Surveying Flux Vacua

J.K.
James Wells

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There is good evidence suggesting that there are many string vacua with no moduli (BP, GKP, KKLT, DD, BDP)

“the landscape”

What is the hope?

- input for model-builders
- motivation for formal theorists

There are two questions we can try to study using the landscape

- “WHAT” question → which string vacuum?
- “WHY” question → why this vacuum and no other

We focus on the “what” question

WHAT question → provides input for theory (model-building)

basic idea ⟹ we are trying to find a needle in a haystack, so it helps to know the distribution of the haystack

Vacua counting ideas can help!

→ Help us locate interesting models and phenomenology within string theory, non-generic selection criteria, etc.

Background (Ashok, Denef, Douglas, GKP, KKLT, ...)

General idea ⟶ look at Type IIB compactified on CY 3-fold

perform orientifold → projects out by worldsheet parity

→this theory preserves 4 supersymmetries (N=1) this theory is also equivalent to a limit of F-theory compactified on a CY 4-fold (Sen)

$$M_4 = \frac{(X_3 \times T^2)}{Z_2}$$

$\rightarrow$we will use both pictures interchangeably

moduli $\rightarrow$ axio-dilaton, complex structure, Kähler

now, add FLUXES to generate $W = \int G \wedge \Omega(z)$

$\rightarrow$ fixes the complex structure moduli and axio-dilaton

$n = \#$ of unprojected complex structure moduli

$4n + 4 = \#$ of total fluxes we can turn on

the only moduli left will be the Kähler moduli

if dilaton weak and size of CY is larger than l_s, then integrating out is consistent

$$V = e^K(|DW|^2 - 3|W|^2) + D^2$$

$$\text{SUSY condition} \Rightarrow D_a W = \partial_a W - W \partial_a K$$

parameterize fluxes with integral basis $\{\Sigma_\alpha\}$ for $H^4(M_4, Z)$

- $N^\alpha \rightarrow$ int. coeff. of G^4 over $\{\Sigma_\alpha\}$
- $\Pi_\alpha(z) = \int \Sigma_\alpha \wedge \Omega_4(z) \rightarrow$ periods

can find a new basis for fluxes using GVW superpotential

$$\downarrow$$

$$G_4 = \bar{W}\Omega_4 - \overline{D^A W} D_A \Omega_4 + \overline{D^0 D^I W} D_0 D_I \Omega_4 + c.c.$$

So, what's the plan of action?

constraints $\rightarrow$ SUSY, RR tadpoles (Gauss' Law for space-filling charge)

$\Downarrow$

We can find a simple parameterization for the fluxes as they appear in W, DW and the tadpole condition

$\Downarrow$

Now we count SUSY flux vacua...

$\downarrow$

- make a choice of $4n + 4$ fluxes N^α which satisfy $Flux - charge = L = \frac{1}{2} N \eta N \leq L_* = O - charge$
- then solve $DW = 0 \rightarrow$ to impose SUSY and fix complex structure moduli and axio-dilaton

so, off we go!

$\Downarrow$

Integrate over fluxes and moduli, imposing $DW = 0$ by a δ function and tadpole cancellation by a step-function

$$
\begin{aligned}
N_{susy}(L \leq L_*) &= \sum_{vac.} \theta(L_* - L) \\
&= \frac{1}{2\pi\imath} \int \frac{d\alpha}{\alpha} e^{\alpha L_*} \mathcal{N}(\alpha)
\end{aligned}
$$

where $\mathcal{N}(\alpha) = \sum_{vac} e^{-\frac{\alpha}{2} N\eta N}$

and the contour is parallel to the imaginary axis, passing 0 to the right

$$
\begin{aligned}
\mathcal{N} &= \sum_N \int_{\mathcal{M}} d^{2n+2}z \, \delta^{2n+2}(DW) |\det D^2 W| e^{-\alpha L} \\
&\sim \int_{\mathcal{M}} d^{2n+2}z \int d^{4n+4}N \, e^{-\frac{\alpha}{2} N\eta N} \delta^{2n+2}(DW) |\det D^2 W|
\end{aligned}
$$

