size: Were they very large distant objects, or were they relatively small, closer objects. Surprisingly, perhaps, Shapley argued that spiral nebulae were part of the Milky Way, while Curtis argued that they were far more distant “island universes.” The outcome of the debate was inconclusive, and it was inconclusive precisely because of the issue of size and distance. No one had any data about how far away (and thus how large) these spiral nebulae were. The issue was resolved three of four years later, by Edwin Hubble….

6. The missing mass problem, or the dark matter problem, is the fact that about 90% of the matter in the universe is not visible to us. That is, its presence cannot be detected, or even inferred, by studying any form of electromagnetic radiation. We also do not know precisely what it is (though we have a number of ideas - MACHO's and WIMP's). The "visible" matter is generally luminous in some part of the electromagnetic spectrum, and we see it either as stars, nebulae, or clouds that obscure other stars or nebula. This visible matter is about 10% of what we can infer must be present, and this inference is made through the laws of gravity. I only asked for two pieces of evidence, but in lectures I gave three:

(i) Flat Rotation curves of galaxies: The fact that stars orbiting the cores of galaxies tend to do so at nearly constant speed, independent of the distance from the core. If most of the mass were concentrated near the center of the galaxy, then one would have a "Keplerian speed profile" with speeds that decrease steadily with distance from the center. The visible matter in a galaxy is strongly concentrated in the middle, but the fact that the rotation curves are flat means that there must be a lot of "dark matter" in the outer part of the galactic disk, and particularly in the halo. This matter is not directly visible to us.

(i) Dynamics of clusters and superclusters of galaxies: Galaxies in a cluster orbit a common in the cluster, and we can, and have, measured the speeds of galaxies in clusters. We can also observe and add up all the visible matter in a cluster of galaxies. We find that the visible mass could not create a gravitational field of sufficient strength to hold the "speeding galaxies" in the cluster. The speeds of the galaxies are sufficiently high and the visible mass sufficiently low that the galaxies would have gone their own way long ago: the cluster would have evaporated. The same is true for the motions of clusters of galaxies within superclusters. We therefore conclude that there must be a lot of dark matter that creates sufficient gravity to hold the clusters and superclusters together. By balancing observed speeds against gravity we learn that about 90% of the mass in a cluster, or supercluster must be dark matter.

(iii) Gravitational Lensing: See Freedman and Kaufmann pp. 608-609 for details. The idea is that gravity can bend and focus rays of light (much as a glass lens can bend and focus light). In particular a large galaxy, or cluster of galaxies, can bend light from a distant source, such as a very distant galaxy or quasar, and create one or more images of that distant object.

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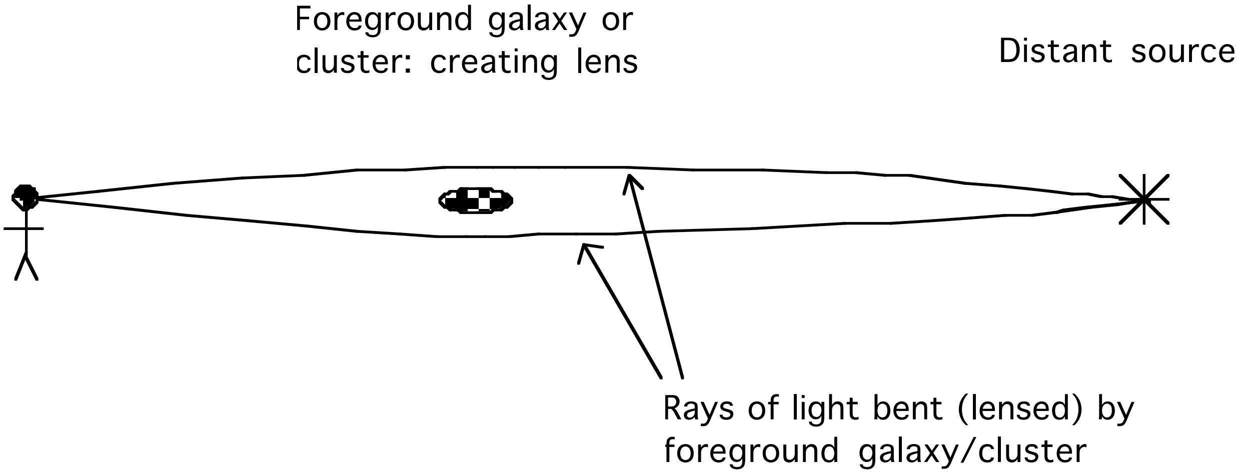


