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Variation of the oscillator strengths for the α emission lines of the one- and two-electrons ions in dense plasma

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We present the results of a detailed theoretical study on the variation of the oscillator strengths for the $1s \rightarrow 2p$ transition of the H-like ions and the $1s^2\,{}^1S \rightarrow 1s2p\,{}^1P$ transition of the He-like ions (or, inversely, the Lyman-α and He_α emission lines, respectively) subject to external plasma which meet the spatial and temporal criteria of the Debye-Hückel (DH) approximation. Our study shows that the resulting oscillator strength decreases for the He_α line for He-like ions, similar to the Lyman-α emission lines for all H-like ions, as the effect of the external plasma increases with the decreasing Debye length D in terms of a reduced Debye length $\lambda_D = Z_{eff} D$. A nearly universal feature is demonstrated for a scaled oscillator strength as a function of the reduced Debye length $\lambda_D = (Z-1)D$ for different He-like ions that meet the same criteria for the DH model. The percentage changes of the oscillator strengths from their plasma-free values are substantially greater than those for the corresponding change for the redshifts of the Lyman-α and He_α emission lines subject to outside dense plasma. Should these general features be demonstrated experimentally, the theoretical procedure presented in this study could easily be applied to extrapolate from a single calculation for one He-like ion to other He-like ions, which could offer an alternative to complement other diagnostic efforts of the dense plasma. *Published by AIP Publishing.* https://doi.org/10.1063/1.5057380

I. INTRODUCTION

The Debye-Hückel (DH) model, based on the classical Maxwell-Boltzmann statistics for an electron-ion collisionless plasma under thermodynamic equilibrium, works in principle mostly for the gas discharged plasmas at relatively low density.[1] It is known that the DH model breaks down for the atomic processes involving electrons from states close to the ionization threshold which are intimately affected by the outside plasma, such as the ones shown by Nantel *et al.*[2] near the series limit. For the dense plasma (e.g., the ones in the sun and sun-like stars), the Debye length is of the order of 10^{-11} m, and consequently, the Debye screening could impact the atomic processes for the astrophysically relevant elements. As a result, it is interesting to note that both the commonly accepted equation-of-state models adopted in the standard solar models, i.e., MHD (Mihalas-Hummer-Dappen) and OPAL (Optical Project at Livermore), include a Debye-Hückel term as the leading Coulomb correction to the free energy.[3,4] Following a detailed assessment of the spatial and temporal criteria of the DH approximation,[5,6] we have shown recently that the DH approximation could be applied to the atomic processes subject to outside dense plasma, involving transitions limited to those between low-lying atomic states, which are dictated by the short-ranged interaction between the innermost atomic orbitals.[5–7] With a much improved experimental energy resolution $\frac{E}{\Delta E}$ (see, e.g., Refs. 8 and 9), together with the agreement we reported recently between the simulated redshift based on the DH model[6,7] and the earlier observed value for the H-like Al^{12+} ion,[8] more extensive measurements for the redshifts subject to dense plasma with a density of $10^{22}\,\mathrm{cm}^{-3}$ or higher could offer an alternative approach to the typical plasma diagnostic efforts based on the change of the line profile due to complicated collisional processes.

In particular, we presented theoretically in Ref. 7 two general features of the redshifts of the spectroscopically isolated $1s2p\,{}^1P \rightarrow 1s^2\,{}^1S$ He_α line of the He-like ions and the $2p \rightarrow 1s$ Lyman-α line of the H-like ions embedded in external dense plasma. The first feature concerns the ratio $R = \Delta\omega_\alpha/\omega_o$ between the redshift $\Delta\omega_\alpha$ due to the external dense plasma and the energy ω_o of the α lines in the absence of the plasma. This ratio R, which will be referred to as the percentage change in our subsequent discussion, turns out to vary as a nearly universal function of a reduced Debye length $\lambda_D = Z_{eff} D$, where D is the Debye length, related to the electron density N_e and temperature T of the outside plasma, or, more conveniently in terms of the Bohr radius a_o by [see, Eqs. (1)–(17) of Ref. 10]

$$D = 1.4048\,(kT/N_e)^{1/2}\,a_o, \tag{1}$$

where the electron energy kT is given in the units of eV and N_e in the units of $1 \times 10^{22}\,\mathrm{cm}^{-3}$. The second general feature is an approximately constant redshift $\Delta\omega_\alpha$ at a given D for He-like ions and H-like ions with Z_{eff} approximately between 5 and 18 based on our critical assessment of the applicability of the DH approximation to atomic processes. These two general features could easily be applied to extrapolate from a

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given single calculated dataset for a H-like or He-like ion to the corresponding data for other ions.

The redshifts for the emission lines involved the inner shell electrons (e.g., the α lines) could be attributed qualitatively to the upward shifts of their energy levels due to an effective screened potential at a distance r from the nuclear charge Z, subject to a charge-neutral outside electron-ion plasma, given by[6,11,12]

$$V_d(r;D) = \begin{cases} V_i(r) = -Z\left(\dfrac{1}{r} - \dfrac{1}{D+A}\right), & r \le A \\ V_o(r) = -Z\left(\dfrac{De^{A/D}}{D+A}\right)\dfrac{e^{-r/D}}{r}, & r \ge A \end{cases} \tag{2}$$

where A is the radius of the Debye sphere. When A approaches zero, the effective screened potential $V_d(r;D)$ takes the form of the usual screened Coulomb potential $V_s = -\frac{Z}{r}e^{-\frac{r}{D}}$. In general, the Debye length D should be greater than the radius A of the Debye sphere, if not much greater, to meet the requirement of the DH approximation.[5] The radius A should also be chosen to ensure that the average radii of the atomic orbitals for the atomic processes of our interest are kept inside the Debye sphere to retain their atomic characteristic instead of overlapping strongly with the outside plasma electrons. Our recent works have shown that the estimates of the atomic data based on the DH approximation depend very much on the choice of A.[6,7] With the exception of a very few limited studies involving the low-lying atomic states, many of the recent atomic structure calculations based on the DH approximation[13–22] and a few on the modified DH approximation[23,24] failed to meet the spatial criterion of the DH approximation, especially those with $A = 0$ when the atomic orbits overlap substantially with the outside plasma electrons.