$|\det D^2 W| \rightarrow$ Jacobian factor from changing $\delta_z(DW)$ to $\delta_{DW}(DW)$

$\downarrow$

now δ fixes fluxes instead of moduli

$$\text{rescale } N \to \frac{N}{\sqrt{\alpha}}$$

$$\Downarrow$$

$$\mathcal{N}(\alpha) \text{ scales as } \alpha^{-2(n+1)}$$

$$\Downarrow$$

$$N_{susy}(L \leq L_*) = \theta(L_*) \tfrac{L_*^{2n+2}}{(2n+2)!} \mathcal{N}(\alpha = 1)$$

Note scaling of $N(L \leq L_*)$ *with* L_*

transform integral from N^α to $X = W$, $Y = DW$ and $Z = D^2W \to$ throws in another Jacobian

$$\Downarrow$$

$$N(L \leq L_*) = \frac{(2\pi L_*)^{2n+2}}{(2n+2)!} |\det \eta|^{-\frac{1}{2}} \int_{\mathcal{M}} d^{2n+2}z \; \det g \, \rho(z)$$

where

$$\rho(z) = \pi^{-2n-2} \int d^2X d^2Z e^{-|X|^2 - |Z|^2} |X|^2 \det M(X, Z, \mathcal{F})$$

- $\mathcal{F}_{IJK}(z) \to$ geom. data of CY $\to$ deriv. of periods $\Pi(z)$
- L_*, n, $\det \eta \to$ constants set only by CY and orientifold
- $g \to$ metric on comp. struc. mod. space $\mathcal{M}$

Now that we can count the vacua, what do we do with it?

We want to study the distribution of gauge group rank among vacua

$\Downarrow$

specifically, net D3-brane charge

from tadpole condition $\rightarrow N_{D3} = L_* - L$

$$N(L \leq L_*) = C_{cy} L_*^{2n+2}$$

$$\downarrow$$

$$N(L \leq L_{**}) = C_{cy} L_{**}^{2n+2}$$

dependence on L_{**} comes only from $\theta(L - L_{**}) \rightarrow$ goes only into rescaling of α

Density $\rightarrow \rho = \frac{\partial N}{\partial L} = (2n+2) C_{cy} L_{**}^{2n+1}$

Each flux vacuum weighted equally

Average Charge

$$\Downarrow$$

$$\langle N_{D3}\rangle = L_* - \frac{1}{C_{cy}L_*^{2n+2}}\int_0^{L_*} dL\, L\frac{\partial N}{\partial L}$$

$$\Downarrow$$

$$\langle N_{D3}\rangle = \frac{L_*}{2n+3} = \frac{\chi(M_4)}{24(2h_-^{2,1}+3)}$$

Note $\rightarrow$ independent of details of CY structure, singularities, etc.

$$\downarrow$$

depends only on $h_-^{2,1}(X_3)$ and $\chi(M_4)$ (background charge)

Let's look at the fraction of vacua with $N_{D3} \geq R$, i.e. $L \leq L_* - R$

$$\downarrow$$

$$\eta = \frac{(L_*-R)^{2n+2}}{L_*^{2n+2}} = \left(1 - \frac{R}{\langle N_{D3}\rangle(2n+3)}\right)^{2n+2} \sim e^{-\frac{R}{\langle N_{D3}\rangle}}$$

$$\text{Fractional density} = -\frac{\partial \eta}{\partial R}$$

$$\Downarrow$$

$$\sigma = \frac{1}{\langle N_{D3} \rangle} e^{-\frac{R}{\langle N_{D3} \rangle}}$$

depends only on $\langle N_{D3} \rangle$, not even on $\chi(M_4)$

CAVEATS

- we weight each "vacuum" equally
- c^N degeneracy due to open string dynamics (Douglas)
- brane/anti-brane pairs $\rightarrow$ change rank, not charge
- continuous flux approximation? (DD,DGKT)
- effective field theory limit? (Banks)