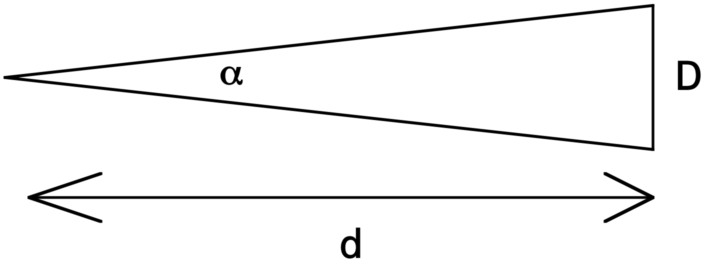
By studying such multiple images, and the bending of light by the large galaxy or cluster, we can get an estimate of the total mass of that large galaxy or cluster. We once again find that there is a “missing mass:” that is, galaxies and clusters must contain a lot of “Dark Matter” that we cannot “see,” or otherwise infer, from studying all the Electromagnetic emissions from that galaxy or cluster .

Only two of these three pieces of observational evidence were requested, and any two of (i), (ii) and (iii) will suffice.

7. There are three galaxies, apart from the Milky way, that can be seen by the human eye: The Large Magellanic Cloud (LMC), The Small Magellanic Cloud (SMC), and under fairly good conditions M31, or the great Andromeda Nebula/Galaxy.

8. This is once again an application of the relationship between size, distance and angular size:

$$D = \frac{\alpha\, d}{57.3}$$



where d is the distance to the object, D is the *size* of the object, and α is the *angular size* in degrees. In this problem you are given the physical size, D = 230,000 l.y., and the distance, d = 2.5 x 10^6 l.y. Thus, the angular size in degrees is:

$$\alpha \approx \frac{57.3\ \mathrm{D}}{\mathrm{d}} \approx \frac{57.3 \times 230{,}000}{2.5 \times 10^6} \approx 5.3^\circ$$

The Moon's angular size, as seen from Earth is about half a degree, and so the angular size of M31 is about ten moon diameters.

9. Most of this can be found in Section 26-3 and Table 26-1 of Freedman and Kaufmann:

Ellipticals:

- Elliptical shape (an oblong blob)
- No disk
- No other obvious structure, except for a dense central nucleus
- Stars orbit center of galaxy in orbits that are randomly oriented in three dimensions
- Devoid of interstellar dust and gas
- Little or no star formation in last 10 billion years
- Stars are old (and therefore small)
- Constitute the majority of galaxies today, if one includes Dwarf ellipticals. If one excludes dwarf ellipticals, then ellipticals account for about 20% of observed galaxies.
- Vast range of sizes - from a few thousand to a few million light years across

Spirals:

- Highly flattened disk of stars, dust and gas
- Thickened central bulge
- Spiral arms
- Extensive star formation (new stars) in disk, and particularly in spiral arms.
- Gas and stars in disk move in circular orbits around the galactic center
- Spherical halos with little or no gas and dust
- Halo stars are all old (and therefore small) - many in globular clusters. Halo stars have randomly oriented orbits in three dimensions
- A minority of galaxies today, if one includes dwarf ellipticals in the overall count. If one excludes dwarf ellipticals, then spirals account for almost 80% of observed galaxies.

10. Matter in spiral galaxies rotates so that the outer parts of the galaxy take longer to complete an orbit than the inner parts. The galaxies also rotate in a sense that tends to “wind up” the spiral arms. Combining these two facts means that spiral arms should indeed tend to get stretched out and wound up around the core. In particular, they should fade away, or disappear relatively rapidly .. after a few revolutions of the galaxy (that is, after a few hundred million to a billion years). The winding dilemma is the fact that observations are not consistent with this conclusion: Observations show too many galaxies with very well-defined spiral arms, which means that spiral arms must persist far longer than the winding motion would seem to allow.

This means that the spiral arms are not simply just made of matter orbiting the core. Indeed, many spiral arms appear to be more like waves in the ocean: Waves sweep through matter, stirring it up, and occasionally dragging it along, but ultimately the wave outruns the matter, leaving the stirred up matter in its wake. This is essentially the density-wave model of spiral arms, which says that spiral arms are the result of compression waves traveling around the galactic disk. These waves consist of regions of higher-than-average density, which therefore have a higher-than-average gravitational attraction for nearby matter and so attract and concentrate matter, and these more concentrated regions of matter are the spiral arms. Such a concentration of matter also enhances the rate of star formation, and so the density waves/spiral arms glow more brightly than neighboring, less dense regions. They are waves in that they move through the matter, concentrating it, but out-running it.

