PHYSICS 153

FINAL EXAM - December 17, 2003

(1)(+25) RELATIVISTIC RELATIONS ARE REQUIRED FOR ALL PARTS OF THIS PROBLEM

An electron and positron are traveling toward each other along the x - axis in an inertial laboratory system.The velocity of the positron is $+0.6c$ relative to the lab system, and the velocity of the electron is $-0.3c$ relative to the lab system.

Calculate the velocity of the positron that would be observed by an observer moving with the electron.

(2) (+25) (Non-relativistic calculations can be used.)

When a clean metal surface is illuminated with light of wavelength 253 nm it is found that the maximum velocity of the emitted electrons is 7.75×10^5 m/s.

a) (+10) What is the metal if it is one of those listed in the table below? Justify your answer with a calculation.

b) (+10) What would the wavelength of the incident light have to be in order to double the maximum velocity of the emitted electrons? Justify your answer with a calculation.

Metal	*K*	*Na*	*Ca*	*Mn*	*Cu*	*Ag*	*Au*	*Ni*
Work Function (eV)	2.1	2.3	3.2	3.8	4.1	4.7	4.8	5.2

(3) (+25)(Homework Problem)

Use the uncertainty principle to estimate the minimum kinetic energy, in eV, of an electron confined to a (one-dimensional) region of size $L = 0.1$ nm.

(5)(+25) Spherical polar coordinates r, θ, φ are used in obtaining solutions to the Schrödinger equation for the hydrogen atom.

a)(+3) Draw a set of orthogonal Cartesian (x, y, z) axes and show (r, θ, φ) on your figure. What are the ranges of r, θ, φ?

b)(+7) One of the 3d wavefunctions for an electron in the hydrogen atom is given by

$$\psi(r,\theta,\varphi) = (const.)\; r^2 e^{-r/3a_0}(\sin\theta\cos\theta\, e^{-i\varphi}) \qquad (\,a_0 = \text{ Bohr radius})$$

In this case what are the values of i) the quantum numbers, ii) the z-component of the orbital angular momentum, iii) the square of the orbital angular momentum, and iv) the energy (in eV)?

c)(+3) In addition to orbital angular momentum, the electron also has an intrinsic angular momentum or "spin" (not included in the above wavefunction). What are the possible values of the z-component, and of the square of the intrinsic angular momentum?

d)(+12) Write an expression for the radial function $R(r)$, and use it to calculate the probability of finding the electron at a distance from the nucleus greater than a_0. You may leave your answer in the form of a definite integral with specified integrand and specified limits.

Useful integral:

$$\int_0^\infty r^n\, e^{-\alpha r}\, dr = \frac{n!}{\alpha^{n+1}} \quad (\alpha > 0,\; n \geqq 0)$$

(4)(+30) A particle of mass m is confined to a one-dimensional box with walls at $x = a$ and $x = b$ where $b > a$. The potential inside the box ($a < x < b$) is $U(x) = 0$, and outside the box $U(x) = \infty$ (i.e. the walls are perfectly rigid). Recall that the time-independent Schrödinger equation is given by