The effect from the external plasma to an otherwise isolated atom offers the opportunity to study the dynamics of the atomic process which is sensitive to the change in atomic wavefunctions due to the changing outside environment. It is the main purpose of this paper to explore such an effect on the transition rate, or the oscillator strength, of the spectroscopically isolated α-lines due to the transition of the inner-most electron of the H-like and He-like ions based on the DH approximation. Similar to what we did on the redshifts of the emission lines of the inner-most atomic electrons, we start our study on the change of the oscillator strengths as D varies and proceed to find out if there is also a general feature that is similar to those we came across for the redshifts. It is well known that for the plasma-free H-like ions, the non-relativistic oscillator strength for each specific bound-bound transition is identical for all H-like ions, i.e., independent of the nuclear charge Z.[13,25] On the other hand, it is expected that the relativistic oscillator strength will vary slightly on the changing Z.[26]

For He-like ions, the non-relativistic oscillator strength for the plasma-free $1s^2\,{}^1S \rightarrow 1s2p\,{}^1P$ transition increases as Z increases. Qualitatively, this increase is due to the increase in the overlapping between the atomic orbitals of the He-like ion in its $1s^2\,{}^1S$ ground state and the excited $1s2p\,{}^1P$ state due to the stronger nuclear attraction as Z increases, i.e., when the relative inward change of the orbital of the upper $2p$ state is slightly greater than that of the lower $1s$ state. Eventually, the oscillator strength approaches approximately an asymptotic value as the relative difference in the size of the ions becomes negligible at large Z. To study the plasma effect on the oscillator strength, one could also start by examining the overlap between the orbitals of the initial and the final states of the transition. As we already understood, the redshift of the α-line for the H-like ions results from the larger upward energy correction $\delta_{1s}(D) = \epsilon_{1s}(D) - \epsilon_{1s}(D=\infty)$ for the $1s$ orbit than that of the $\delta_{2p}(D) = \epsilon_{2p}(D) - \epsilon_{2p}(D=\infty)$ for the $2p$ orbit. On the other hand, the ratio of the energy correction $\zeta_{1s}(D) = \frac{\delta_{1s}(D)}{\epsilon_{1s}(D=\infty)}$ for the $1s$ orbit is actually smaller than the ratio of the energy correction $\zeta_{2p}(D) = \frac{\delta_{2p}(D)}{\epsilon_{2p}(D=\infty)}$ for the $2p$ orbit. As a result, the $2p$ orbit extends further from the nucleus than that of the $1s$ orbit as D decreases and the overlap between the $1s$ and $2p$ orbits decreases as the radial parts of both do not change sign. One should therefore expect a decrease in the oscillator strength as the Debye length decreases or when the plasma effect becomes more pronounced.

In Sec. II, we first focus our discussion on the H-like ions and illustrate analytically that the change in non-relativistic oscillator strength depends explicitly upon the variation of the reduced Debye length. We will also outline the theoretical procedures for the two-electron systems leading to our numerical calculation. Section III presents the results of our study and how the oscillator strengths vary as the functions of reduced Debye length $\lambda_D = Z_{eff}D$, or, the ratio of the temperature T and the electron density N_e of the outside plasma. Finally, in Sec. IV, the implication of the present work is summarized.

II. THEORETICAL PROCEDURE BASED ON THE DEBYE-HÜCKEL APPROXIMATION

For H-like ions, with the reduced Debye length $\lambda_D = ZD$, we define the radius of the Debye sphere A in terms of a size parameter η in unit of a_0, or, $A = (\eta/Z)$. By further expressing r in terms of ρ, or, $\rho = Zr$, the DH potential given by Eq. (2) takes the form of

$$V_d(r;D) \Rightarrow Z^2 V^H(\rho;\lambda_D), \tag{3}$$

where

$$V^H(\rho;\lambda_D) = \begin{cases} V_i^H(\rho;\lambda_D) = -\left(\dfrac{1}{\rho} - \dfrac{1}{\lambda_D+\eta}\right), & \rho \le \eta \\ V_o^H(\rho;\lambda_D) = -\left(\dfrac{\lambda_D\, e^{\frac{\eta}{\lambda_D}}}{(\lambda_D+\eta)}\right)\dfrac{e^{-\frac{\rho}{\lambda_D}}}{\rho}, & \rho \ge \eta \end{cases} \tag{4}$$

with η and ρ in terms of the Bohr radius a_o. The one-electron orbital $\chi_{n\ell}(\rho;\lambda_D)$ for the H-like ions with nuclear charge Z is then generated by

$$\left[-\frac{1}{2}\frac{d^2}{d\rho^2} + \frac{\ell(\ell+1)}{2\rho^2} + V^H(\rho;\lambda_D)\right]\chi_{n\ell}(\rho;\lambda_D) = \epsilon_{n\ell}(\lambda_D)\chi_{n\ell}(\rho;\lambda_D) \tag{5}$$

with its energy given by $E_{n\ell}(\lambda_D) = Z^2\epsilon_{n\ell}(\lambda_D)$.

The non-relativistic oscillator strength $f_{n\ell,n'\ell'}$ for transition between states $n\ell$ and $n'\ell'$ is proportional to the energy separation $\Delta E_{n\ell,n'\ell'} = Z^2\,\delta\epsilon_{n\ell,n'\ell'}$ and the square of the dipole matrix $\langle n\ell|r|n'\ell'\rangle$ between states $n\ell$ and $n'\ell'$, where

$$\delta\epsilon_{n\ell,n'\ell'}(\lambda_D) = \epsilon_{n'\ell'}(\lambda_D) - \epsilon_{n\ell}(\lambda_D), \tag{6}$$

$$\langle n\ell|r|n'\ell'\rangle = \frac{1}{Z} d_{n\ell,n'\ell'}(\lambda_D), \tag{7}$$

and

$$d_{n\ell,n'\ell'}(\lambda_D) = \langle \chi_{n\ell}(\rho,\lambda_D)|\rho|\chi_{n'\ell'}(\rho,\lambda_D)\rangle. \tag{8}$$

As a result, the oscillator strength for all H-like ions subject to the plasma environment can be expressed as a function of λ_D, i.e.,

$$f_{n\ell,n'\ell'} \sim \delta\epsilon_{n\ell,n'\ell'}(\lambda_D)\,(d_{n\ell,n'\ell'}(\lambda_D))^2. \tag{9}$$

From Eq. (6), one is able to show immediately that the ratio, $R = \Delta\omega_\alpha/\omega_o$ between the redshift $\Delta\omega_\alpha$ and the plasma-free energy $\omega_o = \delta\epsilon_{1s,2p}(\infty)$ of the Lyman-α lines of the H-like ions, depends only on the reduced Debye length λ_D or

$$R = 1 - \frac{\delta\epsilon_{1s,2p}(\lambda_D)}{\delta\epsilon_{1s,2p}(\infty)} \tag{10}$$

as discussed earlier. In other words, based on Eqs. (9) and (10), both the oscillator strength and the ratio R vary only as functions of the reduced Debye length λ_D. We note that these two expressions could have been derived from the earlier work,[13] but, they were neither derived nor discussed explicitly. A similar expression to Eq. (9) was presented recently for the modified DH model in Ref. 24.