One of the problems with the density wave theory is that they too will ultimately die away, and yet spiral arms persist longer than even the density wave theory would, by itself, suggest. Density waves need a supply of energy to sustain them, and the problem is to find a source for that energy. One possible source of energy to generate and sustain the density waves is through collisions (or near misses) with other galaxies. Simulations, and indeed observations, show that such events can produce extremely strong, well-defined spiral arms.

There is another theory for explaining spiral arms: Self-propagating star formation. Density waves are expected to produce well-defined spiral arms, whereas some spiral galaxies are flocculent, which means that they have poorly-defined, or rather fuzzy spiral arms. Self-propagating star formation can account for this type of galaxy.

The bottom line is that the true cause of spiral arms is probably a combination of all three factors, and the particular combination varies from galaxy to galaxy.

This question asked you to outline the density wave theory and the role of collisions ... I did not ask for a discussion of self-propagating star formation, I merely mention it here for completeness.

11. MACHOs and WIMPs are possible explanations of Dark Matter. Observations (see answer to question 6) show that about 80% of matter in the universe is dark matter and, as yet, not directly visible to us. We therefore have to examine theoretical possibilities:

(i) MACHOs = Massive Compact Halo Objects. These are normal astronomical objects: Jupiters, brown dwarfs, black dwarfs, neutron stars, black holes .... Very massive objects that do not shine significantly, and hence not visible over large distances. They are believed to populate both the disk and the halo of the galaxy.

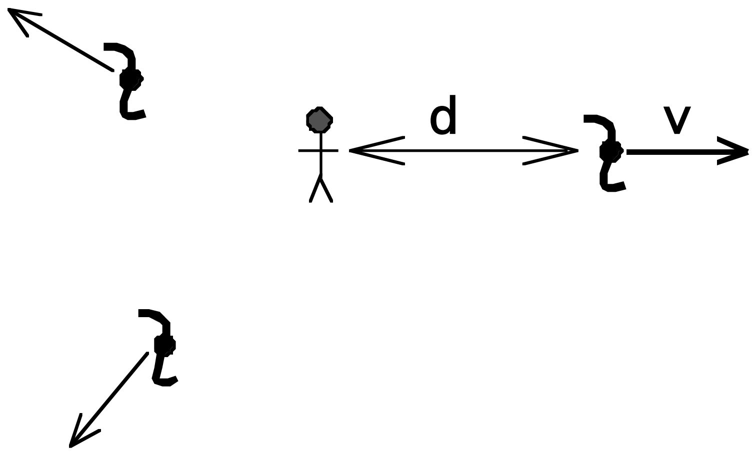
(ii) WIMPS = Weakly Interacting Massive Particles. These are elementary particles that individually have a small mass but are believed to surround galaxies and clusters in vast clouds that extend into the halo. They must interact weakly with matter because if they interacted significantly with matter in the galaxy then they would probably cause some kind of characteristic glow that we could detect.

(iii) Technically, neutrinos are WIMPs because neutrinos are weakly interacting particles and were recently found to have a mass. Moreover, neutrinos are found in vast numbers in the universe. However, neutrinos have an extremely tiny mass (much less than an electron) and are not found in sufficient quantities to account for all the dark matter. WIMP has thus typically come to mean a far more massive type of weakly interacting particle ... that is, one or more species of elementary particle that have masses that are many times that of a proton or neutron.

Recent observations have shown that there are indeed a significant number of MACHOs, but that they can account for *at most* 50% of the total amount of dark matter, and thus the only other viable candidate for the dark matter is WIMPs. Neutrinos cannot account for the balance of 50%, and so astronomy suggests that there are some yet-to-be-discovered weakly interacting massive elementary particles that make up the dark matter. It is hoped that some of these will be created in terrestrial laboratories in the near future.

(iv) **12.** On average, all galaxies are moving away from us (receding) at a speed, **v**, that is directly proportional to their distance from us. This is called the Hubble flow, and it can be expressed:

$$v = H_0 d$$



where $\mathbf{H_o}$ is the Hubble "constant." (Remember that this is only the average motion of galaxies, and individuals may vary.)