Can we see correlations? $\rightarrow$ gauge group rank in small (tree-level) cosmological constant limit

in SUSY vacua, $V = -3|W|^2$

Count vacua with $L \leq L_*$, $|V| \leq |V_*| = 3\lambda_*$

$$\downarrow$$

$$N_{susy}(L \leq L_*, |W|^2 \leq \lambda_*) = \int_{\mathcal{M}} d^{2n+2}z \int_0^{\lambda_*} d\lambda \frac{1}{2\pi\imath} \int \frac{d\alpha}{\alpha} e^{\alpha L_*} \nu(z, \alpha, \lambda)$$

where

$$\nu(z, \alpha, \lambda) = \int d^{4n+4}N \, e^{-\alpha L} \delta(|W|^2 - \lambda) \, \delta^{2n+2}(DW) \, |\det D^2 W|$$

rescaling $N \rightarrow \frac{N}{\sqrt{\alpha}}$ and taking limit $\lambda_* \ll L_*$ ($\lambda_* \rightarrow 0$) gives

$$\Downarrow$$

$$\begin{aligned} N_{susy} &= \frac{2^{2n+3}\pi(2n+2)}{(2n+2)!} L_*^{2n+1} \lambda_* |\det \eta|^{-\frac{1}{2}} \int_{\mathcal{M}} d^{2m}z \det g I(\mathcal{F}) \\ I(\mathcal{F}) &= \int d^{2n}Z e^{-|Z|^2} |\det M'(\mathcal{F}, Z)|^2 \end{aligned}$$

$$N_{susy}(L \leq L_*, |W|^2 \leq \lambda_*) = B_{cy} L_*^{2n+1} \lambda_*$$

$$\Downarrow$$

$$N_{susy}(L \leq L_{**} \leq L_*, |W|^2 \leq \lambda_* \ll L_*) = B_{cy} L_{**}^{2n+1} \lambda_*$$

$$\rho = \frac{\partial N}{\partial L} = (2n+1) B_{cy} L^{2n} \lambda_*$$

$$\langle N_{D3} \rangle = L_* - \frac{1}{B_{cy} L_*^{2n+1} \lambda_*} \int_0^{L_*} dL\, \rho(L) \cdot L$$

$$\Downarrow$$

$\langle N_{D3} \rangle = \frac{L_*}{2n+2}$ **in small c.c. limit**

$$\sigma(R) \sim \frac{1}{\langle N_{D3} \rangle} e^{-\frac{R}{\langle N_{D3} \rangle}}$$

Note $\rightarrow$ $\langle N_{D3} \rangle_{small c.c.}$ is almost the same as $\langle N_{D3} \rangle$, but not quite

$$\frac{\langle N_{D3} \rangle_{small c.c.}}{\langle N_{D3} \rangle} = \frac{2n+3}{2n+2} > 1$$

$\rightarrow$ *slight correlation*

Next step: consider how to count vacua with standard model embeddings

Why? $\rightarrow$ look for what features of the string compactification are important

we may not get the real world out of this

But, we will hopefully get insight into where to look to get realistic models (ie., what types of CY's, associated hidden sectors, exotics, etc.)

We will consider a "technology demonstrator" - Type IIB string theory on a $T^6/Z_2 \times Z_2$ orientifold

$\downarrow$

We will characterize flux vacua issues

$$T^6/Z_2 \times Z_2(\times \Omega R)$$

$$\downarrow$$

$$\alpha : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3)$$

$$\beta : (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3)$$

orientifold involution

$$R : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, -z_3)$$

generates O3-,O7-planes (orientifold fixed planes)

51 complex structure moduli, 3 Kähler moduli

want to wrap D-branes on this manifold

$\rightarrow$ we will look for a Pati-Salam left/right model arising from the gauge theory on D-branes

$$\text{Gauge group} = U(4) \times SU(2)_L \times SU(2)_R$$

$\rightarrow$ embedding of branes determines open string gauge grp., chiral matter

wrapping numbers on $T^6 \rightarrow (n_1, m_1)(n_2, m_2)(n_3, m_3)$

$$\downarrow$$

m =wrapping number on T^2

(IIA dual $\rightarrow$ wrapping no. on a-cycle)

n =magnetic flux on T^2 ($\frac{m}{2\pi}\int F = n$)