For He-like ions, the individual one-electron atomic orbitals are generated from the one-electron Hamiltonian $h_o(r,D)$, i.e.,

$$h_o(r;D) = \frac{p^2}{2m} + V_d(r;D), \tag{11}$$

where p is the momentum of the electron and $V_d(r;D)$ is given by Eq. (2) in DH approximation. The non-relativistic two-electron Hamiltonian for an atom in the plasma environment in the present calculation is expressed in terms of $h_o(r;D)$ as

$$H(r_1,r_2;D) = \sum_{i=1,2} h_o(r_i;D) + \frac{e^2}{r_{12}}, \tag{12}$$

where $r_{12} = |\vec{r_1} - \vec{r_2}|$ represents the separation between electrons 1 and 2.[5] In the present calculation, the radius of the Debye sphere A is expressed in terms of the average radius $\langle r\rangle_{1s}$ of one of the $1s$ electrons in the ground state of He-like ions in the absence of the plasma and a size parameter ζ, i.e.,

$$A = \zeta\langle r\rangle_{1s}, \tag{13}$$

where $\langle r\rangle_{1s} = \langle 1s^2\,{}^1S|r_1|1s^2\,{}^1S\rangle = \langle 1s^2\,{}^1S|r_2|1s^2\,{}^1S\rangle$. The energy $\omega_\alpha(D)$ and the redshift $\Delta\omega_\alpha(D)$ of the He$_\alpha$ line under the external plasma environment in terms of the Debye length D are given, respectively, by Eqs. (7) and (8) of Ref. 7 after diagonalizing the Hamiltonian matrix with a basis set of multi-configuration two-electron orbitals consisting of one-electron orbitals generated from $h_o(r,D)$, or, same as those from Eq. (5), and by following the B-spline based multiconfiguration interaction (BSCI) theoretical and numerical procedures detailed elsewhere.[5,27] The corresponding oscillator strengths f in length and velocity approximation are given explicitly in the BSCI approach by Eqs. (40) and (41) of Ref. 27.

Qualitatively, the less attractive nature of the screened potential $V_d(r;D)$ for an atomic electron in the field of the nuclear charge Z could be understood easily since close to the atomic nucleus, there is a non-negligible presence of the fast free-moving plasma electrons with their relatively high mobility. However, to justify the same Debye screening between two atomic electrons, such as the one applied to most of the recent atomic calculations based on DH approximation,[13–22] one would have to assume a substantial presence of the positive ions between atomic electrons, in spite of the relatively low mobility for the much heavier ions. In addition, since the screened potentials V_d are derived from the Gauss' law by assuming a stationary nuclear charge Z located at $r = 0$, a similar approach could hardly be justified for the fast-moving atomic electrons. Moreover, by including the same Debye screening for the electron-electron interaction, the repulsive force between atomic electrons would be greatly reduced as the plasma effect increases with smaller D. More discussion on the application of Debye screening between atomic electrons will be presented in Sec. III.

We have also carried out in the present study the full relativistic calculation to compare with our non-relativistic results. The relativistic two-electron Hamiltonian for He-like ions in the plasma environment in our calculation is given by

$$H^{DC} = \sum_{i=1,2}\left[c\vec{\alpha}\cdot\vec{p_i} + (\beta-1)mc^2 + V_d(r_i;D)\right] + \frac{e^2}{r_{12}}, \tag{14}$$

where $\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}$ with $k = (1,2,3)$, σ_k is the Pauli 2×2 matrix, and $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ with I the 2×2 unit matrix. Similar to what we outlined in detail in Ref. 7, our calculations were carried out using a revised multi-configuration Dirac-Fock (MCDF) approach which takes the electron correlations into account. The quasi-complete basis scheme[28,29] is adopted to optimize the atomic orbitals (AOs) using the GRASP_JT version based on the earlier GRASP2K codes.[30] Again, $V_d(r;D)$ given by Eq. (2) is employed in Eq. (14) under the DH approximation, instead of the one-electron potential $-Ze^2/r$. All other computational procedures leading to ω_o and $\Delta\omega_\alpha(D)$ are the same as the non-relativistic calculations outlined earlier. The relativistic oscillator strengths are calculated with the procedure detailed elsewhere.[28–30]

III. RESULTS AND DISCUSSION

Table I tabulates the non-relativistic energy separations $\delta\epsilon_{1s,2p}(\lambda_D) = \epsilon_{2p}(\lambda_D) - \epsilon_{1s}(\lambda_D)$ in eV and the corresponding

TABLE I. The energy separations $\delta\epsilon_{1s,2p} = \epsilon_{2p}(\lambda_D) - \epsilon_{1s}(\lambda_D)$ and the non-relativistic oscillator strengths $f(\lambda_D)$ with $\eta = 0, 1, 1.5$, and 2 in units of a_o for the $1s \to 2p$ transition of H-like ions subject to the outside plasma environment.