Running this backwards in time means that at some time in the distant past, all galaxies in the Universe were located in the same place. If the Hubble constant were 100 times smaller then the recession speeds would be 100 times smaller, which means that the rate of expansion would be 100 times slower. Thus the time at which the galaxies were all at the same place (the Big Bang) would have happened 100 times longer ago. Thus if H_o were 100 times smaller, the Universe would be 100 times older.

Similarly, if the Hubble constant were 100 times larger, the expansion would be happening 100 times as fast, and the Universe would have an age 1/100 th of its current value.

In terms of the mathematics:

$$\text{Age of Universe} = \text{Time since Big Bang} \approx \frac{d}{v} = \frac{1}{H_0}$$

Increasing H_o by a factor of 2 thus halves the age of the universe, and halving H_o would double the age.

13. To measure distances between galaxies one generally uses the relationship between brightness, luminosity and distance:

$$b \propto \frac{L}{d^2}$$

To determine the distance one needs to know the luminosity of an object (and then the distance can be figured out by measuring the brightness as seen from Earth). The issue then devolves around finding *standard candles* - i.e. standard objects that are visible in other galaxies for which we know the luminosity.

Variable Stars: Special classes of variable star (Cepheids and RR Lyrae variables) that can be unambiguously identified, and for which there is a known relation between the period of the variation in luminosity, and its peak or average luminosity. So if we measure the period we know the luminosity.

Type Ia Supernovae: Type Ia Supernova explosions are very recognizable, and are visible out to immense distances (hundreds of megaparsecs, and recently to more than a gigaparsec). We also know how the luminosity of the supernova behaves with time. If we catch one of these events (there is typically one in every galaxy every decade - but they may not always be visible from our perspective) then we can figure out its luminosity.

The Tully-Fisher relation: It turns out that the total luminosity of an entire galaxy is very strongly correlated with the speed of the rotation of the disk. The idea is that the higher the speed, the more massive it must be (to create the gravity to hold it together), and the more massive it is, the more stars, and hence the more luminous. The remarkable thing is how well we can estimate total luminosity from the rotation speed of the disk. Thus we use the Doppler effect to measure the rotation speed of the disk, and then we know the total luminosity of the galaxy.

The Hubble Flow: This is the one method for measuring intergalactic distances that does not rely on the brightness-luminosity relation. Hubble noted that the *average* recession speed of galaxies is proportional to distance, d:

$$v_{\text{average recession}} = H_0 d$$

where H_0 is called the Hubble constant. Galaxies can have their own individual peculiar motions within clusters and superclusters, but on average they recede with a speed proportional to their distance from us. The approximate value of H_0 is 72 km/s/Mpc, and so for galaxies at a distance of 50 Mpc, the average recession speed is 50 x 72 km/s = 3600 km/s, which is significantly larger than

13. continued

the individual peculiar motion a galaxy (which is typically only a few hundred km/s). Thus at distances of more than 50 or 100 Mpc, just about all the motion of a galaxy is the Hubble flow, and so by measuring the recession speed or redshift, one is directly measuring distance. Thus, at this distance, redshift is a measure of distance.

14. The data for this question was given in lectures. It may also be found in Freedman and Kaufmann. Since there is some variation in the numbers for the more distant objects, you only need to have things to 20% accuracy.

a) Diameter of Earth (2 x radius) = 12,756 km

$$\frac{12{,}754 \text{ km}}{9.46 \times 10^{12} \text{ km/ly}} \approx 1.35 \times 10^{-9} \text{ ly}$$

b) Distance from Earth to Sun = 1.496×10^8 km

$$\frac{1.496 \times 10^{8} \text{ km}}{9.46 \times 10^{12} \text{ km/ly}} \approx 1.58 \times 10^{-6} \text{ ly}$$

c) Distance from Sun to Pluto = 5.915×10^9 km

$$\frac{5.915 \times 10^{9} \text{ km}}{9.46 \times 10^{12} \text{ km/ly}} \approx 6.25 \times 10^{-4} \text{ ly}$$

d) Distance from Sun to Proxima Centauri

$$= 1.30 \text{ pc} \approx 1.30 \text{ pc} \times 3.26 \text{ ly/pc} \approx 4.24 \text{ ly}$$

e) Thickness of galactic disk near the Sun

$$\approx 600 \text{ pc} \approx 600 \text{ pc} \times 3.26 \text{ ly/pc} \approx 2{,}000 \text{ ly}$$

f) Distance of the Sun from the center of the Galaxy:

$$\approx 8{,}000 \text{ pc} \approx 8000 \text{ pc} \times 3.26 \text{ ly/pc} \approx 25{,}000 \text{ ly}$$

g) Diameter of the Milky Way: 50 kpc, but there could be a 10-20% error in this figure: It depends on how you define the edge....