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi = E\psi$$

a)(+5) In this case the stationary state solutions have the form

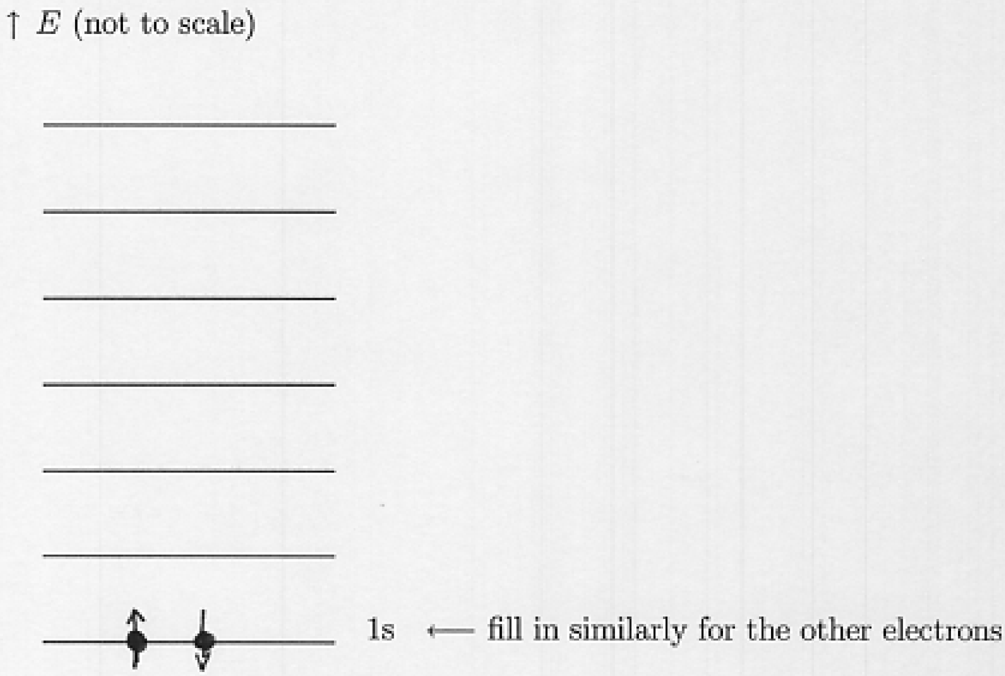
$$\psi(x) = A\sin\{k(x-a)\} \qquad a \le x \le b \qquad (k > 0)$$

Substitute the above expression for $\psi(x)$ into the time-independent Schrödinger equation and derive an expression connecting E and k.
b)(+5) Write the boundary conditions which must be satisfied by $\psi(x)$; use these to find the allowed values of k and the allowed values of E.
c)(+5) The constant A is found to be $A = \sqrt{\frac{2}{b-a}}$. Without doing the actual calculation, explain how this result is obtained. Be as explicit as possible.
d)(+5) For a particle in the first excited state sketch the wavefunction and corresponding probability density, and find, by inspection, the probability that the particle will be found in the middle half of the box [i.e. between $x = a + \frac{1}{4}(b-a)$ and $x = a + \frac{3}{4}(b-a)$]. Your answer should be a dimensionless number. Briefly state your reasoning in arriving at the answer.
e)(+5) For a particle in the first excited state, find the (approximate) probability that the particle will be found in a very small region of size Δx at the center of the box, [i.e. between $x = \frac{1}{2}(b+a) - \frac{1}{2}\Delta x$ and $x = \frac{1}{2}(b+a) + \frac{1}{2}\Delta x$]. Your answer should be a dimensionless number. Briefly state your reasoning in arriving at the answer.
f)(+5) Same as part e) if Δx is not small. You may leave leave your answer in terms of an integral with specific limits and an integrand which is completely specified. (Do not attempt to evaluate the integral.)

Useful relations:

$$\frac{d}{dx}\{\sin(\alpha x + \beta)\} = \alpha\cos(\alpha x + \beta)$$
$$\frac{d}{dx}\{\cos(\alpha x + \beta)\} = -\alpha\sin(\alpha x + \beta)$$

(6)(+25)a)(+10) Write down the electron configuration for the ground state of phosphorus $_{15}$P and complete the diagram already started below, including the letter designations.



b)(+5) When one of the two K-shell electrons in an atom is knocked out leaving a "hole" in the K-shell, an electron in a higher shell can jump into the K-shell hole, leaving a hole in the higher shell, and then an electron in a still higher shell can drop into this hole, etc. For the case of $_{15}$P, show, with downward arrows, on a separate energy level diagram, all the possible characteristic X-ray lines which could result from the possible successive jumps into the holes which are created when an electron jumps into the K-shell hole. (Use the selection rules.). Do not calculate the wavelengths of the X-ray lines.

c)(+10) Using the concept of effective charge, estimate the energy, in eV, and the wavelength in nm, the K_α X-ray for $_{15}$P.

(7)(+25) A sample contains 3 milligrams of pure $^{13}_{7}$N which decays via positron emission into $^{13}_{6}$C with a half-life of 10 minutes.

a)(+13) Calculate the initial activity of the sample in units of Ci.

b)(+6) How long does it take for the activity to drop to one decay per second?

c) At the time determined in part b), how many $^{13}_{7}$N atoms remain?