$\eta = 0$			$\eta = 1$		
λ_D (a_o)	$\delta\epsilon_{1s,2p}$(eV)	$f(\lambda_D)$	λ_D (a_o)	$\delta\epsilon_{1s,2p}$(eV)	$f(\lambda_D)$
10^{12}	10.204	0.4162	10^{12}	10.204	0.4162
80.0	10.197	0.4152	80.0	10.198	0.4152
60.0	10.192	0.4144	60.0	10.194	0.4145
40.0	10.176	0.4124	40.0	10.181	0.4124
30.0	10.155	0.4095	30.0	10.164	0.4097
20.0	10.097	0.4018	20.0	10.117	0.4021
15.0	10.020	0.3915	15.0	10.054	0.3920
10.0	9.810	0.3630	10.0	9.886	0.3644
8.0	9.609	0.3344	8.0	9.725	0.3369
7.0	9.442	0.3097	7.0	9.593	0.3133
6.0	9.191	0.2697	6.0	9.394	0.2756
5.0	8.781	0.1933	5.0	9.070	0.2054
$\eta = 1.5$			$\eta = 2$		
λ_D (a_o)	$\delta\epsilon_{1s,2p}$(eV)	$f(\lambda_D)$	λ_D (a_o)	$\delta\epsilon_{1s,2p}$(eV)	$f(\lambda_D)$
10^{12}	10.204	0.4162	10^{12}	10.204	0.4162
80.0	10.199	0.4153	80.0	10.200	0.4153
60.0	10.196	0.4145	60.0	10.197	0.4147
40.0	10.185	0.4126	40.0	10.189	0.4129
30.0	10.171	0.4099	30.0	10.178	0.4104
20.0	10.133	0.4028	20.0	10.148	0.4038
15.0	10.082	0.3932	15.0	10.109	0.3951
10.0	9.947	0.3672	10.0	10.005	0.3715
8.0	9.819	0.3415	8.0	9.907	0.3484
7.0	9.714	0.3195	7.0	9.827	0.3288
6.0	9.556	0.2847	6.0	9.707	0.2979
5.0	9.300	0.2212	5.0	9.513	0.2426

oscillator strengths $f(\lambda_D)$ with the size parameters $\eta = 0, 1, 1.5$, and $2\,a_o$ for the $1s \to 2p$ transition of the H-like ions subject to outside plasma. As expected, Fig. 1 presents the decreasing oscillator strength $f(\lambda_D)$ as the reduced Debye length $\lambda_D = ZD$ decreases with increasing effect from the outside plasma for three size parameters $\eta = 0, 1.5$, and $2\,a_o$. Qualitatively, this is due to the less overlap between the $1s$ and $2p$ orbitals, when the outer $2p$ orbital moves further away from the nucleus than that of the inner $1s$ orbital as the plasma effect increases with smaller λ_D. Our results with $A = 0$ are consistent with the non-relativistic data shown in Fig. 3 of Ref. 13. With the plasma-free energy separation between the $2p$ and $1s$ states of H-like ions given by $\omega_o = 10.204Z^2$ eV, one leads immediately to the ratio R given by Eq. (10), or effectively the required experimental energy resolution if the redshifts of the Lyman-α are to be measured as discussed in Ref. 7. Figure 2 presents the variation of the percentage change R of the redshift and the percentage change of the oscillator strength f_r of all H-like ions as functions of the reduced Debye length λ_D at a number of size parameters η from zero to $2\,a_o$, where

$$f_r(\lambda_D) = 1 - \frac{f(\lambda_D)}{f(\lambda_D = \infty)}. \qquad (15)$$

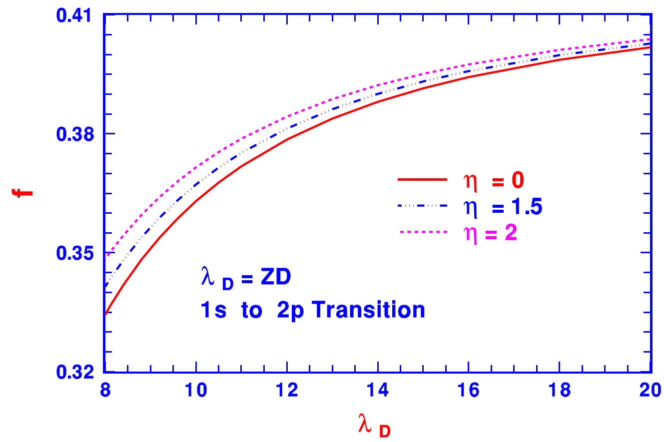

FIG. 1. The non-relativistic oscillator strength f for the $1s \to 2p$ transition for the H-like ions subject to outside plasma with the size parameters of $\eta = 0, 1.5$, and 2 in units of a_o as functions of the reduced Debye length $\lambda_D = ZD$, also in units of a_0.

From Eq. (1), for a given temperature kT, the ratio $\frac{N_e}{Z^2}$ could be expressed in terms of the inverse of λ_D^2. In turn, one could then express the percentage changes R and f_r at a given temperature kT in terms of $\frac{N_e}{Z^2}$ such as the two plots shown in Fig. 3 at $kT = 100$ eV for all H-like ions. Experimentally, with a known resolution, either in energy or in transition rate in terms of the percentage change R or f_r shown in Fig. 3,

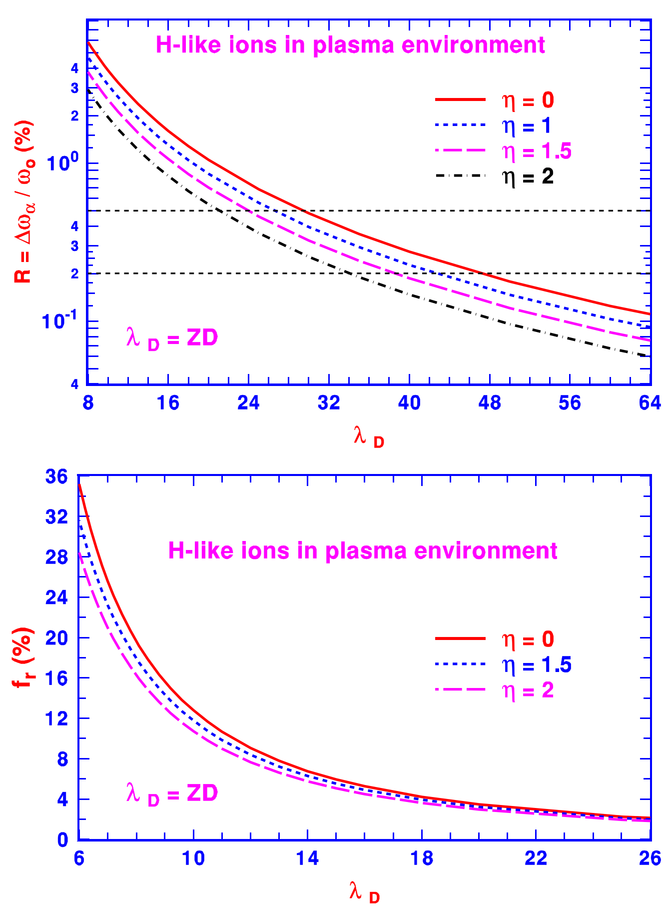

FIG. 2. The percentage change of the redshifts R given by Eq. (10) and the percentage change of the oscillator strengths f_r given by Eq. (15) for the $1s \to 2p$ transition of the H-like ions subject to outside plasma as functions of the reduced Debye length $\lambda_D = ZD$.