$$\approx 50{,}000 \text{ pc} \approx 50{,}000 \text{ pc} \times 3.26 \text{ ly/pc} \approx 160{,}000 \text{ ly}$$

h) Distance to the Andromeda Galaxy: I gave 750 kpc as the distance in class, but there could also be a 10-20% error in this figure. I will stay with 750 kpc

$$\approx 7.50 \times 10^{5} \text{ pc} \approx 7.50 \times 10^{5} \text{ pc} \times 3.26 \text{ ly/pc} \approx 2.4 \times 10^{6} \text{ ly}$$

i) Distance to the Virgo Cluster: I gave a figure of 15 Mpc in class, but again there is some ambiguity in this figure, and values up to 20 Mpc are acceptable.

$$15\text{ Mpc} = 1.5 \times 10^{7}\text{ pc} \approx 1.5 \times 10^{7}\text{ pc} \times 3.26\text{ ly/pc} \approx 4.9 \times 10^{7}\text{ ly}$$

j) Distance to the quasar PC 1247+3406 (see p. 619 of Freedman and Kaufmann)

$$3800\text{ Mpc} = 3.8 \times 10^{9}\text{ pc} \approx 3.8 \times 10^{9}\text{ pc} \times 3.26\text{ ly/pc} \approx 1.2 \times 10^{10}\text{ ly}$$

Tabulating all this

Part	Distance	Distance ratio = Scale Increase
a)	1.35×10^{-9} ly	---
b)	1.58×10^{-5} ly	$\frac{1.58 \times 10^{-5}\text{ ly}}{1.35 \times 10^{-9}\text{ ly}} \approx 11{,}700$
c)	6.25×10^{-4} ly	$\frac{6.25 \times 10^{-4}\text{ ly}}{1.58 \times 10^{-5}\text{ ly}} \approx 39.6$
d)	4.24 ly	$\frac{4.24\text{ ly}}{6.25 \times 10^{-4}\text{ ly}} \approx 6{,}780$
e)	2,000 ly	$\frac{2{,}000}{4.24} \approx 472$
f)	25,000 ly	$\frac{25{,}000}{2{,}000} \approx 12.5$
g)	160,000 ly	$\frac{160{,}000}{25{,}000} \approx 6.4$
h)	2.4×10^{6} ly	$\frac{2.4 \times 10^{6}\text{ ly}}{160{,}000\text{ ly}} \approx 15$
i)	4.9×10^{7} ly	$\frac{4.9 \times 10^{7}\text{ ly}}{2.4 \times 10^{6}\text{ ly}} \approx 20$
j)	1.2×10^{10} ly	$\frac{1.2 \times 10^{10}\text{ ly}}{4.9 \times 10^{7}\text{ ly}} \approx 245$

Overall scale increase (I did not specifically ask for this)

$$\frac{1.2 \times 10^{10}\text{ ly}}{1.35 \times 10^{-9}\text{ ly}} \approx 9 \times 10^{18} \approx 10^{19}$$

15. Limits of various distance measuring methods. I gave figures for these in lectures. Similar data may also be found in figure 26-13 of Freedman and Kaufmann.

Method	Useful Limit
Parallax	Within 100 –200 pc
Spectroscopic Parallax	Within 10,000 pc
Variable stars	Within 20 - 25 Mpc
Tully-Fisher	Within 200 Mpc
Type Ia Supernova explosions	Within about 1000 - 2000 Mpc
Hubble's Law	Useful *beyond* 100 Mpc

- Parallax can thus be used on Proxima Centauri, but not out as far as the thickness of the galactic disk.

- Spectroscopic parallax can thus be used on objects around the galactic core, but not on objects far across the galaxy.

- Variable stars can just be used to the Virgo cluster, but not much further

- Supernovae and the Tully Fisher relation can be used well beyond the Virgo cluster, but not as far as quasars like PC 1247+3406

- Hubble's law cannot be used reliably on the Virgo cluster (it is too close), but starts working well a little beyond that. It works well on distant quasars.

The problem with using the Hubble law for objects that are too close is that the individual motions of galaxies swamp the small average Hubble flow. However, the speed of the Hubble flow increases with distance, and by 100 Mpc the dominant motion of a galaxy is its Hubble flow, and individual motions are relatively small. (This explanation is not required in answering this question, I simply want to remind you of this fact.)