(8)(+25)a)(+12) Calculate the binding energy per nucleon for uranium $^{234}_{92}$U in units of MeV.

b)(+13) Is it possible for $^{234}_{92}$U to undergo spontaneous decay by the emission of an alpha particle? Is it possible for $^{234}_{92}$U to undergo spontaneous decay by the emission of a proton? Justify your answers with a calculation.

Masses: neutron: $m_n = 1.0086654\,\mathrm{u}$. proton: $m_p = 1.0072766\,\mathrm{u}$.

ATOMIC MASSES

Z	element	symbol	A	Atomic Mass,u.
1	Hydrogen	H	1	1.007825
2	Helium	He	2	4.002603
90	Thorium	Th	228	228.028750
		Th	230	230.033131
		Th	232	232.038054
		Th	233	233.041580
91	Proactinium	Pa	233	233.040244
92	Uranium	U	232	232.037168
		U	233	233.039629
		U	234	234.040947
		U	235	235.043925
		U	238	238.050786
93	Neptunium	Np	237	237.048169
			239	239.052932
94	Plutonium	Pu	240	240.053809

CONSTANTS & USEFUL FORMULAS

Avogadro's number	N_A	6.023×10^{23} particles/mole
Bohr radius	a_o	5.292×10^{-11} m
Compton wavelength of electron	$\lambda_C = h/m_e c$	2.426×10^{-12} m
Coulomb constant	$k = 1/4\pi\epsilon_o$	8.988×10^9 N-m^2/C^2
Electron charge	e	1.602×10^{-19} C
Electronvolt	eV	1.602×10^{-19} J
Joule	J	6.242×10^{18} eV
Mass unit	u	1.6604×10^{-27} kg
		$= 931.5$ MeV/c^2
Mass of electron	m_e	9.109×10^{-31} kg
		$= 511$ keV/c^2 $= 0.0005486u$.
Mass of proton	m_p	1.673×10^{-27} kg
		$= 938.3$ MeV/c^2 $= 1.0072766u$.
Mass of neutron	m_n	1.675×10^{-27} kg
		$= 939.6$ MeV/c^2 $= 1.0086654u$.
Planck's constant	h	6.626×10^{-34} J-s
		$= 4.136 \times 10^{-15}$ eV-s
	$\hbar$	1.055×10^{-34} J-s
		$= 6.582 \times 10^{-16}$ eV-s
	hc	1240 eV-nm
Rydberg constant	R	1.097×10^7 m^{-1}
Speed of light	c	2.998×10^8 m/s
		$\cong 3 \times 10^8$ m/s

$E = mc^2 = m_o c^2/\sqrt{1-(\mathrm{v}/c)^2}$

$K = mc^2 - m_o c^2$

$= m_o c^2\left(\frac{1}{\sqrt{1-(\mathrm{v}/c)^2}} - 1\right)$

$$m = \frac{m_o}{\sqrt{1-(\mathrm{v}^2/c^2)}}$$

$p = m\mathrm{v} = m_o \mathrm{v}/\sqrt{1-(\mathrm{v}/c)^2}$

$E^2 = p^2c^2 + m_o^2 c^4$

$$\frac{u}{c} = \frac{pc}{E}$$

$$V_{12} = \frac{V_1 \pm V_2}{1 \pm (V_1 V_2/c^2)}$$

$\Delta\lambda = (h/m_o c)\,(1-\cos\theta)$

$N(t) = N_0\, e^{-\lambda t}$

$R(t) = -\frac{dN(t)}{dt} = R_0 e^{-\lambda t}$

$$\lambda = \frac{\ell n(2)}{t_{1/2}}$$

$1\,\mathrm{Ci} = 3.70 \times 10^{10}\mathrm{s}^{-1}$

$$E_n = \frac{n^2h^2}{8mL^2}$$

$R_0 = 1.5 \times 10^{-15}$m

$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$$E_n = -\frac{1}{2}\,\frac{mk^2z^2e^4}{\hbar^2 n^2}$$