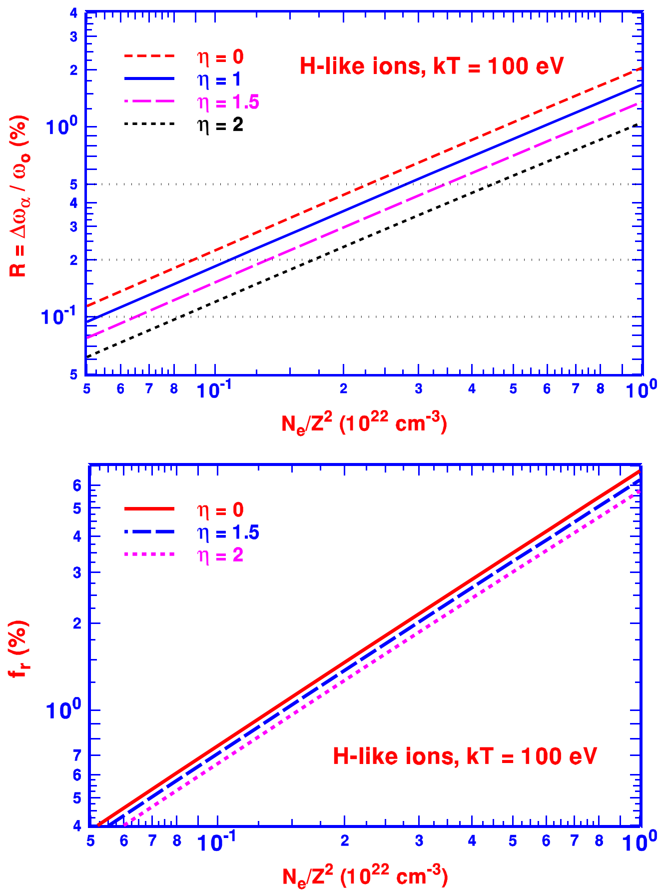

FIG. 3. The percentage change of the redshifts R given by Eq. (10) and the percentage change of the oscillator strengths f_r given by Eq. (15) for the $1s \rightarrow 2p$ transition of the H-like ions subject to outside plasma at $kT = 100\,\text{eV}$ as functions of the plasma density N_e/Z^2 (in units of $10^{22}\,\text{cm}^{-3}$).

one could then estimate the required density for a viable measurement for the redshift or transition rate for any H-like ions with nuclear charge Z. It is interesting to note that the variation of the percentage change f_r as a function of λ_D or $\frac{N_e}{Z^2}$ is a few times greater than the percentage change R for the redshift. Based on the spatial criterion of the DH approximation, together with the agreement between our earlier estimated redshift for the H-like Al ions at 300 eV to the observed data with the scale parameter between $\eta = a_o$ and $2\,a_o$, we might narrow down our choice of the radius of Debye sphere A to be between a_o and $2\,a_o$.

Table II presents the calculated non-relativistic oscillator strengths $f(\lambda_D)$ for the $1s^2\,{}^1S \rightarrow 1s2p\,{}^1P$ transition based on the BSCI approach outlined in Sec. II for four He-like ions, Ar^{16+}, Al^{11+}, Mg^{10+}, and Ne^{8+}, subject to outside plasma and the radius of the Debye sphere $A = \zeta\langle r\rangle_{1s}$ at four values of size parameters $\zeta = 0, 1, 1.5$, and 2. The length and velocity results from our oscillator strength calculation are generally in agreement to 0.1% or better and also in close agreement with the established plasma-free results.[31] Based on the results listed in Table II, similar to Fig. 2 for the H-like ions, it is interesting to show in Fig. 4 that the percentage change f_r as a function of the reduced Debye length $\lambda_D = (Z-1)D$ for the He_α line exhibits approximately a similar general qualitative feature and also substantially greater than the variation of the percentage change R for the redshift. Figure 5 compares the percentage change of the oscillator strength f_r of the He_α emission line of the He-like Ne ions from the present calculation with the earlier calculated results by Saha *et al.*[18] using the less vigorous Slater-type orbitals for the He-like C, B, and Be ions without the Debye screening for the electron-electron interaction. Although the He-like ions studied by Saha *et al.* are those with relatively small Z and not exactly between 5 and 18, the resulting variation of f_r follows qualitatively the general feature of our calculation. Also shown is the calculation for neutral He by Kar and Ho[16] which follows qualitatively the He-like Ne ion result of the present calculation as the difference between the calculations with and without the Debye screening between atomic electrons is still relatively small at Debye lengths which are a few times larger than the average size of the He atom in its ground state. Additionally, the theoretical results by Li *et al.*[20] for the He-like C ion (model1 and model2) are compared with the results from the present calculation and that from Saha *et al.*[18] and Kar and Ho.[16] The one labeled as model1 is calculated by applying the same Debye screening both for the electron from the nucleus and for the interaction between the atomic electrons, whereas the one labeled as model2 is calculated without the Debye screening for the electron-electron interaction. Although according to Li *et al.*, the difference in the resulting oscillator strengths between different calculations is generally small, it is actually fairly substantial when one examines the change from the plasma-free value in terms of f_r as those shown in Fig. 5. This, perhaps, is in part due to the far more involved computational effort when the Debye screening between atomic electrons is included in the calculation.

Following our discussion in Sec. II, we will now briefly present one example which leads to the theoretical estimate that is against the physical intuition due to the substantial reduction of the repulsive force between atomic electrons by including the Debye screening to the electron-electron interaction. Without the Debye screening between atomic electrons, the top plot of Fig. 6 shows that the electron affinity of H^- ion, i.e., the energy difference between the ground state $1s$ of the H atom and the only loosely bound state $1s^2\,{}^1S$ of the H^- ion, approaches zero at a Debye length D around $37.7\,a_o$, based on our calculation with $A = 2.71\,a_o$, i.e., at an average distance of the $1s$ electron from the nucleus for a plasma-free H^- ion.[15] As one would expect, the bound state of H^- will no longer exist subject to outside plasma with D substantially greater than A. In contrast, the bottom plot of Fig. 6 shows that, by including the Debye screening between atomic electrons, Kar and Ho have shown that the theoretically estimated energy of the only bound state of the H^- ion would remain negative at $D = 0.86\,a_o$ with positive non-zero electron affinity (see, Table II of Ref. 14). We should also note that the calculated electron affinity of the H^- ion from both calculations equals 0.0555 Ry at $D = \infty$.

