$$\frac{\partial g_{\mu\nu}}{\partial \lambda} = -2R_{\mu\nu}$$

II. Ricci flow and physics

A. RF = one-loop RG flow of target space metric for 2d σ-model with $B_{\mu\nu}=0$, Φ = constant

$$\lambda = -\tfrac{1}{2}\alpha' \ln \Lambda$$

(N.B.: Unlike RF, RG flow cannot stop at singularities.)

B. Gradient flow on space of metrics

$$\frac{dg^A}{d\lambda} = -G^{AB}\frac{\partial S}{\partial g^B} \qquad g^A \equiv (g_{\mu\nu}(x))$$

metric on space of metrics

of Einstein-Hilbert action

$$S(g) = -\frac{1}{16\pi G_N}\int d^Dx\,\sqrt{g}\,R$$

gives Ricci flow, if we take G_{AB} as

$$G_{AB}\,dg^A dg^B = \frac{1}{32\pi G_N}\int d^Dx\sqrt{g}\left[dg_{\mu\nu}\,dg^{\mu\nu} - \tfrac{1}{2}(dg_\mu{}^\mu)^2\right]$$

reverses flow in Weyl directions — makes them run uphill