(IIA dual $\rightarrow$ wrapping no. on b-cycle)

will put SM branes at orbifold or orientifold fixed points (odd generations)

- N branes at orbifold fixed point $\rightarrow U(\frac{N}{2})$
- N branes at orientifold fixed point $\rightarrow Sp(N)$

chiral matter in bifundamentals $\rightarrow$ string stretching between two branes (also sym. and anti-sym)

$\rightarrow I_{ab} = \#$ of chiral mults. in bifund.

What constrains the branes we can put in?

- **RR tadpoles** (Gauss' Law on space-filling charges)

$$\begin{aligned} \sum N_a n_1 n_2 n_3 &= 16 - 32 N_{flux} \\ -\sum N_a n_1 m_2 m_3 &= 16 \\ -\sum N_a m_1 n_2 m_3 &= 16 \\ -\sum N_a m_1 m_2 n_3 &= 16 \end{aligned}$$

- **NSNS tadpoles** (preserve supersymmetry)(BDL)

$$\sum \tan_2^{-1}(m_i A_i, n_i) = 0 \bmod 2\pi$$

for each brane

- **K-theory** no global $SU(2)$ anomaly (W,CU)

$$\begin{aligned} \sum N_a m_1 m_2 m_3 &= 0 \bmod 4 \\ \sum N_a n_1 n_2 m_3 &= 0 \bmod 4 \\ \sum N_a n_1 m_2 n_3 &= 0 \bmod 4 \\ \sum N_a m_1 n_2 n_3 &= 0 \bmod 4 \end{aligned}$$

$\longrightarrow$ need to satisfy RR and K-theory, but NSNS only for SUSY no discrete B-field turned on

visible sector:

$$
\begin{aligned}
N_{U(4)} &= 8 \quad (1,0)(3,1)(3,-1) \\
N_{Sp(2)_L} &= 2 \quad (0,1)(1,0)(0,-1) \\
N_{Sp(2)_R} &= 2 \quad (0,1)(0,-1)(1,0)
\end{aligned}
$$

why $Sp(2) = SU(2)$?

- no $U(1) \subset U(2)$ anomaly (would require anti-doublets or GS mechanism)
- SUSY everywhere in mod. space $\rightarrow$ fewer constraints

visible sector doesn't satisfy RR tadpoles $\rightarrow$ need hidden sector

- D3 charge of visible sector overshoots $\rightarrow$ need negative contribution from hidden sector
- also need SUSY (will require that only one charge can be negative per brane, with three positive)

to get a SUSY brane with negative charge, need a magnetized D9-brane (all m's and n's nonzero)

Fixing Kähler Moduli

→ there are 3 Kähler moduli

SUSY constraints arise from NSNS tadpoles and from non-perturbative corrections to W (superpotential)

- if we overconstrain → generically, no SUSY sol'n
- if we underconstrain → unfixed moduli

need interplay between constraints

→ we won't deal with non-pert. corrections

visible sector → $\mathcal{A}_2 = \mathcal{A}_3$ (2 moduli unfixed)

brane constraints → 1 or 2 generically

- 1 NSNS tadpole → need 1 constraint from W
- 2 NSNS tadpole → want no constraints from W

One interesting thing to find → model with the most flux

# of flux vacua scale as N_{flux}^{2n+2}

brane construction which allows for the largest number of flux will have the most flux vacua

with many flux vacua, more likely to have "accidental" cancellation of cosmological constant, unification, etc.

↓

not a "statistical prediction," but a good way to find models which are interesting for phenomenology!