Unlike the H-like ions, the oscillator strengths for the $1s^2\,{}^1S \rightarrow 1s2p\,{}^1P$ transition of the He-like ions are no longer identical at given reduced Debye length $\lambda_D = (Z-1)D$ for different He-like ions. This, of course, is expected as the plasma-free oscillator strengths for He-like ions with different Z are different at $\lambda_D = \infty$. In an attempt of searching for

TABLE II. The non-relativistic oscillator strengths $f(\lambda_D)$ for the $1s^2\,{}^1S \rightarrow 1s2p\,{}^1P$ transition of the He-like Ar^{16+}, Al^{11+}, Mg^{10+}, and Ne^{8+} ions subject to the outside plasma environment with the radius of the Debye sphere $A = \zeta\langle r\rangle_{1s}$.

Ar^{16+}	$f(\lambda_D)$				Al^{11+}	$f(\lambda_D)$			
$\lambda_D(a_o)$	$\zeta=0$	$\zeta=1$	$\zeta=1.5$	$\zeta=2$	$\lambda_D(a_o)$	$\zeta=0$	$\zeta=1$	$\zeta=1.5$	$\zeta=2$
93.50	0.7703	0.7704	0.7706	0.7708	90.00	0.7464	0.7464	0.7466	0.7469
46.75	0.7660	0.7663	0.7770	0.7678	45.00	0.7416	0.7420	0.7426	0.7436
25.50	0.7530	0.7542	0.7563	0.7591	21.00	0.7205	0.7222	0.7253	0.7293
21.25	0.7453	0.7469	0.7500	0.7540	18.00	0.7111	0.7135	0.7177	0.7231
17.00	0.7313	0.7340	0.7387	0.7448	16.80	0.7060	0.7087	0.7135	0.7198
15.30	0.7224	0.7257	0.7315	0.7391	14.40	0.6918	0.6956	0.7021	0.7105
13.60	0.7100	0.7143	0.7217	0.7312	13.20	0.6817	0.6863	0.6940	0.7040
12.75	0.7020	0.7069	0.7153	0.7261	12.00	0.6686	0.6742	0.6836	0.6957
11.05	0.6802	0.6870	0.6981	0.7124	10.80	0.6509	0.6580	0.6696	0.6845
9.35	0.6456	0.6555	0.6712	0.6912	9.60	0.6262	0.6356	0.6503	0.6692
8.50	0.6199	0.6322	0.6515	0.6757	8.40	0.5899	0.6028	0.6223	0.6471
Mg^{10+}	$f(\lambda_D)$				Ne^{8+}	$f(\lambda_D)$			
$\lambda_D(a_o)$	$\zeta=0$	$\zeta=1$	$\zeta=1.5$	$\zeta=2$	$\lambda_D(a_o)$	$\zeta=0$	$\zeta=1$	$\zeta=1.5$	$\zeta=2$
93.50	0.7393	0.7393	0.7395	0.7397	90.00	0.7203	0.7204	0.7206	0.7208
52.25	0.7360	0.7363	0.7368	0.7374	54.00	0.7174	0.7177	0.7181	0.7188
27.50	0.7242	0.7252	0.7270	0.7294	27.00	0.7047	0.7057	0.7075	0.7100
19.25	0.7082	0.7103	0.7139	0.7187	18.00	0.6846	0.6870	0.6911	0.6965
15.95	0.6943	0.6973	0.7026	0.7095	15.75	0.6739	0.6770	0.6824	0.6894
13.75	0.6792	0.6834	0.6905	0.6997	13.50	0.6576	0.6619	0.6692	0.6787
12.10	0.6623	0.6678	0.6770	0.6889	11.70	0.6375	0.6434	0.6531	0.6657
11.00	0.6467	0.6535	0.6646	0.6790	10.80	0.6235	0.6306	0.6420	0.6568
10.45	0.6370	0.6446	0.6570	0.6729	9.90	0.6055	0.6142	0.6279	0.6455
9.35	0.6121	0.6220	0.6376	0.6574	9.00	0.5818	0.5926	0.6093	0.6307
8.80	0.5959	0.6073	0.6251	0.6475	8.55	0.5669	0.5791	0.5978	0.6215
8.25	0.5762	0.5896	0.6099	0.6356	8.10	0.5492	0.5632	0.5842	0.6107

a general feature of the oscillator strengths for all He-like ions subject to outside plasma similar to the one given in Ref. 7, we renormalize the plasma-free oscillator strength $f(Z, \lambda_D = \infty)$ for all He-like ions to a constant asymptotic value of 0.825 with a function $N(Z; a, b)$ such that

$$f(Z, \lambda_D = \infty) = 0.825N(Z; a, b), \quad (16)$$

where

$$N(Z; a, b) = \left(1 - \frac{a}{Z} - \frac{b}{Z^2}\right) \quad (17)$$

with the best fitted values of $a = 0.852$ and $b = 3.505$. We then defined a scaled oscillator strength by

$$f_s(Z, \lambda_D) = \frac{f(Z, \lambda_D)}{N(Z; a, b)}. \quad (18)$$

The resulting f_s for the He_α line of four He-like ions, Ar^{16+}, Al^{11+}, Mg^{10+}, and Ne^{8+}, are shown in Fig. 7 for $A = 0$ and $A = 2\langle r\rangle_{1s}$. As anticipated, qualitatively, although the scaled oscillator strength f_s for these four He-like ions are not exactly identical, they vary similarly to the H-like ions shown in Fig. 1 as λ_D decreases. This general feature in scaled oscillator strength f_s, together with the similar percentage change f_r shown in Fig. 4, offers the possibility to extrapolate from a single dataset with a specific Z to generate data for other ions as we pointed out earlier.

