Fall 2004 **PHYSICS 530: General Relativity** **Nick Warner**

Text:	*Spacetime and Geometry* by Sean Carroll
Classes meet:	MW 12:00-1:50; KAP165
Office/phone numbers:	SSC 216D; 740-1146
e-mail:	warner@usc.edu
Office hours:	in SSC216D, MW 2:15–3:30, and by appointment

Course Plan

This is a course on *General* Relativity. My intention is to go from an extremely brief review of special relativity through to a discussion of black holes and cosmology. The first, and slightly larger half of the course will involve the development of the mathematics that we will need to describe gravity: Manifolds, tensor calculus, metrics, covariant derivatives, curvature and geodesics. I will intersperse this with as much physical motivation as possible so that we do not lose sight of our primary goal, which is the description of gravity around planets and stars and the modeling of the universe as a whole. The second part of the course will focus on some of the physical applications: Starting with Einstein's equations, we will discuss black holes, early experimental tests like the precession of perhelion of Mercury, and then move on to cosmology and perhaps some of the bizarre aspects of charged, rotating black holes.

Pre-requisites

This course will be fairly self-contained, and advanced physics is *not* a pre-requisite. I will assume that you are completely comfortable with multi-variable calculus, and a knowledge of the basics of special relativity is highly desirable: I will only spend one to two lectures reminding you of the key ideas of special relativity. It would be useful if (but is not essential that) you have at least seen something the relativistic formulation of Maxwell's E&M.

It is also my intention that this course will be accessible to mathematicians, and so there will not be a lot of dependence on knowledge that is specific to physics.

The Textbook

There are literally hundreds of textbooks on General Relativity, and until recently there has not been a good book that does everything I want to cover. Fortunately, Sean Carroll seems to have produced an excellent text this year. (See page ix of the Preface to *Spacetime and Geometry* to see why I might be making this rather prejudiced remark). However, there are many ways to approach General Relativity, and so there are many other possible sources. I will review some of them below.

I will also hand out *Course Notes*, that is, copies of my detailed notes for each and every lecture. These notes will generally give very complete coverage for each lecture, including all computational details. These will save you from writing frantically while trying to swallow the concepts, and so you will be able to think more during the lecture, and hopefully ask lots of questions.

Grading

I intend to hand out several homework assignments and I will base 85% of your grade upon this homework. I reserve the right to make the last homework assignment cover items from the entire course (*i.e.* I may choose to turn the last homework assignment into a mild “take-home” exam). If I do this, then the last homework/ “take-home” exam will be worth the same as two ordinary homework assignments.

The remaining 15% of your grade will be based upon either a 20-30 minute oral interview, discussion or presentation that will be scheduled at the end of the semester.

Oral Presentations:

There are two possible basic formats for the interview, discussion or presentation: The first is a 20–30 minute interview, discussion that I will schedule with each of you at the end of the semester. In this interview/discussion I may ask you about various parts of the course and might ask you to calculate something that is similar to a homework problem, or I might ask you to read something prior to the meeting, and then give me a brief presentation about what you have learned.

The second, would be to have you select a subject in general relativity, and make a 30 minute blackboard presentation to me, the rest of the class, and whoever else wishes to attend. The presentation can be very focussed (*e.g.* the details of a particular computation), or the presentation can be a survey. The subjects you choose should not be 100% covered in my lectures, but partial overlap is acceptable. You should choose your subject at least 4 weeks prior to the end of the semester, and *clear it with me.* You should also feel free to discuss your presentation with me, your class-mates or anyone else that you think might help. There are some obvious topics: Gravitational waves, the Unruh effect, Hawking radiation, inflation, orbits around rotating black holes, higher-dimensional black holes, the Penrose process, curvature two-forms, spinors, topological invariants, anisotropy in the microwave background .... All of these (and probably many other) topics are natural extensions of the lecture course. Indeed, during the class I will make plenty of comments about broader issues in physics and mathematics, and these might yield additional ideas for topics.

The choice of these two formats depends upon the make-up of the class, and we will discuss this and make the choice by the third week of the semester.

Important Note:

Students who need to request accommodations based on a disability are required to register each semester with the Office of Disability Services and Programs (DSP). In addition a letter of verification to the instructor from DSP is needed for the semester you are enrolled in this course. If you have any questions concerning this procedure, please contact the instructor and DSP at STU 301, 740-0776.

Recommended Texts

- Sean Carroll, *An Introduction to General Relativity: Spacetime and Geometry* (Addison Wesley, 2004). This appears to be a superb text, and contains more than we will get to cover in the course. It does the mathematics rather nicely (though perhaps a little too fast) and then picks some of the most important current topics in General Relativity, and does them in some detail. I will focus on black holes and cosmology, but this book also has chapters on gravitational radiation and Hawking radiation (which, sadly, we will not cover).

- Sean Carroll, *Lecture Notes on General Relativity.* Archive gr-qc/9712019. His pre-book: A completely free set of notes that will cover much of what is in this course, and in a similar style.