Will run a computer search for hidden sectors (1D lines)

- pure D3-,D7-branes $\rightarrow$ SUSY on all Kähler mod. space
- magnetized D9-brane $\rightarrow$ contributes negative D3-charge, positive D7 charges

$\downarrow$

each imposes one NSNS tadpole constraint

$\rightarrow$ we will look for 1 magnetized D9-brane which (along with pure D3-,D7-branes) can satisfy all tadpoles

note that since $\mathcal{A}_2 = \mathcal{A}_3$, we can flip wrapping #'s by $(n_2, m_2) \leftrightarrow (n_3, m_3)$ and still satisfy same NSNS constriant

$\downarrow$

we will include these in search

what we **don't** look for

- points (2 distinct magnetized D9-brane stacks)

Results:

ignoring K-theory constraints: 109 1D surfaces in Kähler moduli space with consistent hidden sectors

$\rightarrow$ all have no $SU(3)_{qcd}$ chiral exotics (consequence of RR tadpole cancellation/cubic anomaly cancellation)

But when we impose K-theory constraint: only 5 1D surfaces with hidden sectors

	magnetized $D9$ brane	N_a	$N_{\tilde{a}}$	$N_{D3,D7_i}$	$n_{\text{flux}}^{\text{max}}$
1	$(-2,1)(-3,1)(-4,1)$	2	2	(40,0,0,0)	1
2	$(-2,1)(-3,1)(-3,1)$	4	-	(16,0,2,2)	0
3	$(-2,1)(-2,1)(-7,2)$	2	0	(0,0,0,6)	0
4	$(-2,1)(-2,1)(-7,2)$	0	2	(0,0,6,0)	0
5	$(-2,1)(-2,1)(-5,1)$	2	2	(24,0,0,0)	0
6	$(-2,1)(-2,1)(-4,1)$	2	2	(8,0,2,2)	0

4 of these have hidden sectors where only $N_{flux} = 0$ is allowed $\rightarrow$ doesn't fix complex structure moduli (the one with flux is the MS model)

we see that the K-theory anomaly constraints are VERY IMPORTANT one more trick up our sleeves: turning on discrete B-field $\rightarrow$ T-dual to having a tilted T^2

$\downarrow$

RR tadpole constraint now IMPLIES K-theory contraint!

if we want a tilted torus $\rightarrow$ invariance under orientifold action implies only one non-trivial choice

$$(n, m)_i \rightarrow (n, m + \tfrac{n}{2})_i$$

tilting even one torus lets an m be half-integer and changes the wrapping numbers of orientifold fixed planes $\rightarrow$ shifts K-theory condition

new K-theory condition is equivalent to RR-tadpole condition

to get visible sector right, can only shift at most two tori

But RR tadpole conditions also shift, because # of orientifold planes decreases for tilted torus case

We are searching for these models now ....

CHIRAL EXOTICS

for no tilted tori or two tilted tori $\rightarrow$ no $SU(4)$ chiral exotics

implied by anomaly cancellation

$\rightarrow$ both tori have the same tilt $\rightarrow$ $I_{a,O} = 0$ $\rightarrow$ no sym. or anti-sym. reps.

$\downarrow$

anomaly cancellation will require # fund. = # anti-fund. $\rightarrow$ imposed by RR tadpole constr.

for one torus tilted $\rightarrow$ $I_{a,O} \neq 0$

$\downarrow$

sym. and anti-sym. reps contribute to anomaly $\rightarrow$ so must fund. + anti-fund.

fund. $-$# anti-fund. $= -24$ $\rightarrow$ regardless of hidden sector!

$\rightarrow$ for one discrete B-field, chiral exotics can be a problem!

Conclusions:

We can learn a LOT about how to build models by studying the "landscape" of vacua

Some lessons:

- K-theory constraints significant
- discrete B-field can solve K-theory constraint, but restrict hidden sector charges
- helpful to have orientifolds where the O3-planes give large negative D3-charge (to compensate for visible sector)
- Need to understand Kähler moduli stabilization

the type of constraints we get from non-pert. corrections to W will tell us what types of SM constructions can be generically realized with no moduli

Future directions

- find "point" solutions (2 magnetized branes) $\rightarrow$ hopefully more constructions
- non-$Sp(2)$ constructions, non-Pati-Salam, non-left/right
- look at more general constructions on more general orientifolds (discrete torsion, Chan-Paton action, etc.)