Figure 8 presents the results based on the relativistic calculation following the procedure outlined in Sec. II for the $1s^2\,{}^1S_{1/2} \rightarrow 1s2p\,{}^1P_{3/2}$ transition of a few He-like ions with Z meeting the criteria for the DH approximation. Again, the scaled oscillator strengths as a function of reduced Debye length λ_D follow a nearly universal variation similar to that shown for the non-relativistic results. Since Z is relatively small and the relativistic result is not expected to be that

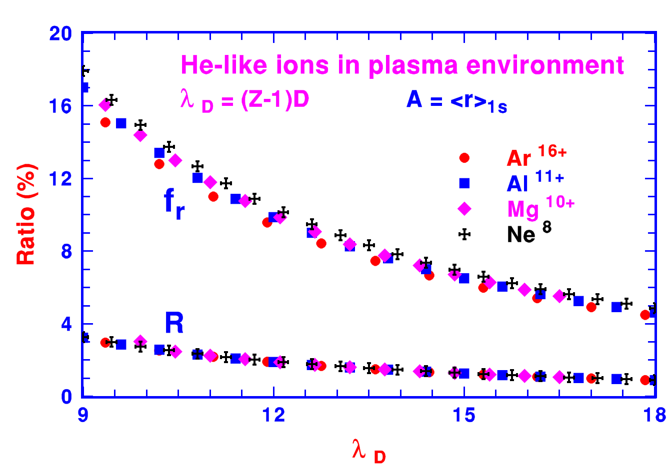

FIG. 4. The percentage change of the oscillator strengths f_r and the corresponding redshifts R for the $1s^2\,{}^1S \rightarrow 1s2p\,{}^1P$ transition for four He-like ions subject to outside plasma as functions of the reduced Debye length $\lambda_D = (Z-1)D$ with $A = \langle r\rangle_{1s}$.

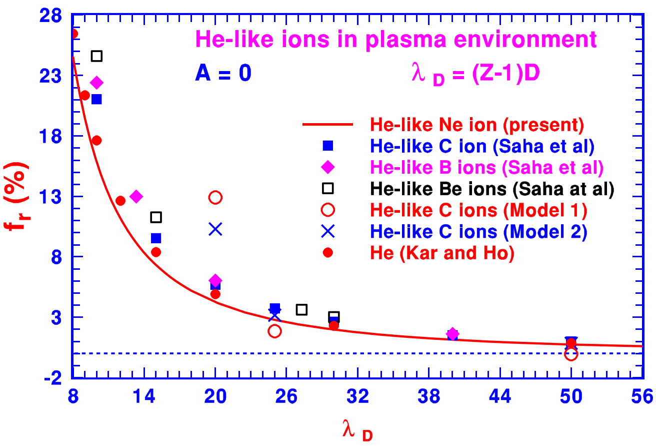

FIG. 5. Comparison between the present calculation and three earlier calculations by Saha *et al.*,[18] Li *et al.*,[20] and Kar and Ho[16] for the percentage change of the non-relativistic oscillator strengths f_r for the $1s^2\,{}^1S \rightarrow 1s2p\,{}^1P$ transition of the He-like ions subject to outside plasma as the function of the reduced Debye length $\lambda_D = (Z-1)D$ with $A = 0$.

much different from the non-relativistic result shown earlier in Fig. 7.

Figure 9 compares the results of the current non-relativistic calculation for f_r with $A = \langle r \rangle_{1s}$ and $2\langle r \rangle_{1s}$ at kT

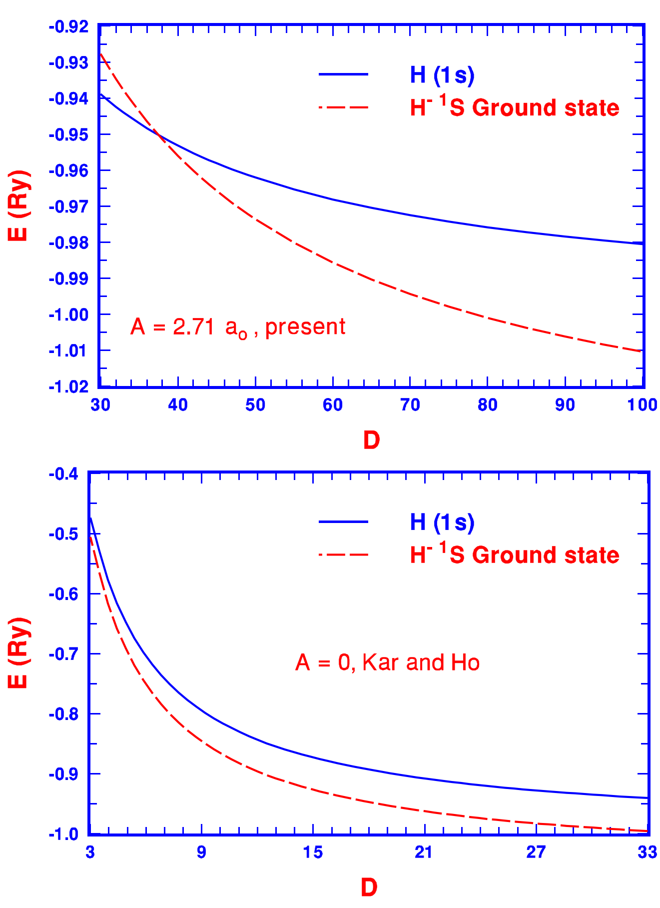

FIG. 6. The energies for the $1s$ state of the H atom and the $1s^2\,{}^1S$ state of the H^- ion as the functions of the Debye length D from the present calculation (top plot) without the Debye screening between atomic electrons and the calculation by Kar and Ho[16] (bottom plot) with the Debye screening between atomic electrons. The electron affinity of the H^- ion is given by the energy separation between the two energy curves.