- James Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Addison Wesley, 2003). A new book that is intended for undergraduate juniors and seniors. The physics is excellent, but because of its primary target audience it takes a long time to get up to speed.
- B.F. Schutz, *A First Course in General Relativity* (Cambridge, 1985). This is a good introductory text, and probably worth obtaining. It focuses on the details of more elementary things, and so would be a good resource for understanding aspects of the course in more detail. It is one of the more elementary of the recommended texts, and it is also the most accessible. The only problems with this book are that it is a bit wordy and does not get far enough into the more advanced topics.
- R. Wald, *General Relativity* (Chicago, 1984). This is the opposite of Shutz: Far too brief on elementary things, but it has a thorough discussions of a number of advanced topics, including black holes, global structure, spinors. It is an excellent text, but rather advanced. It also has the terrible flaw of having no section on cosmology (at least in my edition).
- S. Weinberg, *Gravitation and Cosmology* (Wiley, 1972). A really good physics text: especially strong on astrophysics, cosmology, and experimental tests. An appalling book when it comes to the underlying mathematical structures. There is also no discussion of black holes.
- C. Misner, K. Thorne and J. Wheeler, *Gravitation* (Freeman, 1973). The "telephone book." A terrible read in large quantities, but useful if you need to look something up. It is filled with horrible "Wheelerisms," and has a chatty style that I dislike intensely. It is, however, a good resource to dip into when you need to find something.
- R. D'Inverno, *Introducing Einstein's Relativity* (Oxford, 1992) I only recently came across this book, and I almost discarded at Chapter 2. It introduces some nonsense called k-calculus in special relativity, and it uses the (very annoying) metric convention in which the spatial metric is negative. That said, the book appears to cover most of the material in this course, and what I read in later chapters seemed pretty good.
- S. Hawking and G. Ellis, *The Large-Scale Structure of Space-Time* (Cambridge, 1973) An advanced book all about global techniques and singularity theorems. It goes over the classic work of Hawking and Penrose, in which they showed that black holes are an essential part of nature. It is also fairly unreadable and one or two of the "proofs" are wrong......
- L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields* (Pergamon, 1975). Solid, no-frills, authoritative and lots of computation. It is vintage Landau and Lifshitz, and it is the opposite of Misner, Thorne and Wheeler.
- B. Schutz, *Geometrical Methods of Mathematical Physics* (Cambridge, 1980). Another good book by Schutz, this one covering some mathematical points that are left out of the GR book (but at a very accessible level). Included are discussions of Lie derivatives, differential forms, and applications to physics other than GR.

Tentative Course Outline

1. Special Relativity: A Review of Key Concepts
   (a) Invariance of the speed of light and the Lorentz tansform
   (b) Proper times and distances.
   (c) Four-vectors
   (d) Scalars and Lorentz tensors
2. Manifolds
   (a) Formal definitions
   (b) Examples
3. Tenors and Differential Forms
   (a) Tangent vectors, differentials
   (b) Tensors and their transformations
   (c) Mapping of tensors: Push-forwards and pull-backs
   (d) Differential forms
   (e) The exterior derivative
4. The Metric
   (a) Metrics and signatures
   (b) Examples
   (c) Raising and lowering of indices
5. The Physical Meaning of the Metric
   (a) Proper time
   (b) The principle of equivalence
   (c) Gravitational time dilation and red-shifts
6. Tensor Densities and Hodge Duality
   (a) Tensors densities
   (b) The ϵ-symbol
   (c) Duality of differential forms
   (d) The δ-operator
7. Covariant Derivatives and Affine Connections

8. Geodesics
    (a) The geodesic equation and its variational principle
    (b) Local inertial frames
    (c) Normal coordinates
    (d) Parallel transport
9. Curvature
    (a) Origins and meaning of curvature
    (b) The Riemann tensor and its properties
    (c) Geodesic deviation and tidal forces
    (d) The Ricci tensor and scalar
10. Symmetries
    (a) Isometries and Killing vectors
    (b) Lie derivatives
    (c) Nöther's Theorem and conserved quantities
    (d) Example: Angular momentum
11. Embeddings, Hypersurfaces and Integration
    (a) Surfaces in manifolds
    (b) How to integrate covariantly on a manifold
    (c) Gauss' Law
12. Energy-Momentum Tensors and Action Principles
13. Einstein's Equations
14. Weak Fields,
    (a) The Newtonian limit
    (b) Gravitational Waves
15. The Schwarzschild Metric and Black Holes
    (a) Spherical symmetry
    (b) Birkhoff's theorem
    (c) Relativistic orbits
    (d) The precession of perihelion
    (e) The deflection of light by the Sun

    (f) Global structure and geodesic completeness
    (g) Event horizons and singularities

16. Rotating/Charged Black Holes *(if time permits)*
    (a) Charged objects: The Reissner-Nordström metric
    (b) Rotating objects: The Kerr-Newman metric

17. Cosmology
    (a) Homogeneity and isotropy
    (b) The Robertson-Walker metric
    (c) The Friedman equations
    (d) Cosmological parameters
    (e) Cosmological red-shift
    (f) The Hubble flow and the age of the universe
    (g) The cosmological constant and dark energy
    (h) The microwave background and decoupling

Disclaimer

The foregoing is a rough plan, and it may be too optimistic. The precise details depend upon who attends the class. The variable balance lies between the physical and mathematical aspects of the subject: I thus reserve the right to re-distribute, re-order, add or discard material as the course progresses and develops.