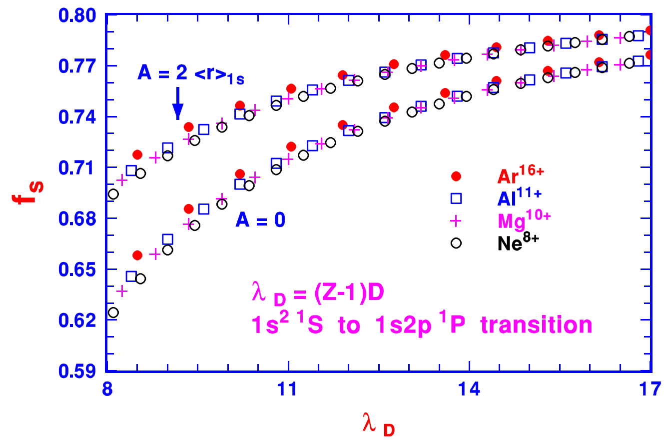

FIG. 7. The non-relativistic scaled oscillator strength f_s defined by Eq. (18) for the $1s^2\,{}^1S \rightarrow 1s2p\,{}^1P$ transition subject to outside plasma as functions of the reduced Debye length $\lambda_D = (Z-1)D$ with $A = 0$ and $A = 2\langle r \rangle_{1s}$ for four He-like ions.

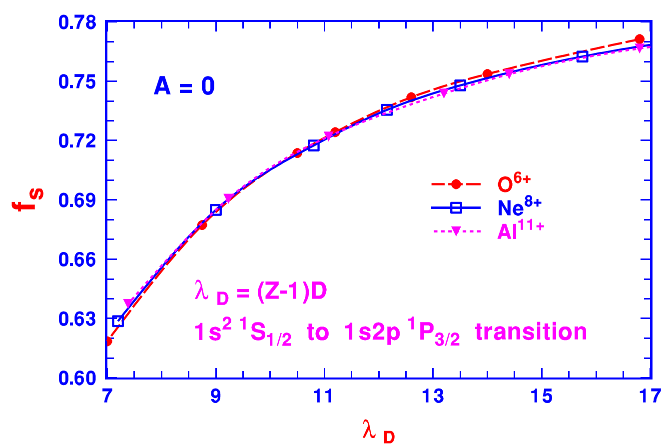

FIG. 8. The relativistic scaled oscillator strength f_s for the $1s^2\,{}^1S_{J=1/2} \rightarrow 1s2p\,{}^1P_{3/2}$ transition subject to outside plasma as functions of the reduced Debye length $\lambda_D = (Z-1)D$ with $A = 0$ for three He-like ions. The fitted coefficient a of Eq. (17) is 0.757, which is slightly different from 0.852 of the non-relativistic calculation.

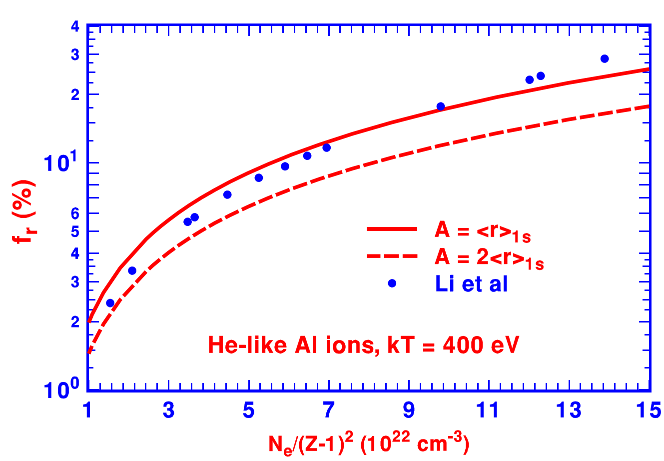

FIG. 9. Comparison between the results of the present non-relativistic calculation based on the DH approximation and the recent calculation by Li *et al.*[32] based on the ion sphere approximation for the percentage change f_r of the oscillator strength for the $1s^2\,{}^1S \rightarrow 1s2p\,{}^1P$ transition of the He-like Al ions subject to outside plasma at $kT = 400\,\text{eV}$ as functions of the plasma density $N_e/(Z-1)^2$ (in units of $10^{22}\,\text{cm}^{-3}$).

= 400 eV for He-like Al ions with the results of the calculation based on the ion sphere (IS) model by Li *et al.*[32] It is interesting to see that the rate of change in oscillator strength f_r from the IS calculation varies from values close to the ones from our DH calculation with $A = 2\langle r\rangle_{1s}$ to that with $A = \langle r\rangle_{1s}$ as the density N_e increases. This is consistent with the fact that the radius of the ion sphere R_o is inversely proportional to the density $N_e^{1/3}$, i.e., R_o varies approximately from $2a_o$ to a_o as N_e increases. In other words, it supports what we pointed out earlier that a judicious application of the DH approximation should keep the radius of Debye sphere A greater than 0 to meet the spatial criterion of the DH approximation.

IV. CONCLUSION

The best known quantitatively observed data on the redshift of the α line from atomic ions embedded in external plasma are the laser-generated H-like Al^{12+} at approximately $kT = 300$ eV and a density of (5–10) × 10^{23} cm^{-3} with a measured value of 3.7 ± 0.7 eV.[8] This measured result is confirmed first by a simulated QMIT redshift at 3.5 eV and a density 8×10^{23} cm^{-3}.[33] It is also in agreement with the recent calculation we presented based on the DH model with the size parameters between $\eta = a_o$ and $\eta = 2\ a_o$.[6] For the H-like Al^{12+} ion with a redshift of 3.7 ± 0.7 eV, the ratio R should have a value range approximately between 0.17% and 0.25% from Fig. 2 or a required experimental energy resolution $E/\Delta E$ of 600 or better at the plasma-free energy around 1724.5 eV for the Lyman-α line. With the availability of $E/\Delta E$ up to 5000 for the monochromator at energies from 500 to 1000 eV and a bit lower for energy up to 2000 eV with the SXR (soft x-ray) instrument at the Linac Coherent Light Source (LCLS) free electron laser (FEL),[9] there is a good possibility that an additional quantitative measurement could be carried out for a critical test of the theoretical results on the variation of the redshifts and the transition rates discussed in this paper. Such observed data would offer a more reliable determination of the size parameters η and ζ which, in turn, could more accurately characterize the radius of the Debye sphere. Should the general features reported in this paper be demonstrated by the experimental measurement, they could be applied easily to extrapolate from a well-characterized dataset for a single H-like or He-like ion to generate data for other ions and facilitate additional experimental measurements for other H-like or He-like ions. The quantitatively measured redshifts and the variation of the transition rates of the spectroscopically isolated α emission lines of the H-like and He-like ions, together with the general features presented in this paper, could lead to a reliable and effective alternative to complement the current diagnostic effort of the dense plasmas relied mostly on the change of the spectral profiles of the emission lines due to the complicated collisional process.[2,8,33,34]

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